Decisions with Multiple Agents: Game Theory

Alice Gao

Lecture 24

Based on work by K. Leyton-Brown, K. Larson, and P. van Beek

Learning Goals

By the end of the lecture, you should be able to

- Determine dominant-strategy equilibria of a 2-player normal form game.
- Determine pure-strategy Nash equilibria of a 2-player normal form game.
- Determine Pareto optimal outcomes of a 2-player normal form game.
- Calculate a mixed strategy Nash equilibrium of a 2-player normal form game.

Outline

Prisoner's dilemma

		Bob: cooperate	Bob: defect
Alice	cooperate	$(-1, -1)$	$(-3, 0)$
	defect	$(0, -3)$	$(-2, -2)$

CQ: Prisoner's dilemma - DSE

CQ: Which outcome, if any, is a **dominant strategy equilibrium**?

(A) (cooperate, cooperate)

(B) (cooperate, defect)

(C) (defect, cooperate)

(D) (defect, defect)

(E) There is no dominant strategy equilibrium.

		Bob	
		cooperate	defect
Alice	cooperate	$(-1, -1)$	$(-3, 0)$
	defect	$(0, -3)$	$(-2, -2)$

CQ: Prisoner's dilemma - NE

CQ: How many of the four outcomes are **pure-strategy Nash equilibria**?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

		Bob: cooperate	Bob: defect
Alice	cooperate	$(-1, -1)$	$(-3, 0)$
	defect	$(0, -3)$	$(-2, -2)$

CQ: Prisoner's dilemma - Pareto optimality

CQ: How many of the four outcomes are **Pareto optimal**?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

		Bob: cooperate	Bob: defect
Alice	cooperate	$(-1, -1)$	$(-3, 0)$
	defect	$(0, -3)$	$(-2, -2)$

Matching quarters

		Bob	
		heads	tails
Alice	heads	$(1, 0)$	$(0, 1)$
	tails	$(0, 1)$	$(1, 0)$

Alice wants the two coins to match whereas Bob wants the two coins to mismatch.

CQ: Matching quarters - NE

CQ: How many of the four outcomes are **pure-strategy Nash equilibria**?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

		Bob: heads	Bob: tails
Alice	heads	$(1, 0)$	$(0, 1)$
	tails	$(0, 1)$	$(1, 0)$

Dancing or concert?

		Bob	
		dancing	*concert*
Alice	*dancing*	$(2, 1)$	$(0, 0)$
	concert	$(0, 0)$	$(1, 2)$

Alice and Bob want to sign up for an activity together. They both prefer to sign up for the same activity. However, Alice prefers dancing over going to a concert whereas Bob prefers going to a concert over dancing.

CQ: Why is a player willing to mix between two actions?

Consider a 2-player normal form game and fix Bob's strategy. Alice is willing to play heads 60% of the time and tails 40% of the time. Which of the following statements is true?

(A) Alice's expected utility of playing heads is greater than her expected utility of playing tails.

(B) Alice's expected utility of playing heads is less than her expected utility of playing tails.

(C) Alice's expected utility of playing heads is same as her expected utility of playing tails.

CQ: Dancing or concert - mixed-strategy NE

CQ: At the mixed strategy Nash equilibrium, with what probability does **Alice go dancing**?

(A) 0 (B) 1/3 (C) 2/3 (D) 1

		Bob: *dancing*	Bob: *concert*
Alice	*dancing*	$(2, 1)$	$(0, 0)$
	concert	$(0, 0)$	$(1, 2)$

CQ: Dancing or concert - mixed-strategy NE

CQ: At the mixed strategy Nash equilibrium, with what probability does **Bob go dancing**?

(A) 0 (B) 1/3 (C) 2/3 (D) 1

		Bob	
		dancing	*concert*
Alice	*dancing*	$(2, 1)$	$(0, 0)$
	concert	$(0, 0)$	$(1, 2)$

Revisiting the Learning Goals

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