Decisions with Multiple Agents: Game Theory

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Lecture 23

Based on work by K. Leyton-Brown, K. Larson, and P. van Beek

Outline

Learning Goals

By the end of the lecture, you should be able to

- Determine dominant-strategy equilibria of a 2-player normal form game.
- Determine pure-strategy Nash equilibria of a 2-player normal form game.
- Determine whether one outcome Pareto dominates another outcome of a game. Determine Pareto optimal outcomes of a 2-player normal form game.
- Calculate a mixed strategy Nash equilibrium of a 2-player normal form game.

CQ: Prior knowledge w/ GT and MD

CQ: Have you learned Game Theory and/or Mechanism Design in another course?

(A) Yes

(B) No

Decision making with multiple agents

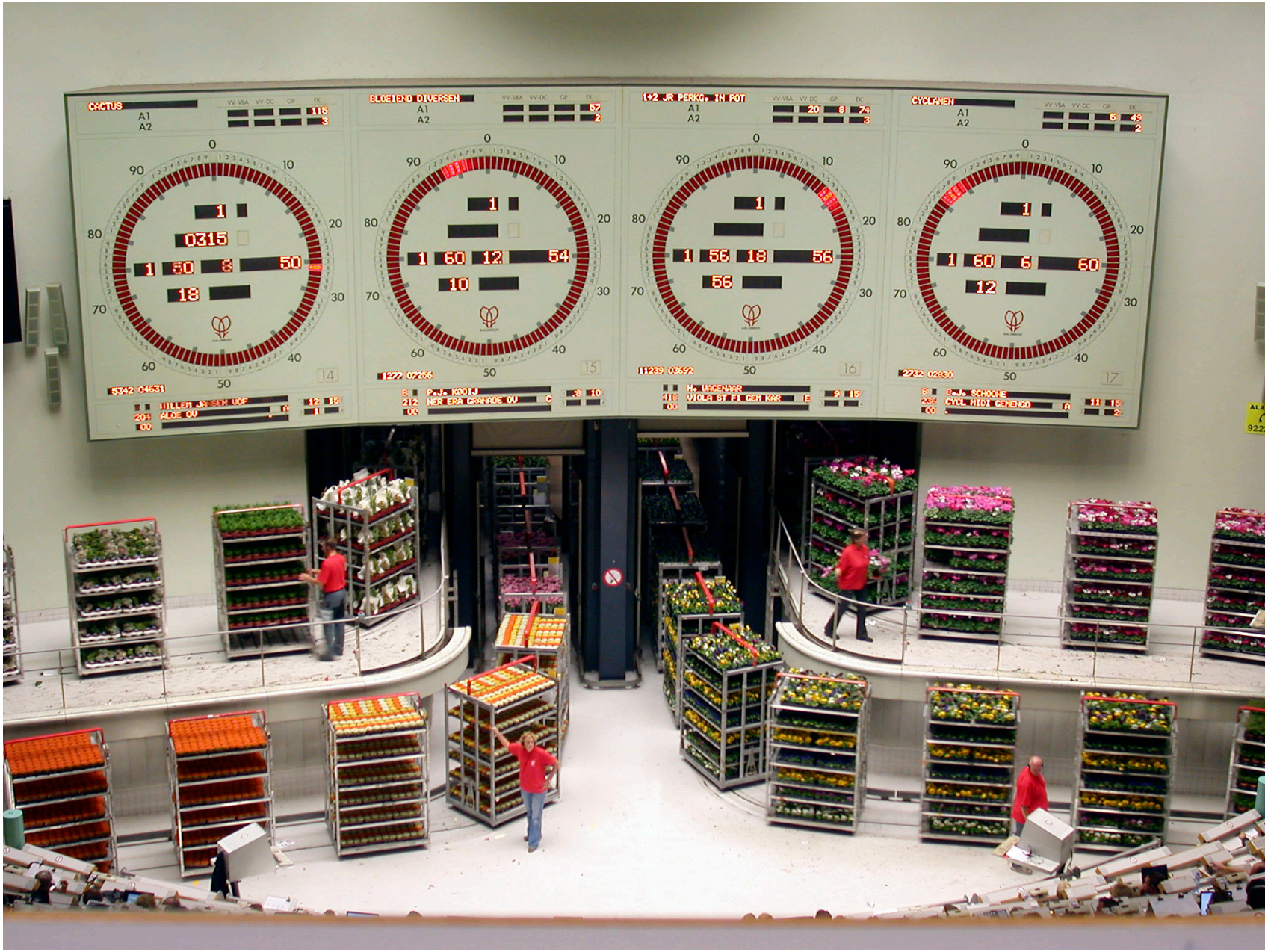
- Decision making in a multi-agent environment.
- When making a decision, each agent needs to take into account of the other agents' behaviour.

What is a game?

Game Theory

A game is a mathematical model of a strategic scenario.

Dutch flower auction



Matching problems

Examples: medical residency matching, school choice, and organ transplant, etc.

Crowdsourcing

Examples: 99 Designs, Topcoder, Duolingo, uwflow.com

Game Theory vs Mechanism Design

- Game theory: Given a game, how would agents play it?
- Mechanism design: How should we design the rules of the game so that the agents will behave the way we want them to?

The multi-agent framework

- Each agent decides what to do based on
  - their information about the world
  - their information about other agents
  - their utility function
- The outcome depends on the actions of all agents.

Relationship between utility functions

A game can be

- cooperative where agents have a common goal.
- competitive where agents have conflicting goals.
- or somewhere in between.

CQ: Home or dancing?

		Bob: *home*	Bob: *dancing*
Alice	*home*	$(0, 0)$	$(0, 1)$
	dancing	$(1, 0)$	$(2, 2)$

CQ: Home or dancing?
What do you think the players will do?

		Bob: *home*	Bob: *dancing*
Alice	*home*	$(0, 0)$	$(0, 1)$
	dancing	$(1, 0)$	$(2, 2)$

(A) (home, home)

(B) (home, dancing)

(C) (dancing, home)

(D) (dancing, dancing)

CQ: Home or dancing - DSE

CQ: Which of the following statements is correct?

		Bob: *home*	Bob: *dancing*
Alice	*home*	$(0, 0)$	$(0, 1)$
	dancing	$(1, 0)$	$(2, 2)$

(A) (home, home) is the only dominant strategy equilibrium.

(B) (dancing, dancing) is the only dominant strategy equilibrium.

(C) (dancing, home) or (home, dancing) is the only dominant strategy equilibrium.

(D) This game has more than one dominant strategy equilibrium.

(E) This game has no dominant strategy equilibrium.

CQ: Dancing or running - DSE

		Bob	
		dancing	*running*
Alice	*dancing*	$(2, 2)$	$(0, 0)$
	running	$(0, 0)$	$(1, 1)$

CQ: Dancing or running

What do you think the players will do?

		Bob: *dancing*	Bob: *running*
Alice	*dancing*	$(2, 2)$	$(0, 0)$
	running	$(0, 0)$	$(1, 1)$

(A) (dancing, dancing)

(B) (dancing, running)

(C) (running, dancing)

(D) (running, running)

CQ: Dancing or running - DSE

CQ: Which of the following statements is correct?

		Bob	
		dancing	*running*
Alice	*dancing*	$(2, 2)$	$(0, 0)$
	running	$(0, 0)$	$(1, 1)$

(A) (dancing, dancing) is the only dominant strategy equilibrium.

(B) (running, running) is the only dominant strategy equilibrium.

(C) (dancing, running) or (running, dancing) is the only dominant strategy equilibrium.

(D) This game has more than one dominant strategy equilibrium.

(E) This game has no dominant strategy equilibrium.

Nash equilibrium

- Won Nobel prize in Economics.
- One-page paper on Nash equilibrium and 26-page PhD thesis.
- Every finite game has at least one Nash equilibrium. (It may not be a pure strategy equilibrium though.)

CQ: Dancing or running - NE

CQ: Which of the following is correct about the game below? Consider only pure-strategy Nash equilibria.

		Bob	
		dancing	*running*
Alice	*dancing*	$(2, 2)$	$(0, 0)$
	running	$(0, 0)$	$(1, 1)$

(A) (dancing, dancing) is the only Nash equilibrium.

(B) (running, running) is the only Nash equilibrium.

(C) (dancing, dancing) and (running, running) are both Nash equilibria.

(D) This game has more than two Nash equilibria.

CQ: Dancing or running - Pareto optimality

CQ: How many of the four outcomes are **Pareto optimal**?

		Bob: *dancing*	Bob: *running*
Alice	*dancing*	$(2, 2)$	$(0, 0)$
	running	$(0, 0)$	$(1, 1)$

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

Revisiting the Learning Goals

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