Independence

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Lecture 13

Based on work by K. Leyton-Brown, K. Larson, and P. van Beek

Outline

Learning Goals

By the end of the lecture, you should be able to

- Given a description of a domain or a probabilistic model for the domain, determine whether two variables are independent.
- Given a description of a domain or a probabilistic model for the domain, determine whether two variables are conditionally independent given a third variable.

The Holmes Scenario

Mr. Holmes lives in a high crime area and therefore has installed a burglar alarm. He relies on his neighbors to phone him when they hear the alarm sound. Mr. Holmes has two neighbors, Dr. Watson and Mrs. Gibbon.

Unfortunately, his neighbors are not entirely reliable. Dr. Watson is known to be a tasteless practical joker and Mrs. Gibbon, while more reliable in general, has occasional drinking problems.

Mr. Holmes also knows from reading the instruction manual of his alarm system that the device is sensitive to earthquakes and can be triggered by one accidentally. He realizes that if an earthquake has occurred, it would surely be on the radio news.

(Unconditional) Independence

Definition ((unconditional) independence)

X and Y are (unconditionally) independent iff

$$\forall x, \forall y, P(X = x | Y = y) = P(X = x)$$

$$\forall x, \forall y, P(Y = y | X = x) = P(Y = y)$$

$$\forall x, \forall y, P(X = x \wedge Y = y) = P(X = x)\, P(Y = y)$$

Learning Y's value doesn't affect your belief about X.

Conditional Independence

Definition (conditional independence)

X and Y are conditionally independent given Z if

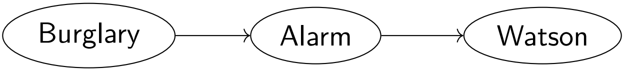
$$\forall x, \forall y, \forall z, P(X = x | Y = y \wedge Z = z) = P(X = x | Z = z).$$

$$\forall x, \forall y, \forall z, P(Y = y | X = x \wedge Z = z) = P(Y = y | Z = z).$$

$$\forall x, \forall y, \forall z, P(Y = y \wedge X = x | Z = z) = P(Y = y | Z = z) P(X = x | Z = z).$$

Learning Y's value doesn't affect your belief about X, knowing the value of Z.

Burglary, Alarm and Watson



$P(B) = 0.1$

$P(A|B) = 0.9$ $\quad$ $P(A|\neg B) = 0.1$

$P(W|B \wedge A) = 0.8$ $\quad$ $P(W|B \wedge \neg A) = 0.4$

$P(W|\neg B \wedge A) = 0.8$ $\quad$ $P(W|\neg B \wedge \neg A) = 0.4$

CQ Unconditional Independence

CQ: Are Burglary and Watson independent?

$$\text{Burglary} \longrightarrow \text{Alarm} \longrightarrow \text{Watson}$$

$P(B) = 0.1$ $\quad$ $P(A|B) = 0.9$ $\quad$ $P(A|\neg B) = 0.1$

$P(W|B \land A) = 0.8$ $\quad$ $P(W|B \land \neg A) = 0.4$

$P(W|\neg B \land A) = 0.8$ $\quad$ $P(W|\neg B \land \neg A) = 0.4$

(A) Yes

(B) No

(C) I don't know.

CQ: Conditional Independence

CQ: Are Burglary and Watson conditionally independent given Alarm?

$P(B) = 0.1$ $\quad$ $P(A|B) = 0.9$ $\quad$ $P(A|\neg B) = 0.1$

$P(W|B \wedge A) = 0.8$ $\quad$ $P(W|B \wedge \neg A) = 0.4$

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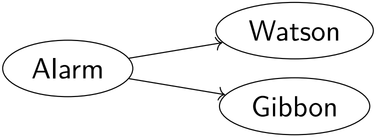
(A) Yes

(B) No

(C) I don't know.

Alarm, Watson and Gibbon



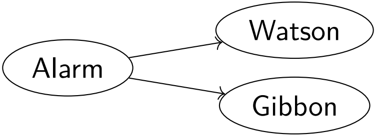
$P(A) = 0.1$ $\qquad$ $P(W|A) = 0.8$ $\qquad$ $P(W|\neg A) = 0.4$

$P(G|W \wedge A) = 0.4$ $\qquad$ $P(G|W \wedge \neg A) = 0.1$

$P(G|\neg W \wedge A) = 0.4$ $\qquad$ $P(G|\neg W \wedge \neg A) = 0.1$

CQ Unconditional Independence

CQ: Are Watson and Gibbon independent?



$P(A) = 0.1$ $\quad P(W|A) = 0.8$ $\quad P(W|\neg A) = 0.4$

$P(G|W \wedge A) = 0.4$ $\quad P(G|W \wedge \neg A) = 0.1$

$P(G|\neg W \wedge A) = 0.4$ $\quad P(G|\neg W \wedge \neg A) = 0.1$

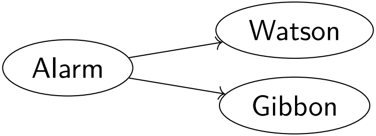
(A) Yes

(B) No

(C) I don't know.

CQ Conditional Independence

CQ: Are Watson and Gibbon conditionally independent given Alarm?

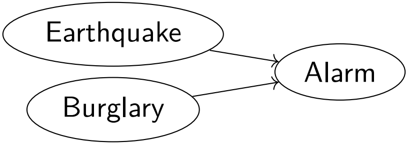


$P(A) = 0.1$ $\quad P(W|A) = 0.8$ $\quad P(W|\neg A) = 0.4$

$P(G|W \wedge A) = 0.4$ $\quad P(G|W \wedge \neg A) = 0.1$
$P(G|\neg W \wedge A) = 0.4$ $\quad P(G|\neg W \wedge \neg A) = 0.1$

(A) Yes
(B) No
(C) I don't know.

Earthquake, Burglary, and Alarm



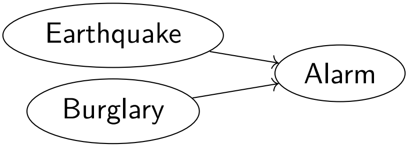
$P(E) = 0.1$ $\quad$ $P(B|E) = 0.2$ $\quad$ $P(B|\neg E) = 0.2$

$P(A|B \wedge E) = 0.9$ $\quad$ $P(A|B \wedge \neg E) = 0.8$

$P(A|\neg B \wedge E) = 0.2$ $\quad$ $P(A|\neg B \wedge \neg E) = 0.1$

CQ Unconditional Independence

CQ: Are Earthquake and Burglary independent?



$P(E) = 0.1$ $\quad$ $P(B|E) = 0.2$ $\quad$ $P(B|\neg E) = 0.2$

$P(A|B \wedge E) = 0.9$ $\quad$ $P(A|B \wedge \neg E) = 0.8$

$P(A|\neg B \wedge E) = 0.2$ $\quad$ $P(A|\neg B \wedge \neg E) = 0.1$

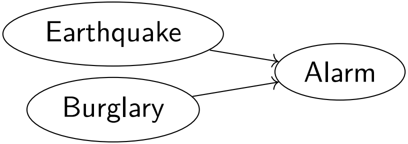
(A) Yes

(B) No

(C) I don't know.

CQ: Conditional Independence

CQ: Are Earthquake and Burglary conditionally independent given Alarm?



$P(E) = 0.1$ $\quad$ $P(B|E) = 0.2$ $\quad$ $P(B|\neg E) = 0.2$

$P(A|B \land E) = 0.9$ $\quad$ $P(A|B \land \neg E) = 0.8$

$P(A|\neg B \land E) = 0.2$ $\quad$ $P(A|\neg B \land \neg E) = 0.1$

(A) Yes

(B) No

(C) I don't know.

Revisiting the Learning Goals

By the end of the lecture, you should be able to

- Given a description of a domain or a probabilistic model for the domain, determine whether two variables are independent.
- Given a description of a domain or a probabilistic model for the domain, determine whether two variables are conditionally independent given a third variable.