Solution: SAMPLE SOLUTIONS

Contents

1 Learning Goals **1**

2 The Mail Delivery Robot **2**

3 Practice Questions **2**

1 Learning Goals

By the end of the exercise, you should be able to:

- Model a one-off decision problem by constructing a decision network containing nodes, arcs, conditional probability distributions, and a utility function.
- Choose the best action by evaluating a decision network.
- Determine the best action given a change in the utility function of a decision network.

2 The Mail Delivery Robot

The robot must choose its route to pickup the mail. There is a short route and a long route. The long route is slower, but on the short route the robot might slip and fall (an accident happens). The robot can put on pads. This won't change the probability of an accident, but it will make it less severe if it happens. Unfortunately, the pads add weight and slow the robot down. The robot would like to pick up the mail quickly with little/no damage.

Based on the problem description, let's assume that:

- If the robot goes on the long route, no accident occurs.
- If the robot goes on the short route, an accident occurs with a fixed probability q.

3 Practice Questions

Question 1:

We define:

- A: whether an accident occurs or not.
- P: whether the robot puts on pad.
- S: whether the robot chooses the short route.

What are the random variables?

What are the decision variables?

Solution: A is a random variable. P and S are decision variables.

Question 2:

Complete the following partial decision network by drawing some lines to connect some nodes based on the problem description. Also, write down the conditional probabilities of accident given the route.

$$P(A|\neg S) =$$
$$P(A|S) =$$

Accident

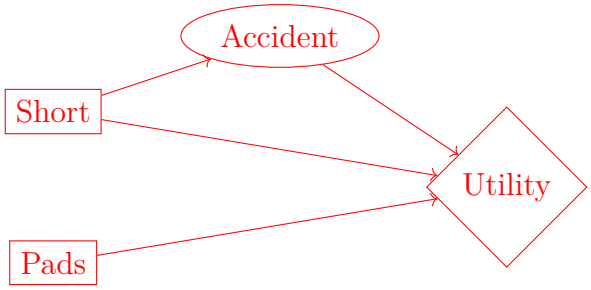
Short

Utility

Pads

Solution:

$$P(A|\neg S) = 0$$
$$P(A|S) = q$$



Question 3:

- If the robot chooses the long route and no accident happens, which of wearing and not wearing pads gives the robot higher utility?
- If the robot chooses the short route and no accident happens, which of wearing and not wearing pads gives the robot higher utility?
- If the robot wears pads and no accident happens, which of choosing the short route and choosing the long route gives the robot higher utility?
- If the robot does not wear pads and no accident happens, which of choosing the short route and choosing the long route gives the robot higher utility?
- If the robot chooses the short route and an accident happens, which of wearing and not wearing pads gives the robot higher utility?

Solution:

$$U(\neg P \wedge \neg S \wedge \neg A) > U(P \wedge \neg S \wedge \neg A)$$
$$U(\neg P \wedge S \wedge \neg A) > U(P \wedge S \wedge \neg A)$$
$$U(P \wedge S \wedge \neg A) > U(P \wedge \neg S \wedge \neg A)$$
$$U(\neg P \wedge S \wedge \neg A) > U(\neg P \wedge \neg S \wedge \neg A)$$
$$U(P \wedge S \wedge A) > U(\neg P \wedge S \wedge A)$$

Questions 4:

Here is the definition of the robot's utility function. Calculate the expectation value of $U(\neg P, \neg S)$, $U(\neg P, S)$, $U(P, \neg S)$, $U(P, S)$. Determine the best decisions for the robot. How do these decisions depend on the value of q?

	State	$U(w_i)$
$\neg P, \neg S, \neg A$	w_0 slow, no weight	
$\neg P, \neg S, A$	w_1 impossible	
$\neg P, S, \neg A$	w_2 quick, no weight	
$\neg P, S, A$	w_3 severe damage	
$P, \neg S, \neg A$	w_4 slow, extra weight	
$P, \neg S, A$	w_5 impossible	
$P, S, \neg A$	w_6 quick, extra weight	
P, S, A	w_7 moderate damage	

Solution:

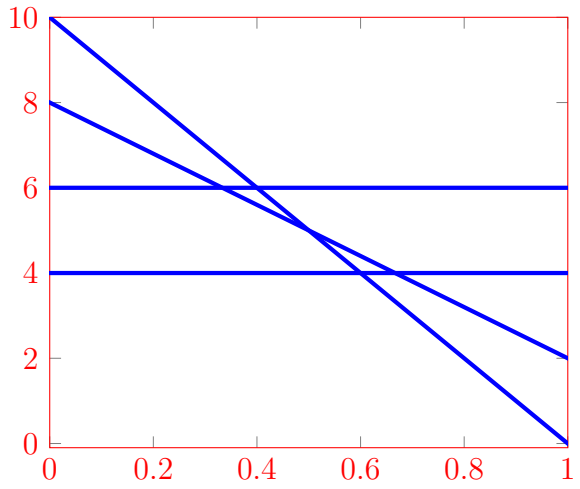
	State	$U(w_i)$
$\neg P, \neg S, \neg A$	w_0 slow, no weight	6
$\neg P, \neg S, A$	w_1 impossible	
$\neg P, S, \neg A$	w_2 quick, no weight	10
$\neg P, S, A$	w_3 severe damage	0
$P, \neg S, \neg A$	w_4 slow, extra weight	4
$P, \neg S, A$	w_5 impossible	
$P, S, \neg A$	w_6 quick, extra weight	8
P, S, A	w_7 moderate damage	2

$$
\begin{aligned}
EU(\neg P, \neg S) =& P(w_0|\neg P \wedge \neg S) * U(w_0) \\
&+ P(w_1|\neg P \wedge \neg S) * U(w_1) \\
=& P(\neg P \wedge \neg S \wedge \neg A|\neg P \wedge \neg S) * U(w_0) \\
&+ P(\neg P \wedge \neg S \wedge A|\neg P \wedge \neg S) * U(w_1) \\
=& P(\neg A|\neg P \wedge \neg S) * U(w_0) \\
&+ P(A|\neg P \wedge \neg S) * U(w_1) \\
=& P(\neg A|\neg S) * U(w_0) \\
&+ P(A|\neg S) * U(w_1) \\
=& (1)(6) + (0)(-) \\
=& 6
\end{aligned}
$$

$$
\begin{aligned}
EU(\neg P, S) =& P(w_2|\neg P \wedge S) * U(w_2) \\
&+ P(w_3|\neg P \wedge S) * U(w_3) \\
=& P(\neg P \wedge S \wedge \neg A|\neg P \wedge S) * U(w_2) \\
&+ P(\neg P \wedge S \wedge A|\neg P \wedge S) * U(w_3) \\
=& P(\neg A|\neg P \wedge S) * U(w_2) \\
&+ P(A|\neg P \wedge S) * U(w_3) \\
=& P(\neg A|S) * U(w_2) \\
&+ P(A|S) * U(w_3) \\
=& (1-q)(10) + (q)(0) \\
=& 10 - 10q
\end{aligned}
$$

$$
\begin{aligned}
EU(P, \neg S) =& P(w_4|P \wedge \neg S) * U(w_4) \\
&+ P(w_5|P \wedge \neg S) * U(w_5) \\
=& P(P \wedge \neg S \wedge \neg A|P \wedge \neg S) * U(w_4) \\
&+ P(P \wedge \neg S \wedge A|P \wedge \neg S) * U(w_5) \\
=& P(\neg A|P \wedge \neg S) * U(w_4) \\
&+ P(A|P \wedge \neg S) * U(w_5) \\
=& P(\neg A|\neg S) * U(w_4) \\
&+ P(A|\neg S) * U(w_5) \\
=& (1)(4) + (0)(-) \\
=& 4
\end{aligned}
$$

$$
\begin{aligned}
EU(P, S) =& P(w_6|P \wedge S) * U(w_6) \\
&+ P(w_7|P \wedge S) * U(w_7) \\
=& P(P \wedge S \wedge \neg A|P \wedge S) * U(w_6) \\
&+ P(P \wedge S \wedge A|P \wedge S) * U(w_7) \\
=& P(\neg A|P \wedge S) * U(w_6) \\
&+ P(A|P \wedge S) * U(w_7) \\
=& P(\neg A|S) * U(w_6) \\
&+ P(A|S) * U(w_7) \\
=& (1-q)(8) + (q)(2) \\
=& 8 - 6q
\end{aligned}
$$



no pad, long, 6

pad, long, 4

pad, short, $8 - 6q$

no pad, short, $10 - 10q$

What should the robot do?

- If $q \leq 2/5$, then wear no pad and go the short route.
- If $q > 2/5$, then wear no pad and go the long route.