The if-then-else inference rule.

```
    ⦇ P ⦈
(1)
    if (B) {
 (1)   ⦇ (P ∧ B) ⦈      if-then-else
 (3)     C1
       ⦇ Q ⦈           <justify based on subproof>
    } else {
       ⦇ (P ∧ (¬B)) ⦈   if-then-else
 (3)     C2                                (2)
       ⦇ Q ⦈           <justify based on subproof>
    }
                (2)
    ⦇ Q ⦈              if-then-else
```

Complete the annotations for the 2 subproofs

```
⦇ (P ∧ B) ⦈

    C1

⦇ Q ⦈


⦇ (P ∧ (¬B)) ⦈

    C2

⦇ Q ⦈
```

Example of "if-then-else"

$\langle\!\langle$ true $\rangle\!\rangle$

if $(x > y)$ { (alternatively, $\langle\!\langle (x>y) \rangle\!\rangle$)

$\langle\!\langle \text{true} \land (x > y) \rangle\!\rangle$ if-then-else

$\langle\!\langle ((x>y) \land (x=x)) \lor ((x \le y) \land (x=y)) \rangle\!\rangle$ implied (A)

max = x;

$\langle\!\langle ((x>y) \land (max=x)) \lor ((x \le y) \land (max=y)) \rangle\!\rangle$ assignment

} else {

$\langle\!\langle \text{true} \land (\neg(x>y)) \rangle\!\rangle$ (alternatively, $\langle\!\langle (\neg(x>y)) \rangle\!\rangle$) if-then-else

$\langle\!\langle ((x>y) \land (y=x)) \lor ((x \le y) \land (y=y)) \rangle\!\rangle$ implied (B)

max = y;

$\langle\!\langle ((x>y) \land (max=x)) \lor ((x \le y) \land (max=y)) \rangle\!\rangle$ assignment

}

$\langle\!\langle ((x>y) \land (max=x)) \lor ((x \le y) \land (max=y)) \rangle\!\rangle$ if-then-else

Proof of implied (A)

Assume $(x > y)$ is true.

$(x=x)$ is true by def. of =. By def. of $\land$, $(x>y) \land (x=x)$ is true. By def of $\lor$, $(((x>y) \land (x=x)) \lor ((x \le y) \land (x=y)))$ is true.

Steps to follow:

① Annotate the program.

② Prove any implied's.

The if–then inference rule

⦃ P ⦄

if (B) {

⦃ $(P \wedge B)$ ⦄   if–then

C

⦃ Q ⦄   <justify based on subproof>

}

⦃ Q ⦄   if–then

implied $((P \wedge (\neg B)) \rightarrow Q)$

Complete the annotations for the subproof.

⦃ $(P \wedge B)$ ⦄

C

⦃ Q ⦄

Example of "if-then".

$(\!|\ \text{true}\ |\!)$

if (max < x) {

$\quad (\!|\ (max < x)\ |\!)$ — if-then

$\quad (\!|\ (x \geq x)\ |\!)$ — implied (a)

$\quad max = x;$

$\quad (\!|\ (max \geq x)\ |\!)$ — assignment

}

$(\!|\ (max \geq x)\ |\!)$ — if-then

implied (b): $((\neg(max < x)) \rightarrow (max \geq x))$

Proof of implied (a):

Assume $(max < x)$ is true. $(x \geq x)$ is true by def. of $\geq$.

Proof of implied (b):

Assume $(\neg(max < x))$. By def of $\neg$, we have that $(max \geq x)$