Predicate Logic: Soundness and Completeness of Natural Deduction

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Lecture 17

Outline

Learning goals

By the end of this lecture, you should be able to:

- Define soundness and completeness.
- Prove that an inference rule is sound or not sound.
- Prove that a semantic entailment holds using the soundness and completeness theorems.
- Show that no natural deduction proof exists for a semantic entailment using the soundness and completeness theorems.

CQ Choosing concrete formulas

True/False: Let α be a Predicate formula. There exists an interpretation and environment under which α is true.

(A) True

(B) False

(C) Not enough information

CQ A set of formulas is unsatisfiable

Suppose that a set of formulas Σ is unsatisfiable.

Which of the following is correct?

(A) For every pair (I, E), at least one formula in Σ is false.

(B) For one pair (I, E), at least one formula in Σ is false.

(C) For every pair (I, E), at least one formula in Σ is a contradiction.

(D) For one pair (I, E), at least one formula in Σ is a contradiction.

(E) None of the above

CQ Proving unsatisfiability

We want to prove that a set of formulas Σ is unsatisfiable. See the beginning of our proof below:
Consider any interpretation and environment (I, E). Consider two cases.

1. At least one formula in Σ is false under (I, E).
2. ...

What is the other case?

(A) Every formula in Σ is true under (I, E).
(B) Every formula in Σ is false under (I, E).
(C) At least one formula in Σ is true under (I, E).
(D) At least one formula in Σ is false under (I, E).
(E) None of the above

Revisiting the learning goals

By the end of this lecture, you should be able to:

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