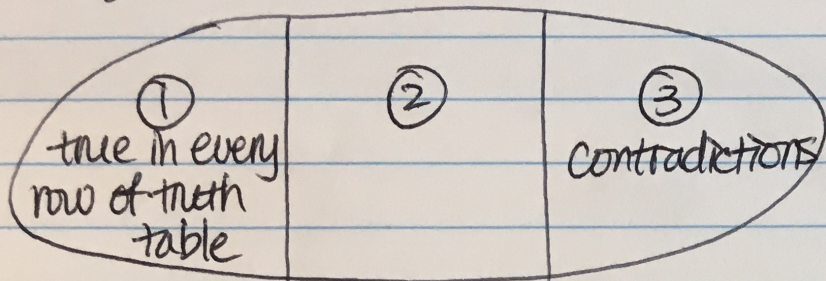


# Tautology, Contradiction, Satisfiable

Sept 19



Answer: (d). a and b are true. c is false

set 3, contradictions,  
false in every row of the truth table.

set 2., contingencies,  
true in at least one row, false in at least one  
row.

set 1, tautologies,  
true in every row of the truth table.

sets ① and ② satisfiable.  
true in at least one row.

①  
tautologies  
true in every row

②  
contingencies  
true in at least  
one row.  
false in at least  
one row.

③  
contradictions  
false in every row

satisfiable  
true in at least one row.

## Tautology, Contradiction, Satisfiable.

Sept 19.

$$(p \vee q) \leftrightarrow ((p \wedge (\neg q)) \vee ((\neg p) \wedge q)) \quad (\text{Slide 22 example 3})$$

Note that this formula is a biconditional.

The left side is an inclusive OR, and the right side is an exclusive OR.

Can this formula be true?

- A biconditional is true when both sides of it have the same truth value. So we need the inclusive & exclusive OR's to be both true or both false.

- When  $p$  and  $q$  have different truth values, both OR's are true.

So this formula can be true & is not a contradiction.

Can this formula be false?

- A biconditional is false when both sides have different truth values. So we need the two OR's to be different.

- The two OR's have different truth values when

$$p \equiv q \equiv T.$$

$$(p \vee q) \equiv T \text{ and } ((p \wedge (\neg q)) \vee ((\neg p) \wedge q)) \equiv F.$$

So this formula can be false and is not a tautology.

It's satisfiable but not a tautology.

Tautology, Contradiction, Satisfiable.

Sept 19.

$$(((p \wedge q) \rightarrow r) \wedge (p \rightarrow q)) \rightarrow (p \rightarrow r)$$

Is this formula a contradiction? Can it be true?

This formula is a giant conditional.

The easiest way to make a conditional true is to make the premise false.

The premise is a conjunction. The easiest way to make a conjunction false is to make one of the two formulas false. Let's make  $(p \rightarrow q)$  false by  $p \equiv T, q \equiv F$ .

When  $p \equiv T, q \equiv F, r \equiv T$  or  $F$ , the formula is true.  
So it is not a contradiction.

Is this formula a tautology? Can it be false?

Proof:

To make it false, we need the premise to be true and the conclusion to be false.

To make the conclusion  $(p \rightarrow r)$  false, we need  $p \equiv T$  and  $r \equiv F$ .

The formula becomes true.

$$(((T \wedge q) \rightarrow F) \wedge (T \rightarrow q)) \rightarrow F$$

$$\equiv ((q \rightarrow F) \wedge q) \rightarrow F$$

$$\equiv ((\neg q) \wedge q) \rightarrow F$$

$$\equiv (F \rightarrow F)$$

$$\equiv T$$

This shows that it's impossible to make the formula false, so it's a tautology.

QED