The basic rules of natural deduction:

	introduction	*elimination*

$\wedge$

$$\frac{\phi \quad \psi}{\phi \wedge \psi} \wedge i \qquad\qquad \frac{\phi \wedge \psi}{\phi} \wedge e_1 \qquad \frac{\phi \wedge \psi}{\psi} \wedge e_2$$

$\vee$

$$\frac{\phi}{\phi \vee \psi} \vee i_1 \qquad \frac{\psi}{\phi \vee \psi} \vee i_2$$

$$\frac{\phi \vee \psi \quad \boxed{\begin{array}{c}\phi \\ \vdots \\ \chi\end{array}} \ \boxed{\begin{array}{c}\psi \\ \vdots \\ \chi\end{array}}}{\chi} \vee e$$

$\rightarrow$

$$\frac{\boxed{\begin{array}{c}\phi \\ \vdots \\ \psi\end{array}}}{\phi \rightarrow \psi} \rightarrow i \qquad\qquad \frac{\phi \quad \phi \rightarrow \psi}{\psi} \rightarrow e$$

$\neg$

$$\frac{\boxed{\begin{array}{c}\phi \\ \vdots \\ \bot\end{array}}}{\neg \phi} \neg i \qquad\qquad \frac{\phi \quad \neg \phi}{\bot} \neg e$$

$\bot$ (no introduction rule for $\bot$) $\dfrac{\bot}{\phi} \bot e$

$\neg\neg$ $\dfrac{\neg\neg\phi}{\phi} \neg\neg e$

Some useful derived rules:

$$\frac{\phi \rightarrow \psi \quad \neg \psi}{\neg \phi} MT \qquad\qquad \frac{\phi}{\neg\neg\phi} \neg\neg i$$

$$\frac{\boxed{\begin{array}{c}\neg\phi \\ \vdots \\ \bot\end{array}}}{\phi} PBC \qquad\qquad \frac{}{\phi \vee \neg\phi} LEM$$

Figure 1.2. Natural deduction rules for propositional logic.