Propositional Logic: The basic rules of natural deduction.

mimic our reasoning process

adding connective to formula — introduction   *removing connective from formula* — elimination.

conjunction rules

$\wedge$

premises *inference line*

$$\dfrac{a \quad b}{(a \wedge b)} \quad \wedge i$$

Conclusion   *name of the rule*

$$\dfrac{(a \wedge b)}{a} \qquad \dfrac{(a \wedge b)}{b} \quad \wedge e$$

disjunction rules

$\vee$

$$\dfrac{a}{(a \vee b)} \qquad \dfrac{b}{(a \vee b)} \quad \vee i$$

proof by cases

$$\dfrac{(a \vee b) \quad \begin{array}{c} a \\ \vdots \\ c \end{array} \quad \begin{array}{c} b \\ \vdots \\ c \end{array}}{c} \quad \vee e$$

implication rules

$\rightarrow$

subproof *assumption* *additional formulas* *conclusion*

$$\dfrac{\begin{array}{c} a \\ \vdots \\ b \end{array}}{(a \rightarrow b)} \quad \rightarrow i$$

modus ponens

$$\dfrac{a \quad (a \rightarrow b)}{b} \quad \rightarrow e$$

negation rules

$\neg$

proof by contradiction.

$$\dfrac{\begin{array}{c} a \\ \vdots \\ \bot \end{array}}{(\neg a)} \quad \neg i$$

bottom

$$\dfrac{a \quad (\neg a)}{\bot} \quad \neg e / \bot i$$

contradiction rules

$\bot$

contradiction

$$\dfrac{a \quad (\neg a)}{\bot} \quad \bot i / \neg e$$

same rule two names

$$\dfrac{\bot}{a} \quad \bot e$$

double negation rules

$\neg\neg$

$$\dfrac{(\neg(\neg a))}{a} \quad \neg\neg e$$

Some useful derived rules

$$\frac{(a \to b) \quad (\neg b)}{(\neg a)} \quad \text{MT (modus tollens)}$$

$$\frac{a}{(\neg(\neg a))} \quad \neg\neg i$$

$$\frac{\boxed{\begin{array}{c} (\neg a) \\ \vdots \\ \bot \end{array}}}{a} \quad \text{PBC (proof by contradiction)}.$$

$$\frac{}{(a \lor (\neg a))} \quad \text{LEM (law of excluded middle)}$$

Writing a natural deduction proof.

① Write down all the premises.
② Write down the conclusion.
③ Apply elimination rules to the premises?
④ Apply introduction rules to produce the conclusion?

* which rule should you apply first?

Subproof:
① You should create a subproof only if you are using it to apply a rule. If you don't know which rule you are applying, don't create a subproof.

② When you create a subproof,
    ⓐ fill in the assumption.
    ⓑ fill in the conclusion.
    ⓒ fill in the middle.

③ Inside a subproof, you can use all the formulas that have appeared above.

④ Outside a subproof, you cannot use any individual formula in the subproof. You can only use the subproof as a whole.