Today, you are going to work in teams to solve logical deduction problems. The problems will involve translating English sentences to formulas, proving semantic entailment, and constructing natural deduction proofs.

Ground rules:

1. If you don't think working on natural deduction proofs in groups is helpful for you, feel free to leave. If you choose to stay, you must follow our rules.

2. Put away any electronic device (phone, tablet, computer). You don't need them.

3. Form teams of 3-4 people.

4. We will give your team one of three problems to start with.

5. Each problem has three parts: Translation, Prove/disprove semantic entailment, Construct a natural deduction proof.

6. As a team, complete as many problems as you can.

7. Once you complete a problem, show it to Joe. Joewill accept your solution if and only if every team member can answer Joe's questions about it.

8. For each problem, the first three teams handing in a complete and clear solution will get one chocolate per team member.

9. This is a team effort. Please ask questions, answer questions, and help each other.

   If you want to go fast, go alone! If you want to go far, go together!

10. Last, but not least, HAVE FUN! =)

What we (the course staff) will do:

- If you have a question about the problem or want to check your solutions, come see any of Joe and Stephanie.

- If you want to submit a clear and complete solution to win prizes, come see Joe.

The onnagata problem

Consider the following argument, drawn from an article by Julian Baggini. The onnagata are male actors portraying female characters in kabuki theatre.

Premise 1: If women are too close to femininity to portray women, then men must be too close to masculinity to play men, and vice versa.

Premise 2: And yet, if the onnagata are correct, women are too close to femininity to portray women and yet men are not too close to masculinity to play men.

Conclusion: Therefore, the onnagata are incorrect, and women are not too close to femininity to portray women.
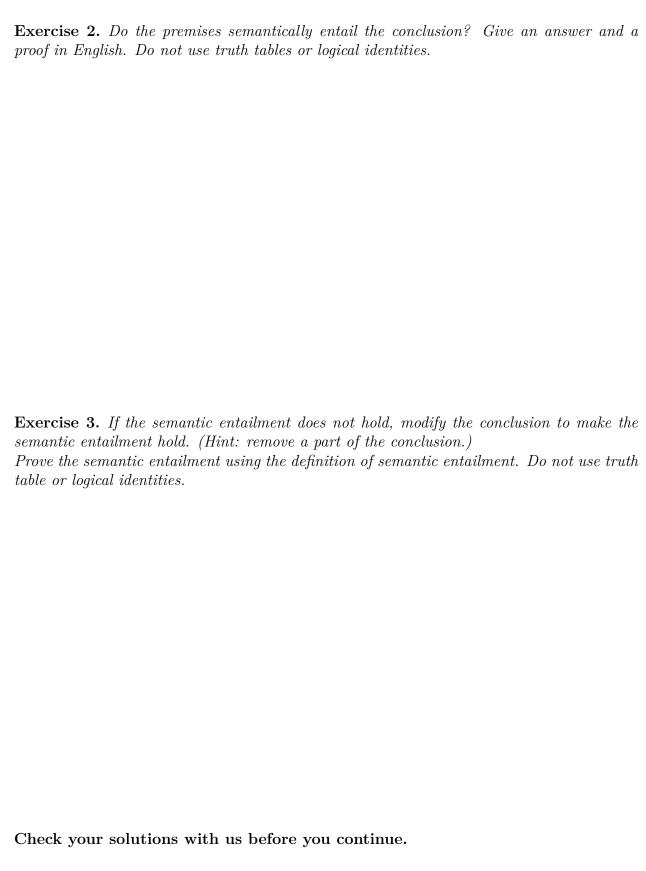
Relevant articles:

- http://www.butterfliesandwheels.org/2004/tu-quoque/

- https://www.theguardian.com/stage/2004/aug/21/theatre

Exercise 1. *Translate the argument into propositional formulas. Define your own propositions.*
Note: We do not have natural deduction rules for $\leftrightarrow$. When you need the formula $(a \leftrightarrow b)$, use the formula $((a \rightarrow b) \land (b \rightarrow a))$ instead.

Check your solutions with us before you continue.

Exercise 2. *Do the premises semantically entail the conclusion? Give an answer and a proof in English. Do not use truth tables or logical identities.*

Exercise 3. *If the semantic entailment does not hold, modify the conclusion to make the semantic entailment hold. (Hint: remove a part of the conclusion.)*
Prove the semantic entailment using the definition of semantic entailment. Do not use truth table or logical identities.

Check your solutions with us before you continue.

Exercise 4. *Give a natural deduction proof from the premises to the (modified) conclusion. If you want to use any rule that is not a basic rule of natural deduction, check with us to see if you can use it.*

The superman problem

Consider the following argument about the existence of Superman.

If Superman were able and willing to prevent evil, he would do so.
If Superman were unable to prevent evil he would be impotent.
If Superman were unwilling to prevent evil, he would be malevolent.
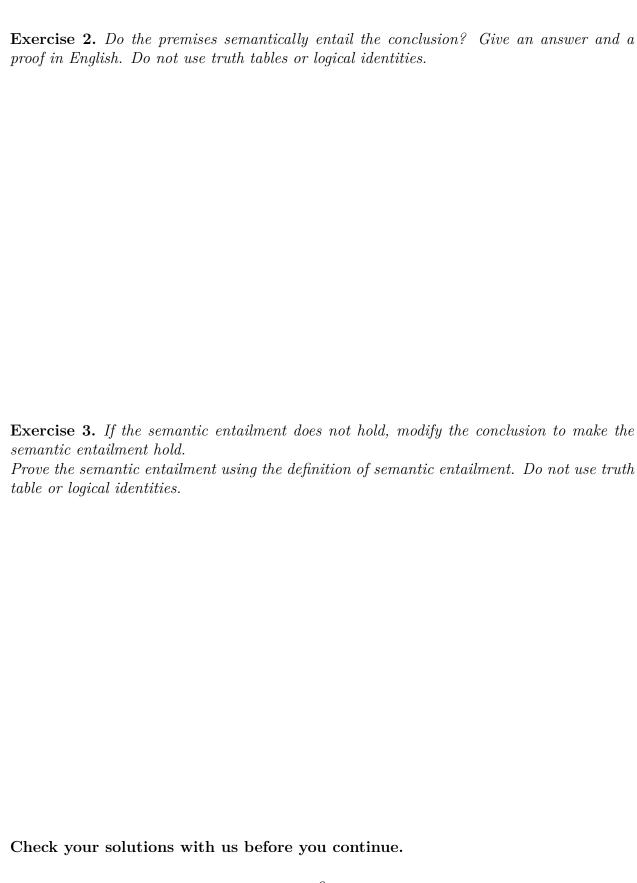Superman does not prevent evil.
If Superman exists, he is neither impotent nor malevolent.
Therefore, Superman exists.

Exercise 1. *Translate the argument into propositional formulas. Define your own propositions.*
Note that we do not have natural deduction rules for $\leftrightarrow$. When you need the formula $(a \leftrightarrow b)$, use the formula $((a \rightarrow b) \wedge (b \rightarrow a))$ instead.

Check your solutions with us before you continue.

Exercise 2. *Do the premises semantically entail the conclusion? Give an answer and a proof in English. Do not use truth tables or logical identities.*

Exercise 3. *If the semantic entailment does not hold, modify the conclusion to make the semantic entailment hold.*
Prove the semantic entailment using the definition of semantic entailment. Do not use truth table or logical identities.

Check your solutions with us before you continue.

Exercise 4. *Give a natural deduction proof from the premises to the (modified) conclusion. If you want to use any rule that is not a basic rule of natural deduction, check with us to see if you can use it.*

The knights and knaves problem

A very special island is inhabited only by knights and knaves. Knights always tell the truth, and knaves always lie. A knight invited a newcomer to the island and told the newcomer the following facts about five inhabitants: Alice, Bob, Peggy, Rex and James.

- It is not the case that Bob is a knight or Peggy is a knight.

- James and Bob are both knights if and only if Rex is a knight.

- If Rex is a knave, then Alice is a knave.

Using the power of logic, the newcomer concluded that Alice is a knight.

Exercise 1. *Translate the argument into propositional formulas. Define your own propositions.*
Note that we do not have natural deduction rules for $\leftrightarrow$. When you need the formula $(a \leftrightarrow b)$, use the formula $((a \rightarrow b) \wedge (b \rightarrow a))$ instead.

Check your solutions with us before you continue.

Exercise 2. *Do the premises semantically entail the conclusion? Give an answer and a proof in English. Do not use truth tables or logical identities.*

Exercise 3. *If the semantic entailment does not hold, modify the conclusion to make the semantic entailment hold.*
Prove the semantic entailment using the definition of semantic entailment. Do not use truth table or logical identities.

Check your solutions with us before you continue.

Exercise 4. *Give a natural deduction proof from the premises to the (modified) conclusion. If you want to use any rule that is not a basic rule of natural deduction, check with us to see if you can use it.*