CSC 363H1/5 S **Assignment # 4**

Due: By 11:00am on Monday 9 April. **Worth**: 10%

1. [30 pts] Recall that **coNP** is the class of complements of languages in **NP**.

   (a) By analogy with the definition of **NP** completeness, we way that a language **coNP**=complete if

   - B is in **coNP**, and
   - B is **coNP**-hard, i.e., for all $A \in$**coNP**, $A \leq_p B$.

   Show that for any language A, A is **coNP** complete if and only if $\overline{A}$ is **NP** complete.

   (b) Show that if $\mathbf{P} = \mathbf{NP}$ then $\mathbf{P} = \mathbf{coNP}$.

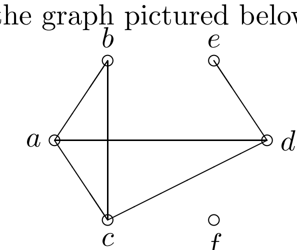
   (c) Show that if there is any language that is both **NP** complete and **coNP** complete, then NP = coNP.

2. [20 pts] Show that A_{TM} is **NP**-hard. Recall that

$$A_{TM} = \{\langle M, w\rangle | \ \ M \text{ is a deterministic TM that accepts string } w\}.$$

   Hint: use the original definition of NP-hardness, not a reduction from other known NP-hard problems). What is the running time of your reduction function? (be more specific than just "polynomial").

3. [25 pts] A "core set" in an undirected graph $G = (V, E)$ is a subset of the vertices $V' \subseteq V$ such that every vertex outside V' is connected to some vertex in V' (*i.e.*, $\forall v \in V - V' \ \exists u \in V' : (v, u) \in E$). For example, if G is the graph pictured below, then $\{a, d, f\}$ is a core set in G but $\{a, d\}$ is not.



   The language COR-SET is $\{\langle G, k\rangle | G$ is a graph that has a core set of size $k\}$. Show that COR-SET is **NP** complete. (*Hint*: Think of the connection to SET-COVER that was shown to be NP-complete in class:

   SET-COVER = $\{\langle U; S_1, S_2, \ldots, S_m(S_i \subseteq U); k\rangle$ | there is a subcollection of k of the S_i whose union is $U\}$.

4. [25 pts] Let COR-SET-SEARCH be the search problem associated with COR-SET. Here are the formal specifcations.

   Input: A graph G, and a number k.
   Output: A core set of size k , or an answer "NONE" if no such set exists.

   Show that COR-SET is self-reducible. That is, given a polynomial time algorithm to COR-SET, show the a polynomial time algorithm to solve COR-SET-SEARCH.