CSC 2411H - Assignment 3

Due Mar 21, 2005

1. (a) Determine the payoff of the row player when both players play the optimal mixed strategies for the game described by the matrix

$$\begin{pmatrix} 5 & 6 \\ 7 & 4 \end{pmatrix}$$

   (b) Consider a zero-sum game of two players, where player I has m strategies and player II has $n \geq m$ startegies. Prove or disprove: Player II has an optimal mixed strategy that uses no more than m of her pure strategies (in other words her strategy vector is supported on no more than m coordinates).

2. There are n boxes in front of you, all empty but one which contains a prize. A *probe* is the act of openning a box to see if it contains the prize. Your goal is to find the prize while minimizing the number of probes. Clearly, a deterministic algorithm needs $n - 1$ probes in the worst case .

   (a) Give a randomized algorithm that performs only $(n + 1)/2$ probes in expetation.

   (b) Use Yao's principle to show that this is tight.

3. Consider the following optimization problem: Let n be even and let c be a positive vector in $\mathbb{R}^n$, find

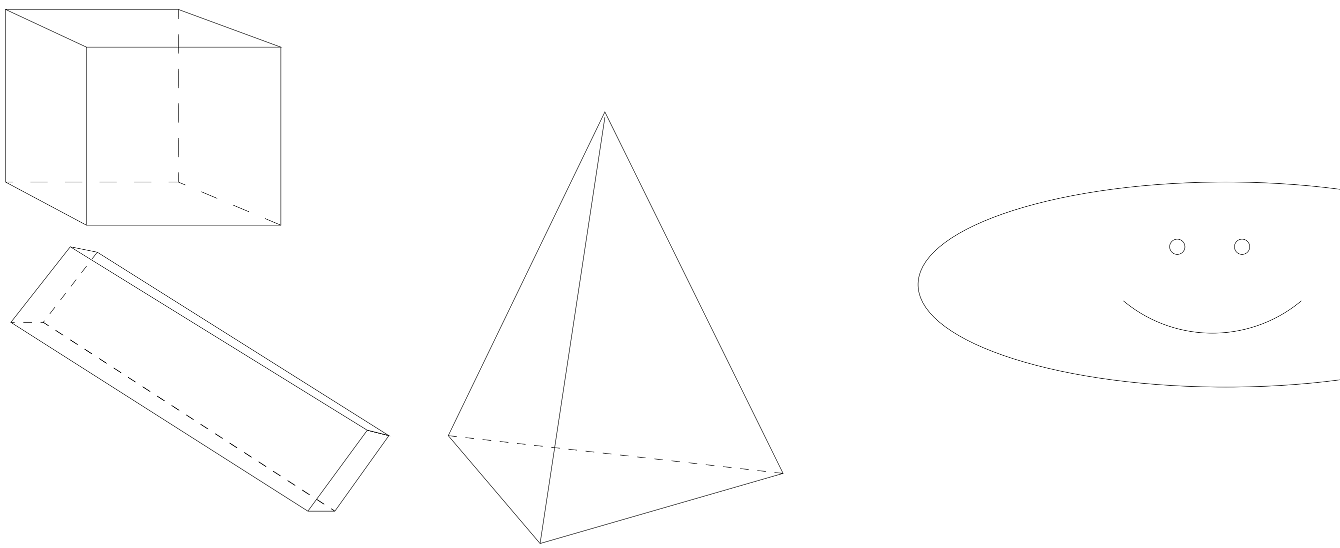
$$\min\{\langle c, x\rangle | x \geq 0 \text{ and } \sum_{i \in S} x_i \geq 1 \quad \forall S \subset \{1, 2, \ldots, n\} \text{ of size } n/2\}.$$

   This can be thought of as an LP with exponentially many constraints.

   (a) Show that the Elliposid algorithm can be used to get a poly(n)-time algorithm for teh problem.

(b) Show that in fact, a simpler ($O(n \log n)$ time) algorithm for the problem exists.

4. Why does the Ellipsoid Algorithm use ellipsoids?... Can you think of other families of geometric bodies in $\mathbb{R}^n$ that will also do? State precisely which properties of the ellipsoid are used in the definition and in the analysis of the algorithm. Using that, try to think whether the cubes, boxes (the last two not necessarily axis aligned), balls, simplices (a simplex is the convex hull of $n+1$ affinely independent points). The case of simplices is fairly involved and you are only required to give the relevant analysis in two dimensions.



5. Give an example in which the Interior Point Algorithm described in class (Ye's method) will make a move that increases the duality gap ($\sum_i x_i s_i$). Concentrate on a primal step. Hint : You should look for a case in which $\sum_i d_i s_i < 0$ where d is the vector dfined in class (why?). You may ignore the centralizing step $x \mapsto (1, 1, \ldots, 1)^t$.

6. (a) Show that semidefinite programs may have irrational solutions. This shows that in some sense, semidefinite programming cannot be solved exactly and can only be approximated.

   (b) Let G be an undirected graph with n vertices. We wish to assign to each vertex x a vector v_x so that (i) v_x is a unit vector and the sum $\sum_{xy \in E} \langle v_x, v_y \rangle$ is minimized. Express the problem as a semidefinite program.