A2-Q4 [12 Marks]

(a) [4 Marks]

$R_1 = (a+b+c)^* cba(a+b+c)^*$: Every string in $L(R_1)$ contains cba and each string that contains cba consists of some arbitrary substring in $\{a,b,c\}^*$ followed by cba, followed by some arbitrary substring in $\{a,b,c\}^*$.

(b) [4 Marks]

$R_2 = (a+c)^*(bS)^*$, where $S = \varepsilon + a(a+c)^*$ represents all strings over $\{a,c\}$ that do not begin with c. No string in $L(R_2)$ contains bc because anything that comes after a b can not start with c. Also, any string that doesn't contain bc is in $L(R_2)$ because any such string can be written as $x_0bx_1bx_2...bx_{n-1}bx_n$ where x_0 contains any arbitrary combination of a and c and other x_i contain combinations of a and c that doesn't start with c. So, the x_0 is constructed by $L((a+c)^*)$ and the rest is constructed by $L((bS)^*)$.

(c) [4 Marks]

$R_3 = R_2abS(bS)^*$, where S is defined in the previous part. Every string in $L(R_3)$ contains ab but doesn't contain bc because no string in R_2 contains bc and no string in $L(S(bS)^*)$ starts with c. Moreover, every string that contains ab but doesn't contain bc can be written as $x.ab.y$ where x is any string that doesn't contain bc and y is any string that doesn't contain bc and doesn't start with c. x is constructed by $L(R_2)$ as shown in the previous part and y is constructed by $L(S(bS)^*)$ (according to previous part).