

EXAMPLE 3.9

The following is a formal description of $M_1 = (Q, \Sigma, \Gamma, \delta, q_1, q_{\text{accept}}, q_{\text{reject}})$, the Turing machine that we informally described (page 139) for deciding the language $B = \{w\#w \mid w \in \{0,1\}^*\}$.

- $Q = \{q_1, \dots, q_{14}, q_{\text{accept}}, q_{\text{reject}}\}$,
- $\Sigma = \{0,1,\#\}$, and $\Gamma = \{0,1,\#,x,\sqcup\}$.
- We describe δ with a state diagram (see the following figure).
- The start, accept, and reject states are q_1 , q_{accept} , and q_{reject} .

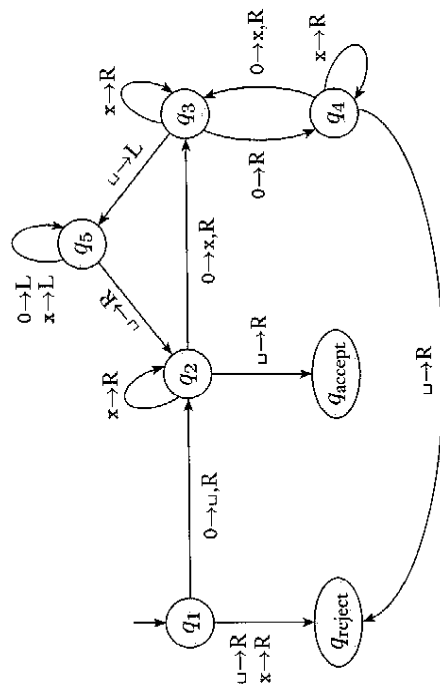


FIGURE 3.8
State diagram for Turing machine M_2

In this state diagram, the label $0 \rightarrow \sqcup, R$ appears on the transition from q_1 to q_2 . The label signifies that, when in state q_1 with the head reading 0, the machine moves to state q_2 , writes $\sqcup$, and moves the head to the right. In other words, $\delta(q_1, 0) = (q_2, \sqcup, R)$. For clarity we use the shorthand $0 \rightarrow R$ in the transition from q_4 , to mean that the machine moves to the right when reading 0 in state q_4 , doesn't alter the tape, so $\delta(q_4, 0) = (q_4, 0, R)$.

This machine begins by writing a blank symbol over the leftmost 0 on the tape so that it can find the left-hand end of the tape in stage 4. Whereas we don't normally use a more suggestive symbol such as # for the left-hand end of the tape, we use a blank here to keep the tape alphabet, and hence the state transitions, small. Example 3.11 gives another method of finding the left-hand end of the tape.

Next we give a sample run of this machine on input 0000. The starting configuration is $q_1 0000$. The sequence of configurations the machine enters appears as follows; read down the columns and left to right.

q_1 0000	$\sqcup q_3$ x0x $\sqcup$	$\sqcup x q_5$ xxx $\sqcup$
$\sqcup q_2$ 000	$q_5 \sqcup x 0 x \sqcup$	$\sqcup q_5$ xxx $\sqcup$
$\sqcup x q_3$ 00	$\sqcup q_2$ x0x $\sqcup$	$q_5 \sqcup$ xxx $\sqcup$
$\sqcup x 0 q_4$ 0	$\sqcup x q_2$ 0x $\sqcup$	$\sqcup q_2$ xxx $\sqcup$
$\sqcup x 0 x q_3$	$\sqcup x x q_3$ x $\sqcup$	$\sqcup x q_2$ xx $\sqcup$
$\sqcup x 0 q_5$ x $\sqcup$	$\sqcup x x x q_3$	$\sqcup x x q_2$ x $\sqcup$
$\sqcup x q_5$ 0x $\sqcup$	$\sqcup x x q_5$ x $\sqcup$	$\sqcup x x x q_2$
		$\sqcup x x x \sqcup q_{\text{accept}}$

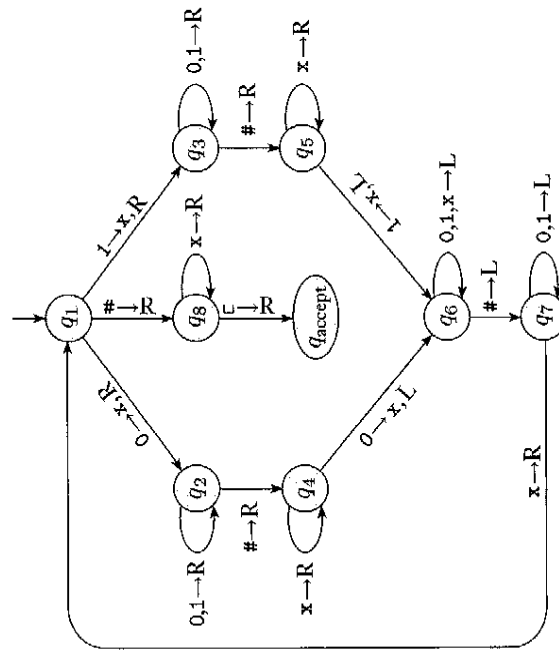


FIGURE 3.10
State diagram for Turing machine M_1

In Figure 3.10, which depicts the state diagram of TM M_1 , you will find the label $0,1 \rightarrow R$ on the transition going from q_3 to itself. That label means that the machine stays in q_3 and moves to the right when it reads a 0 or a 1 in state q_3 . It doesn't change the symbol on the tape.

Stage 1 is implemented by states q_1 through q_6 , and stage 2 by the remaining states. To simplify the figure, we don't show the reject state or the transitions going to the reject state. Those transitions occur implicitly whenever a state lacks an outgoing transition for a particular symbol. Thus, because in state q_5 no outgoing arrow with a # is present, if a # occurs under the head when the machine is in state q_5 , it goes to state q_{reject} . For completeness, we say that the head moves right in each of these transitions to the reject state.