problem set 6 (not to be submitted)
CSC236, Fall 2008

1. (this is the most challenging question in this set. Take it seriously!!) Consider a limited regular expressions that is defined by using the same recursive definition as for regular expressions except we omit the rule that the expression can be the "epsilon" symbol. In other words, we are using the rules $R = \emptyset$, $R = \sigma$ for $\sigma \in \Sigma$, $R = ST$, $R = S + T$, or $R = (S)^*$ where S, T can also be obtained by these rules. We call such an expression "ϵ-free regular expression".

   Show that every regular expression R is equivalent (do you remember what 'equivalent' means?) to a ϵ-free regular expression R' or R is equivalent to $\epsilon + R'$ where R' is an ϵ-free regular expression. For example $(01 + 10 + \epsilon)0^*$ is equivalnt to $(01 + 10)0^* + 0^*$ (do you see why?)

   For your proof you should use structural induction.

2. if L_1 and L_2 are regular languages. Is it true that $L_1 \setminus L_2 = \{x \in \Sigma^* : x \in L_1, x \notin L_2\})$ is regular? Prove or give a counter-example.

3. if L_1, L_2, L_3 are regular is $L_1 \cap (L_2 \cup L_3)$ regular? Prove or give a counter-example.

4. Prove or give a counter-example: if $L_1 \cup L_2$ are regular, is it always the case that L_1 and L_2 are regular?

5. In class we presented the language $L = \{x \in \{0,1\}^* : Z(x) - N(x) = 0\}$, where $Z(x)$ counts the number of zeros in x and $N(x)$ counts the number of ones in x. We showed that L is not regular. Consider the similar looking language $L_7 = \{x \in \{0,1\}^* : Z(x) - N(x)$ is divisible by $7\}$. Is L regular?

6. Show that any finite language is regular. You may want to use induction (on what? why is it allowed?) or simply give a direct proof.