



Understanding the Origins of Bias in Word Embeddings

Marc-Etienne Brunet
Colleen Alkalay-Houlihan
Ashton Anderson
Richard Zemel

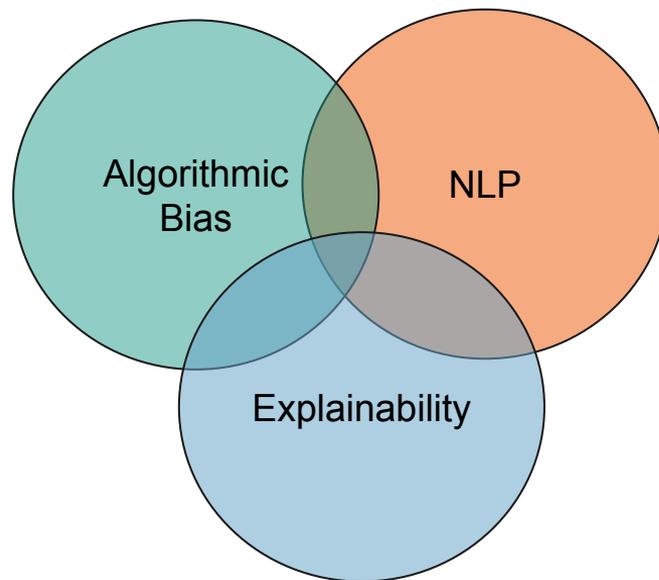
Introduction

Graduate student at U of T (Vector Institute)

Work at the intersection of Bias, Explainability, and Natural Language Processing

Collaborated with Colleen Alkalay-Houlihan

Supervised by Ashton Anderson and Richard Zemel



UNIVERSITY OF
TORONTO



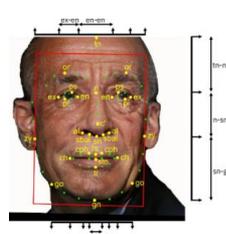
VECTOR
INSTITUTE

INSTITUT
VECTEUR

Many Forms of Algorithmic Bias

For example:

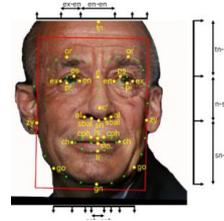
- Facial Recognition
- Automated Hiring
- Criminal Risk Assessment
- Word Embeddings



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How can we **attribute** the **bias** in word embeddings **to** the individual **documents** in their training corpora?

> **Background**

Method Overview

Critical Details

Experiments



Word Embeddings: Definitions in Vector Space

Definitions encode **relationships** between words

lead·er

/ˈlɛdər/ 

noun

1. the person who leads or commands a group, organization, or country.
"the leader of a protest group"
synonyms: chief, head, principal, boss; More

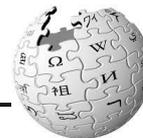
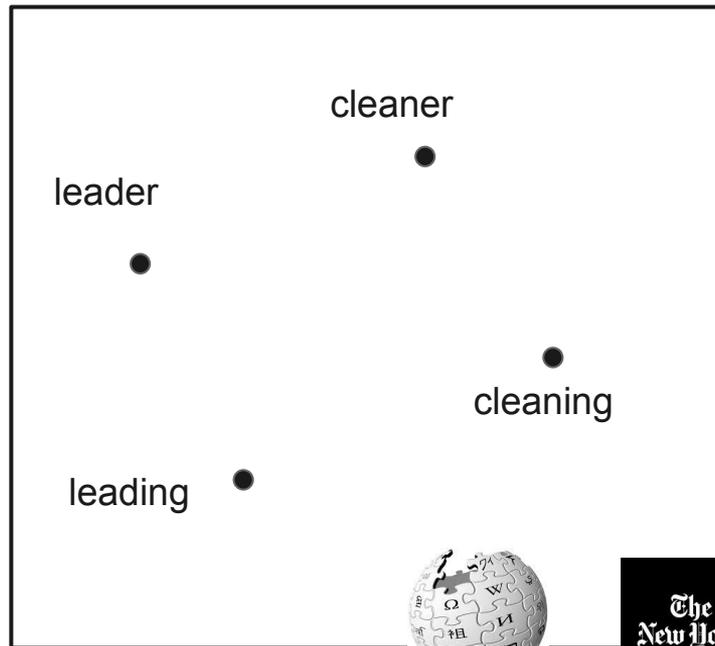
cleaning

/ˈklɛniŋ/ 

noun

noun: **cleaning**

- the action of making something clean, especially the inside of a house.
"the housekeeper will help with the cleaning"



WIKIPEDIA
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The
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Times

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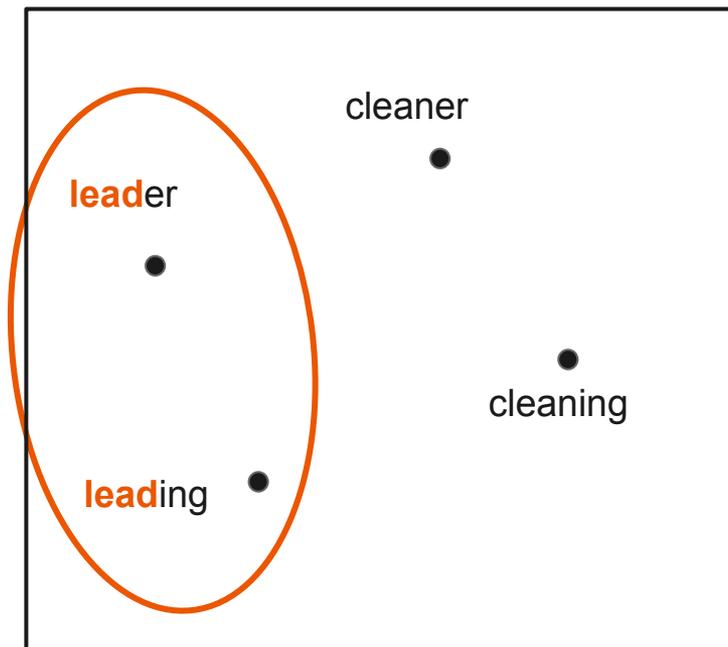
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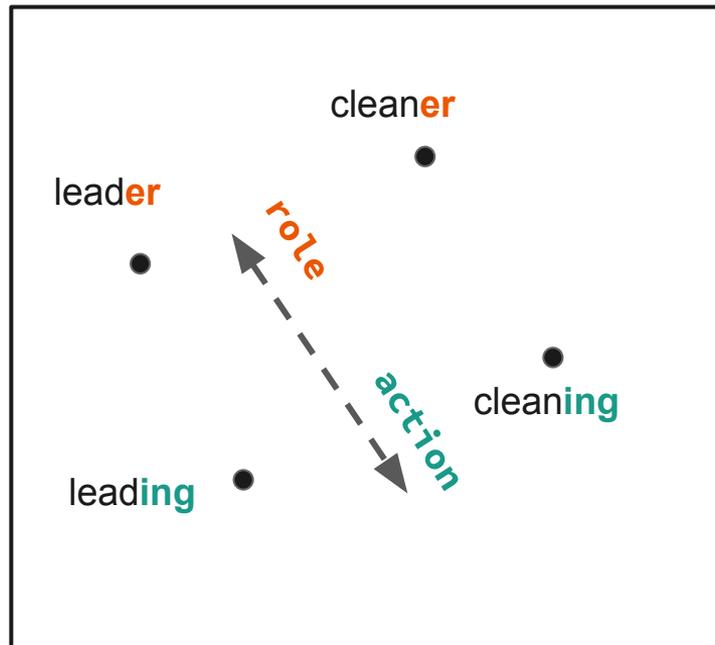
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Problematic Definitions in Vector Space

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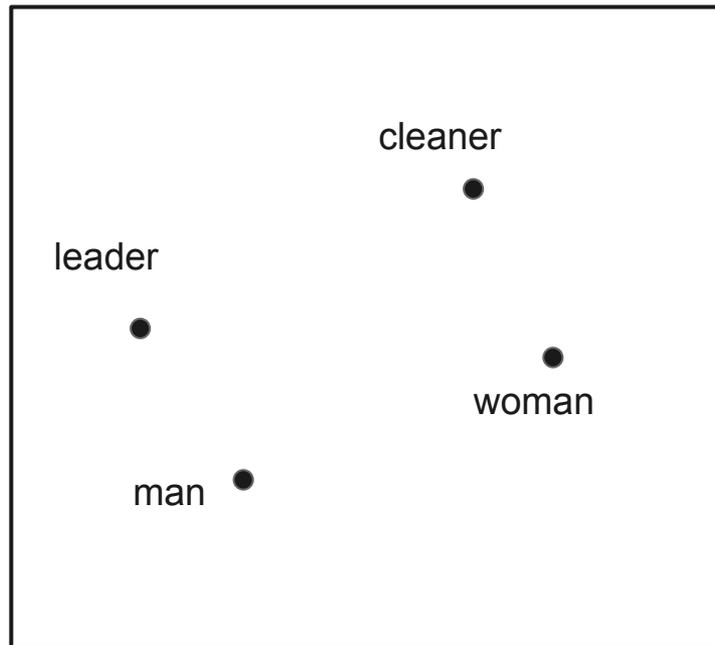
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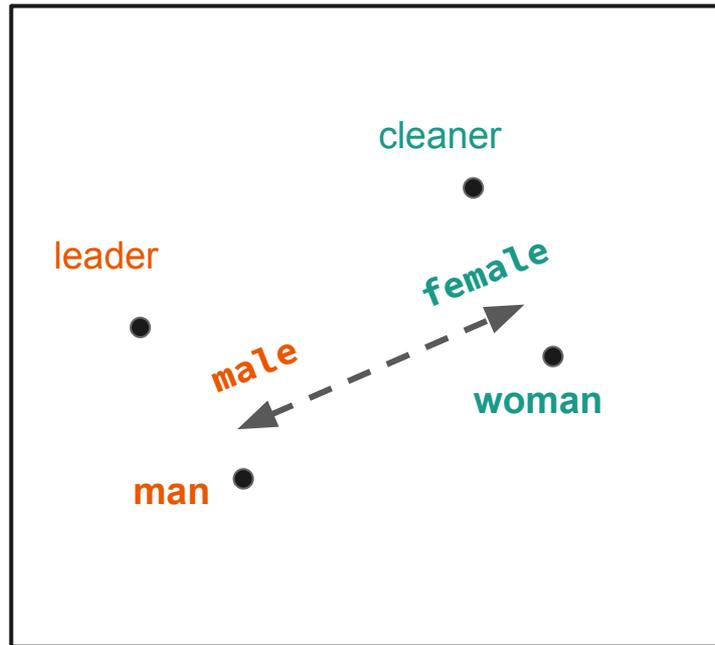
noun **man**

1. ~~the person~~ who leads or commands a group, organization, or country.
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synonyms: chief, head, principal, boss; [More](#)

cleaning
/'klēniŋ/ 

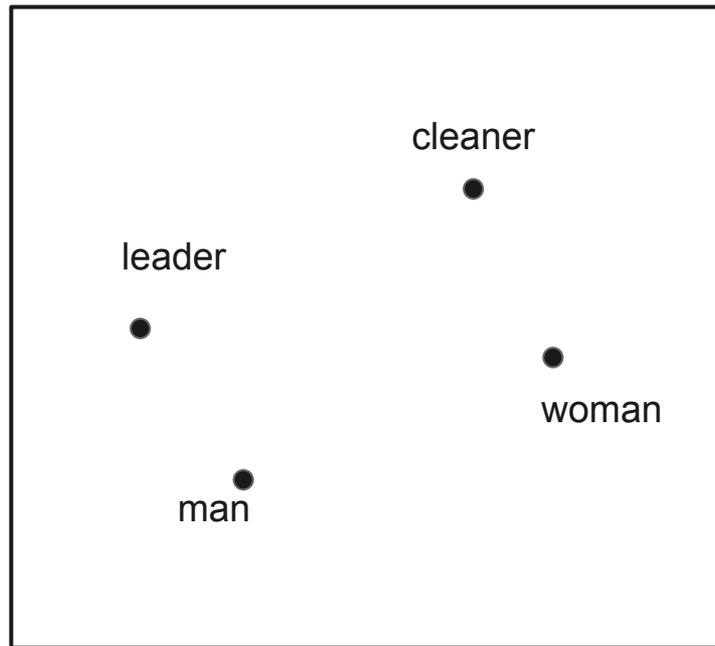
noun
noun: cleaning **a woman**

the action of making something clean, especially the inside of a house.
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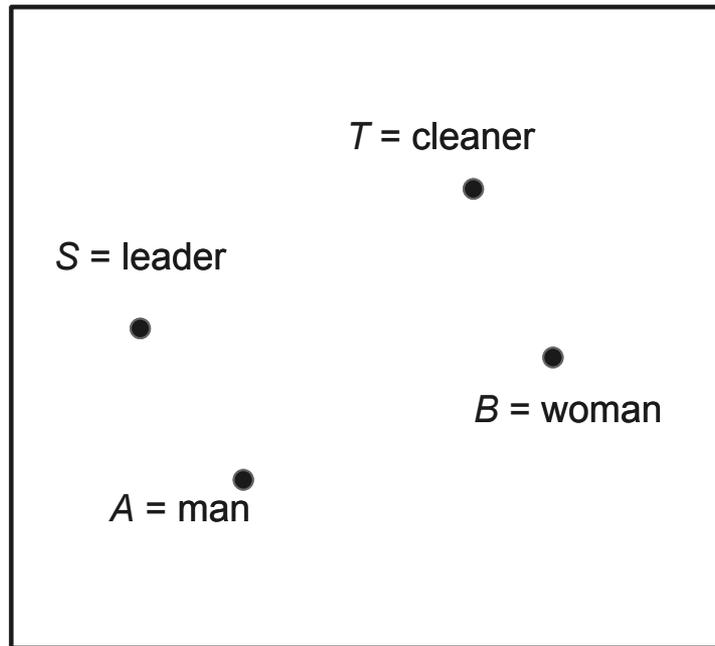
Measuring Bias in Word Embeddings

How can we **measure** bias in word embeddings?



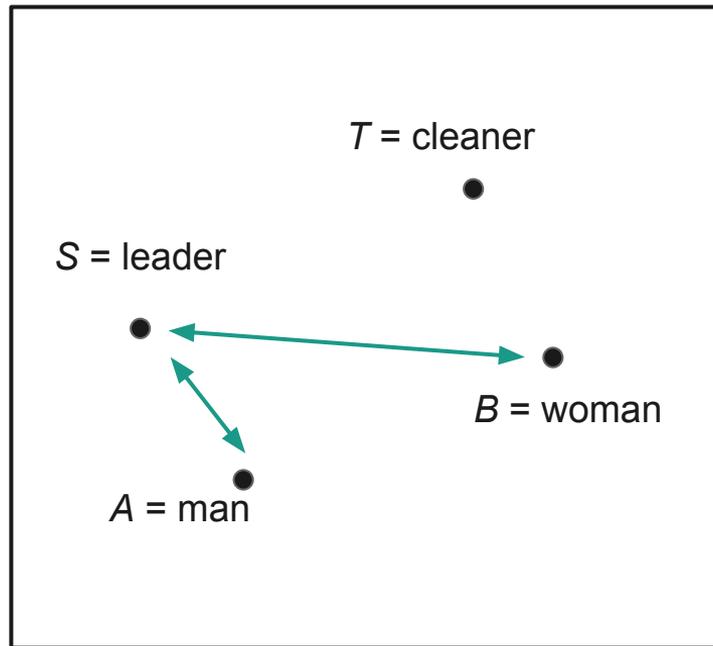
Measuring Bias in Word Embeddings

Implicit Association Test
(IAT)



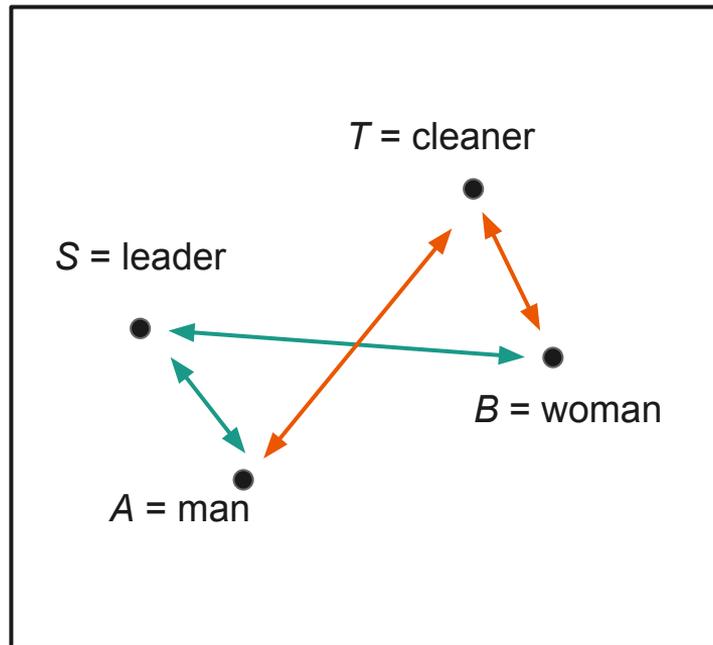
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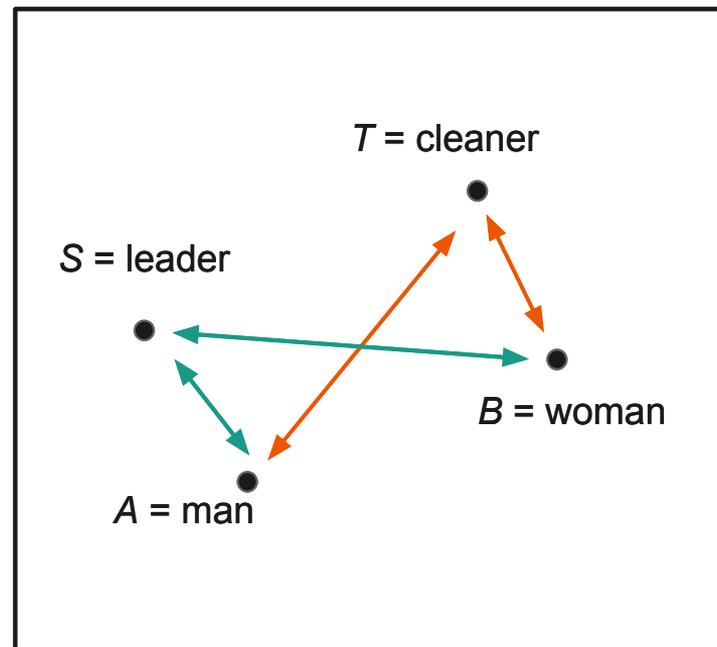
Measuring Bias in Word Embeddings

Implicit **A**ssociation Test
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Word **E**mbedding **A**ssociation Test
(WEAT)

$$\text{Association}_{S,A} \approx \sum_{S,A} \cos(s,a)$$



Measuring Bias

WEAT on popular corpora matches IAT study results

Target Words	Attribute Words	IAT		WEAT	
		effect size	p-val	effect size	p-val
Flowers v.s. Insects	Pleasant v.s. Unpleasant	1.35	1.0E-08	1.5	1.0E-07
Math v.s. Arts	Male v.s. Female Terms	0.82	1.0E-02	1.06	1.8E-02
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Aylin Caliskan, Joanna J. Bryson, Arvind Narayanan (Science 2017)

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“Semantics derived automatically from language corpora
contain human-like biases”

Aylin Caliskan, Joanna J. Bryson, Arvind Narayanan (Science 2017)

Background

➤ **Method Overview**

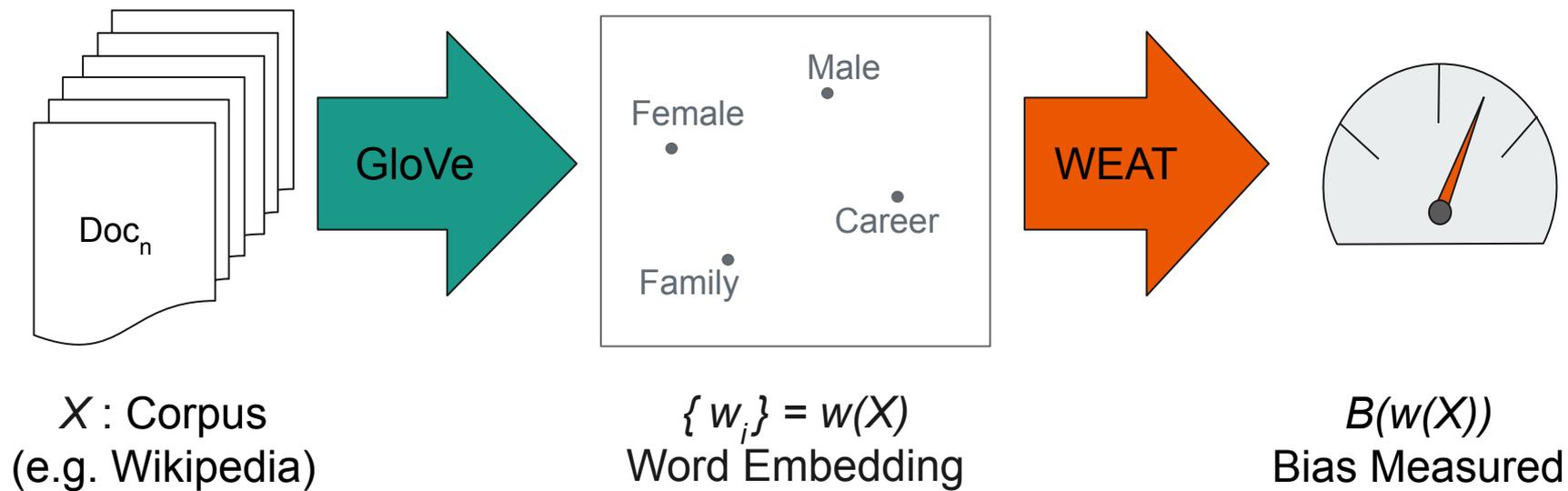
Critical Details

Experiments

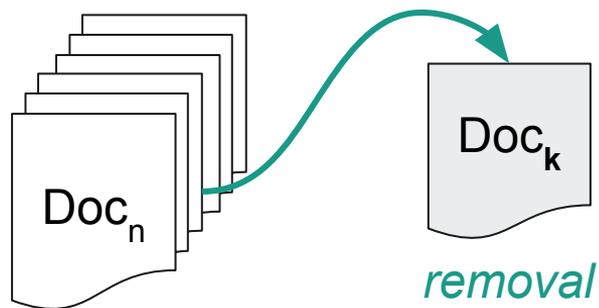


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From Word2Bias



Differential Bias



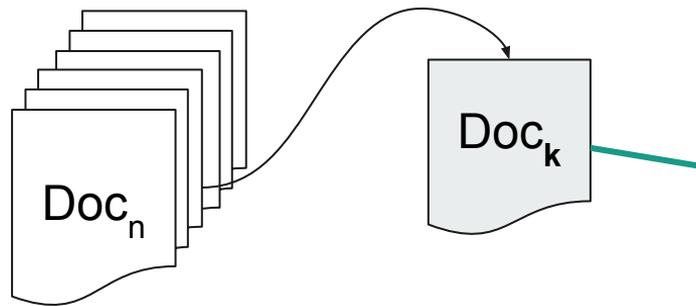
$$X = \sum_{i=1}^n X^{(i)} \quad \tilde{X} = X - X^{(k)}$$

Idea: Consider the differential contribution of each document



$$\Delta B = B(w(X)) - B(w(\tilde{X}))$$

Differential Bias



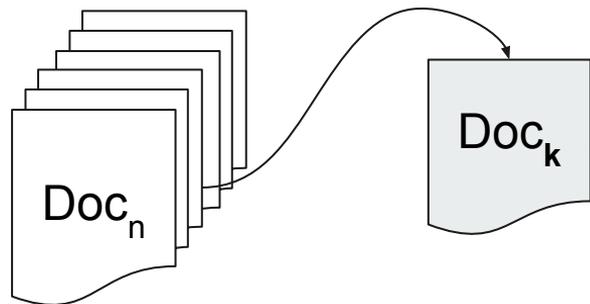
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Bias Attributed

Document ID	ΔB
1	-0.0014
2	0.0127
...	...
k	0.0374
...	...
n	0.0089

Differential Bias

Analyse Metadata?

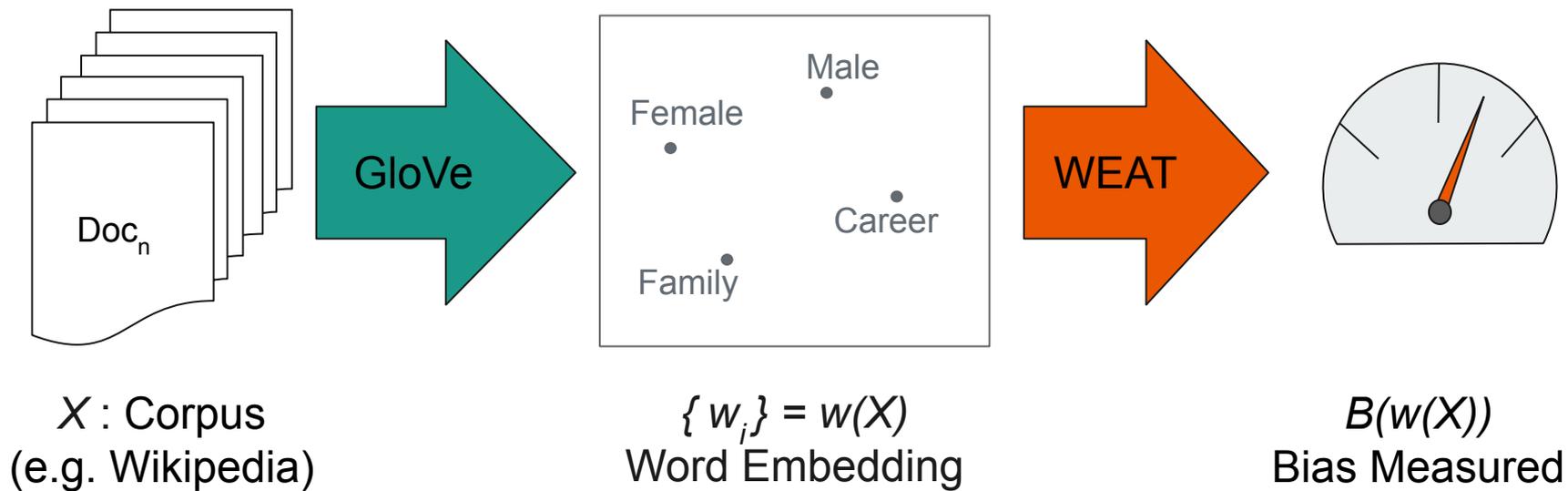


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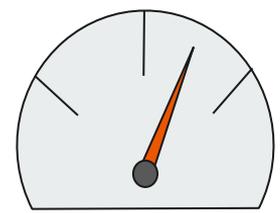
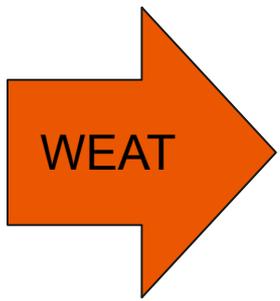
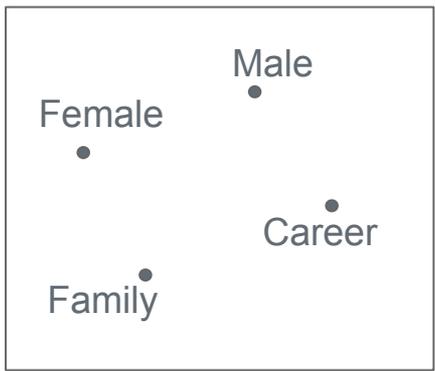
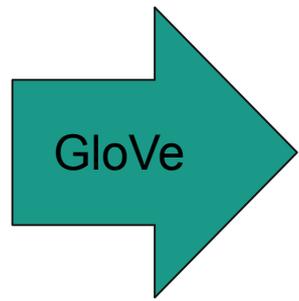
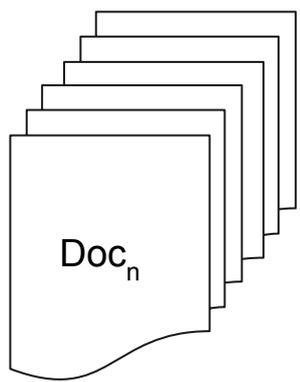
Bias Gradient

$$\nabla_X B(w(X)) = \nabla_w B(w) \nabla_X w(X)$$



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X : Corpus
(e.g. Wikipedia)

$\{w_i\} = w(X)$
Word Embedding

$B(w(X))$
Bias Measured

Background

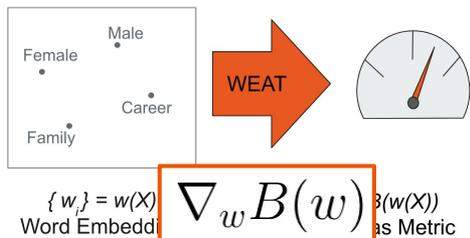
Method Overview

> **Critical Details**

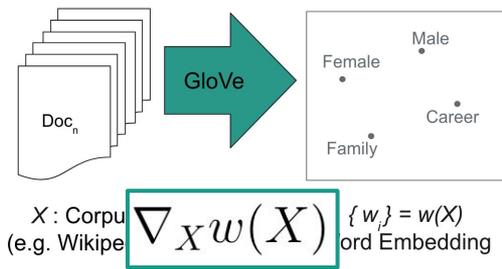
Experiments



Computing the Components



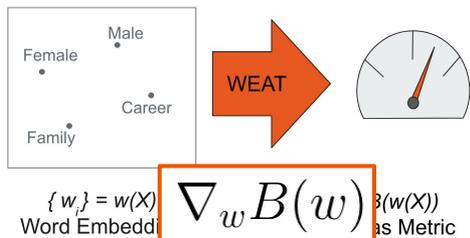
Fast & Easy: Math, Automatic Differentiation, or two evaluations of $B(w)$.



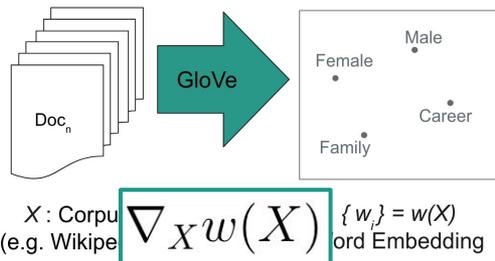
Slow & Hard: Differentiate through an entire training procedure:

- Leave-one-out retraining? (*time-bound*)
- Backprop? (*memory-bound*)
- Approximate using **Influence Functions**
Koh & Liang (ICML 2017)

Computing the Components



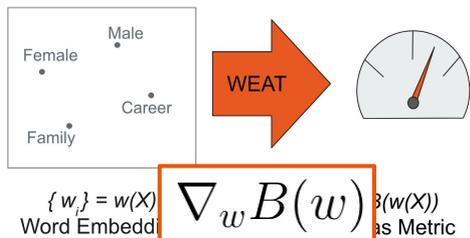
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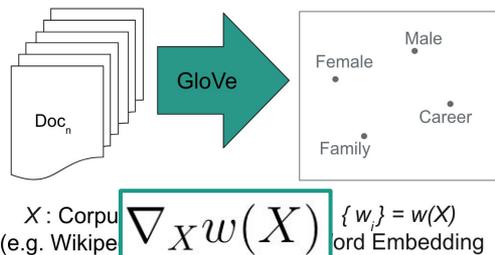
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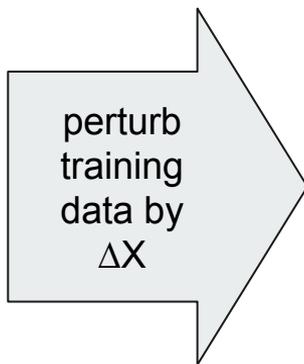
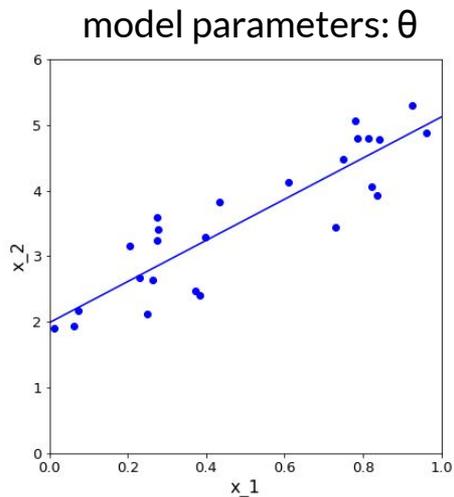


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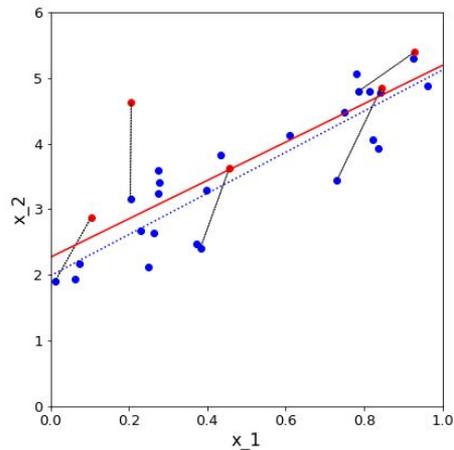
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Influence Functions

Give us a way to approximate the change in model parameters



new model params: $\tilde{\theta} \approx \text{infl_func}(\theta, \Delta X)$



Influence Functions

$$\tilde{\theta} \approx \theta^* - \underbrace{\frac{1}{n} H_{\theta^*}^{-1}}_{\text{Inverse Hessian}} \sum_{k \in \delta} [\nabla_{\theta} L(\tilde{z}_k, \theta^*) - \nabla_{\theta} L(z_k, \theta^*)]$$

Inverse Hessian
(GloVe: **2VD** x **2VD** matrix)

$$V = |\text{vocab}| \quad w_i \in \mathbb{R}^D$$

2VD can easily be > **10⁹**

Applying Influence Functions to GloVe

GloVe
Loss:

$$J(X, w, u, b, c) = \sum_{i=1}^V \sum_{j=1}^V f(X_{ij})(w_i^T u_j + b_i + c_j - \log X_{ij})^2$$

word vectors \nearrow

other params
(**treat as const**)

Applying Influence Functions to GloVe

Gradient of
Pointwise
Loss

$$\nabla_w L(X_i, w) = \left(\underbrace{0, \dots, 0}_{D(i-1)}, \underbrace{\nabla_{w_i} L(X_i, w)}_D, \underbrace{0, \dots, 0}_{D(V-i)} \right)$$

VD dimensions

Hessian becomes **block diagonal!**

(V Blocks of D by D)

Allows us to apply influence function approximation to **one word vector at a time!**

Algorithm: Compute Differential Bias

$w^*, u^*, b^*, c^* = \text{GloVe}(X)$ # Train embedding
for doc **in** corpus **do**
 $\tilde{X} = X - X^{(k)}$ # Subtract coocs from doc k
 for word i **in** doc \cap WEAT words
 # Only need change in WEAT word vectors
 $\tilde{w}_i = w_i^* - \frac{1}{V} H_{w_i}^{-1} [\nabla_{w_i} L(\tilde{X}_i, w) - \nabla_{w_i} L(X_i, w)]$
 end for
 $\Delta_{\text{doc}} B \approx B_{\text{weat}}(w^*) - B_{\text{weat}}(\tilde{w})$
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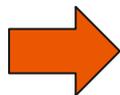
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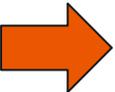
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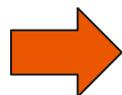
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> Experiments





Objectives of Experiments

1. Assess the accuracy of our influence function approximation
2. Identify and analyse most bias impacting documents

WEAT

S = Science	T = Arts
A = Male	B = Female

S = Instruments	T = Weapons
A = Pleasant	B = Unpleasant

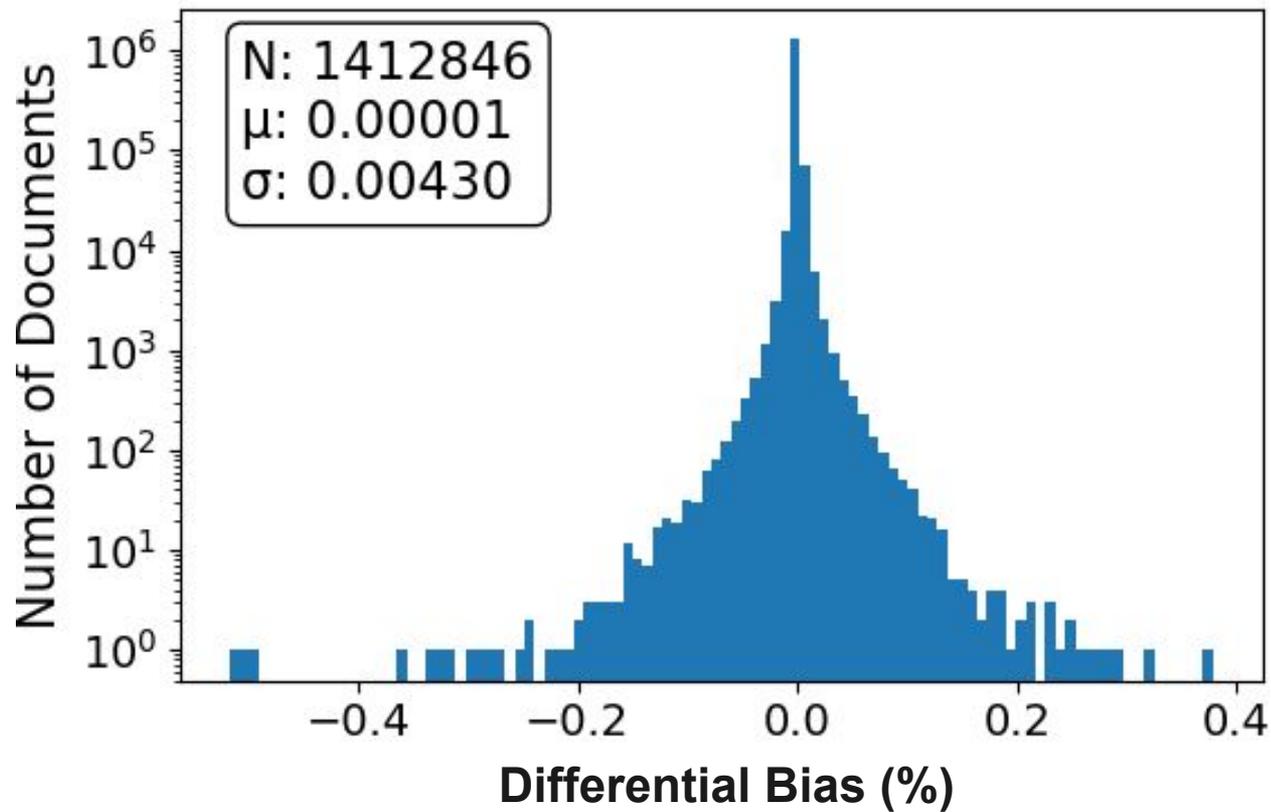
Corpora



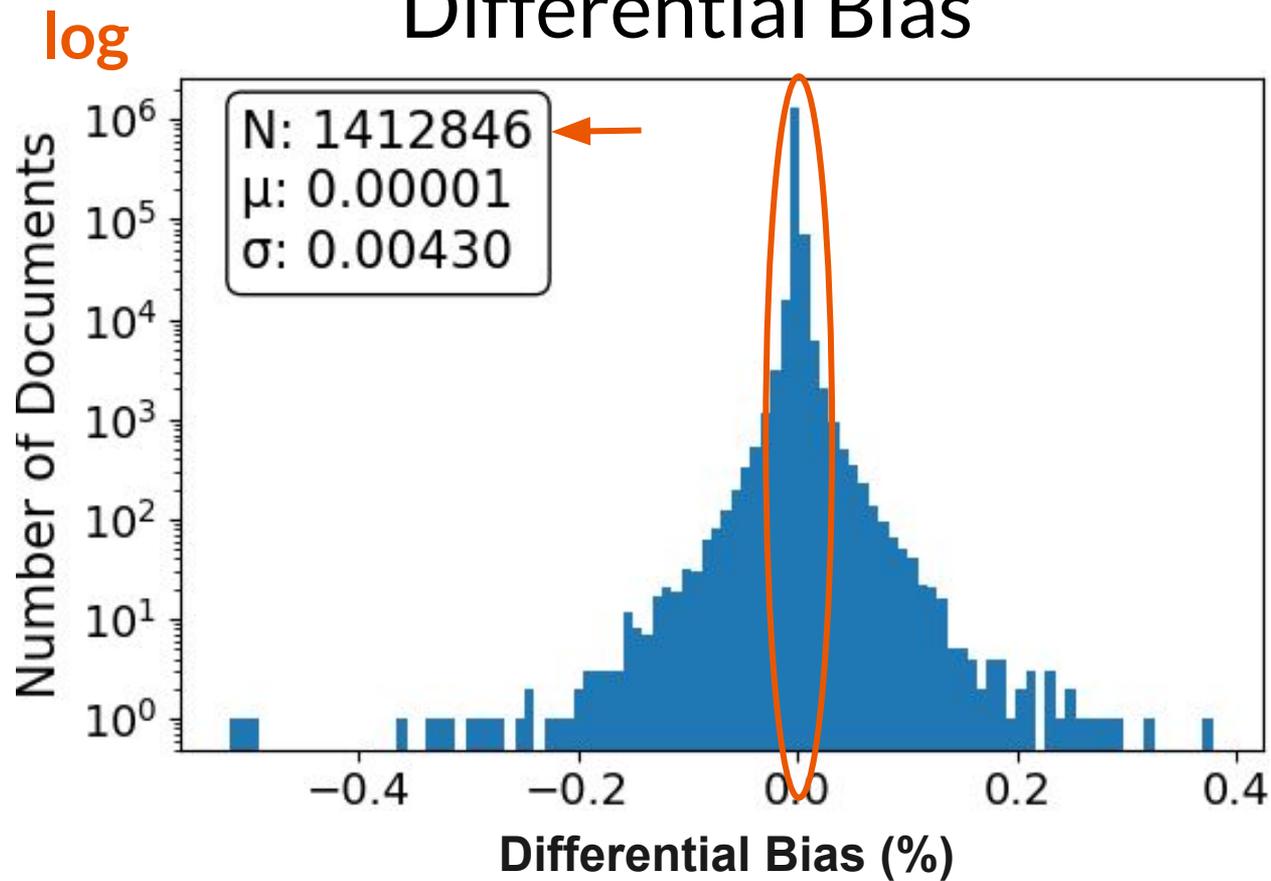
WIKIPEDIA
The Free Encyclopedia

The image shows the logo for The New York Times, consisting of the words "The New York Times" in a white, traditional gothic-style font, centered within a solid black square. The entire logo is enclosed in an orange oval.

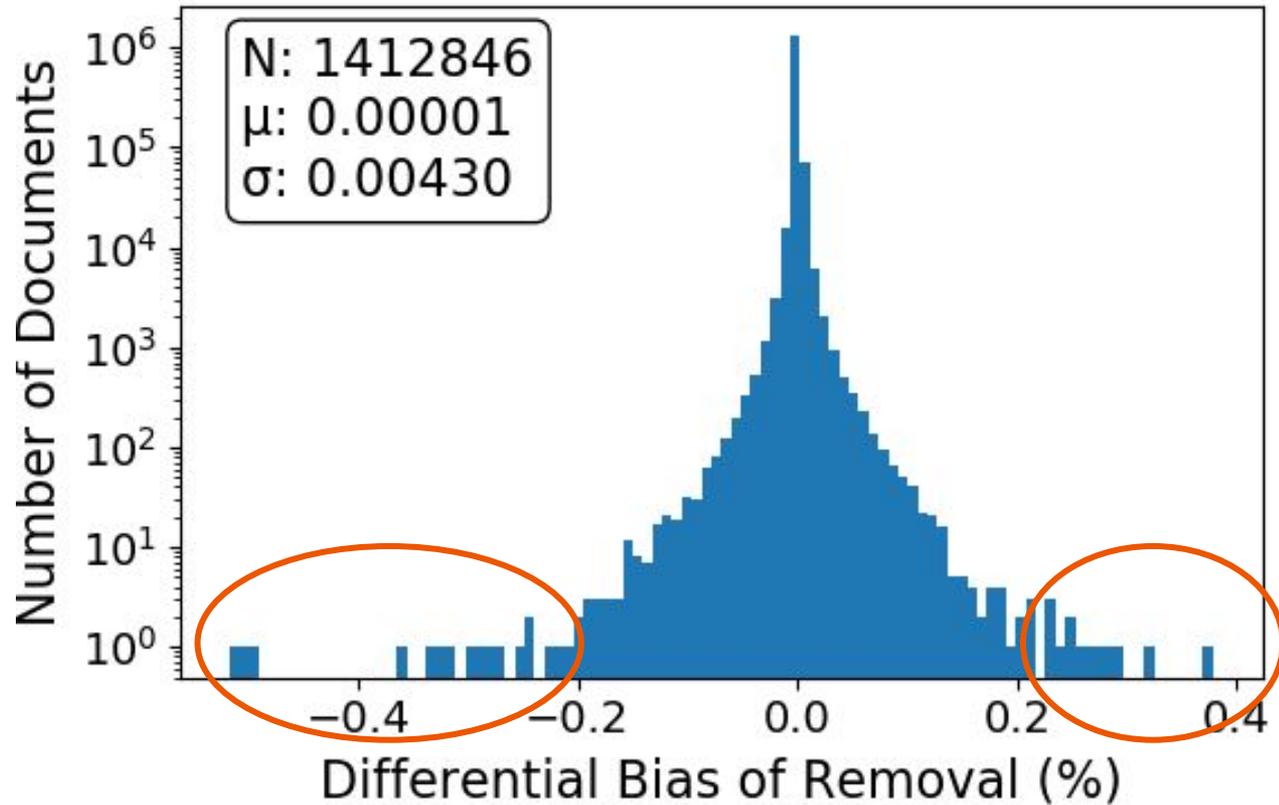
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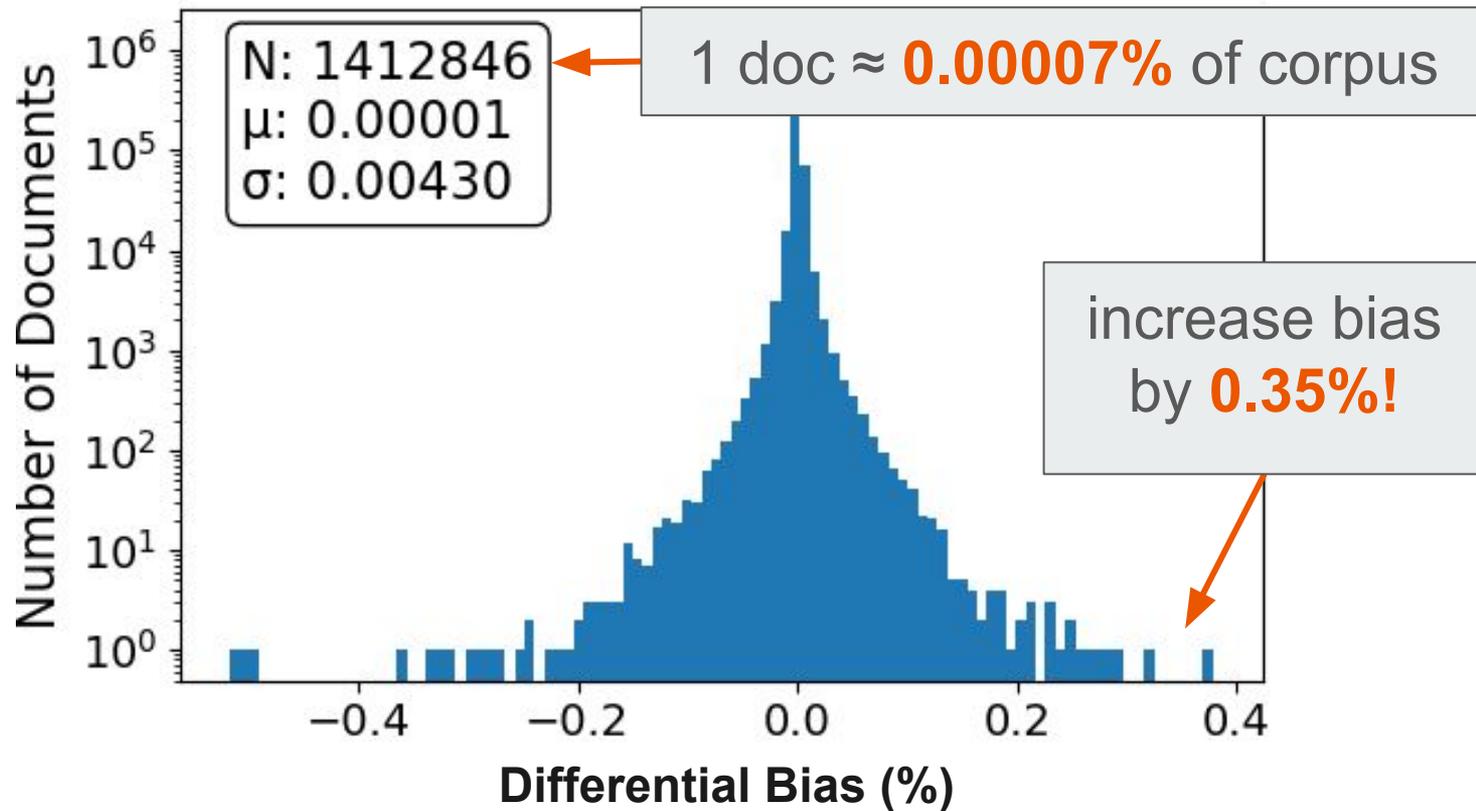
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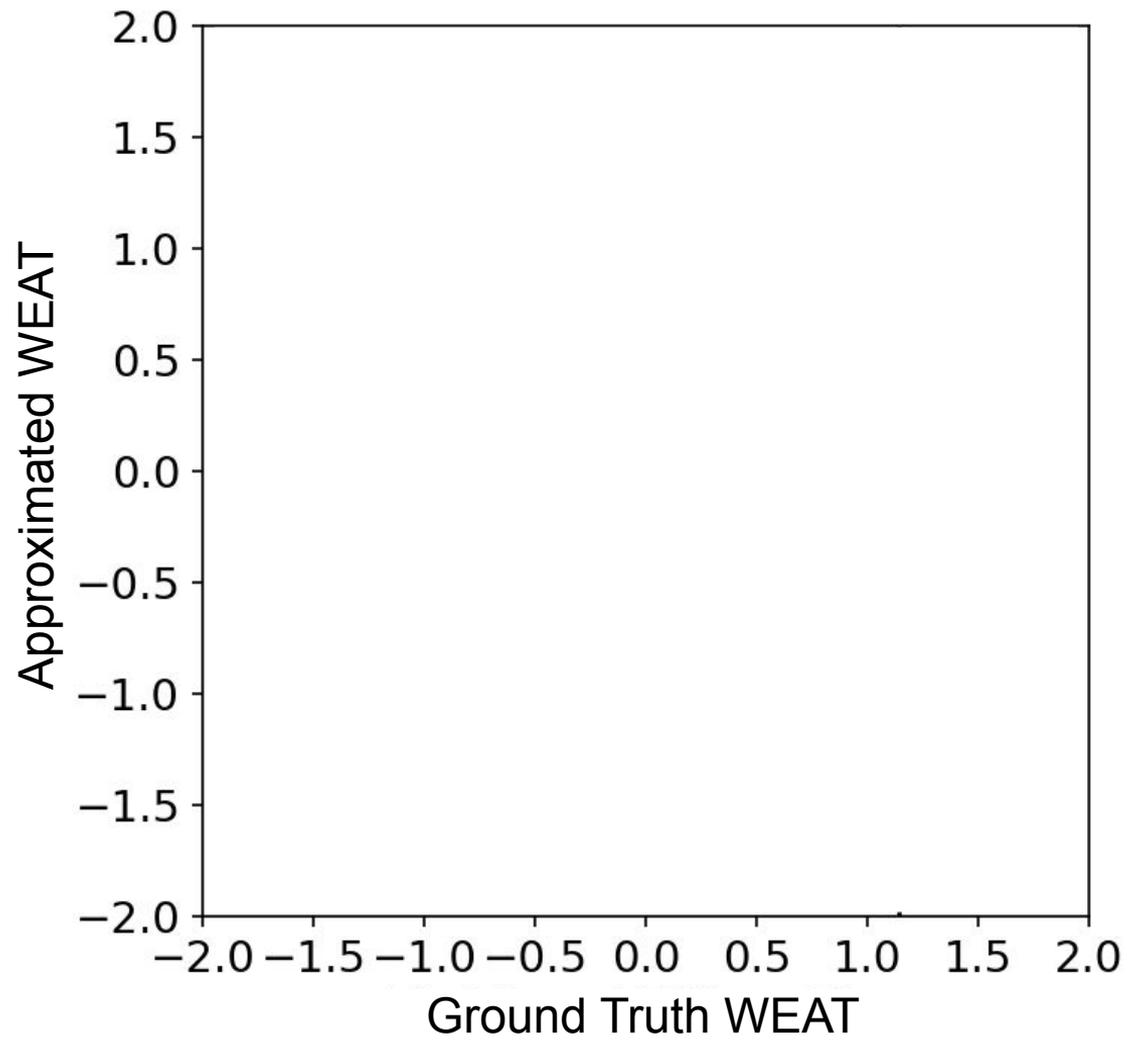


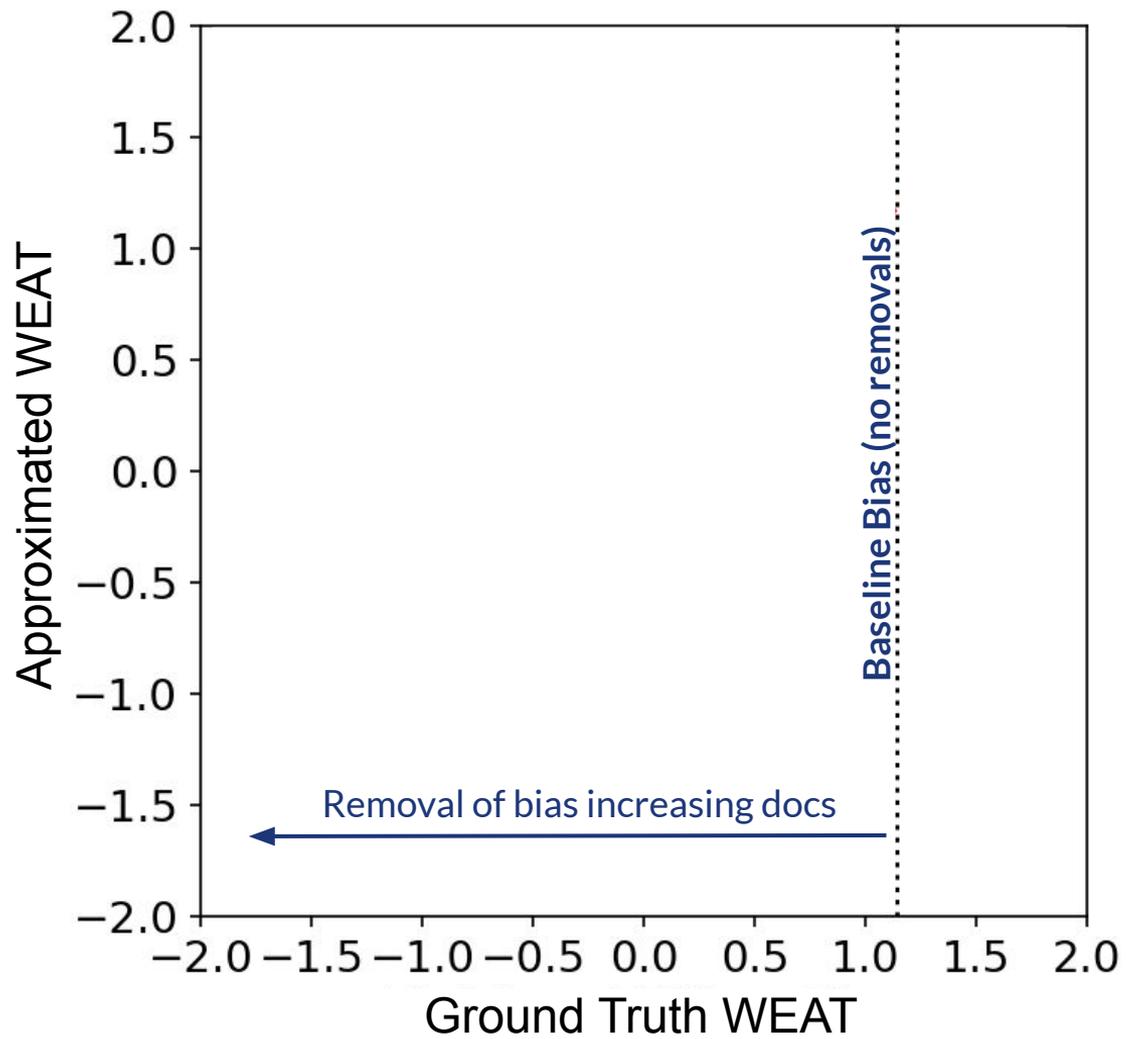
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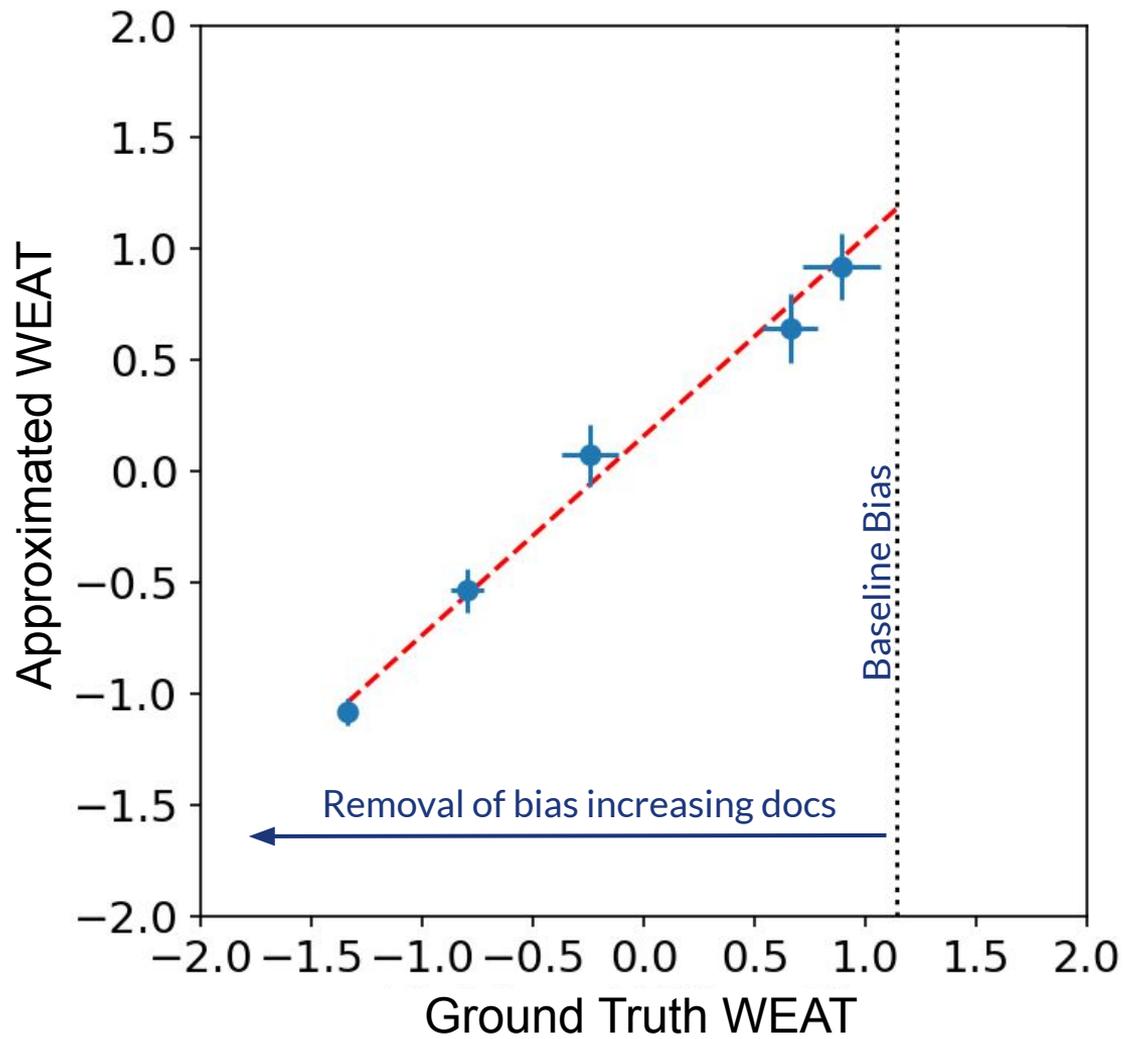


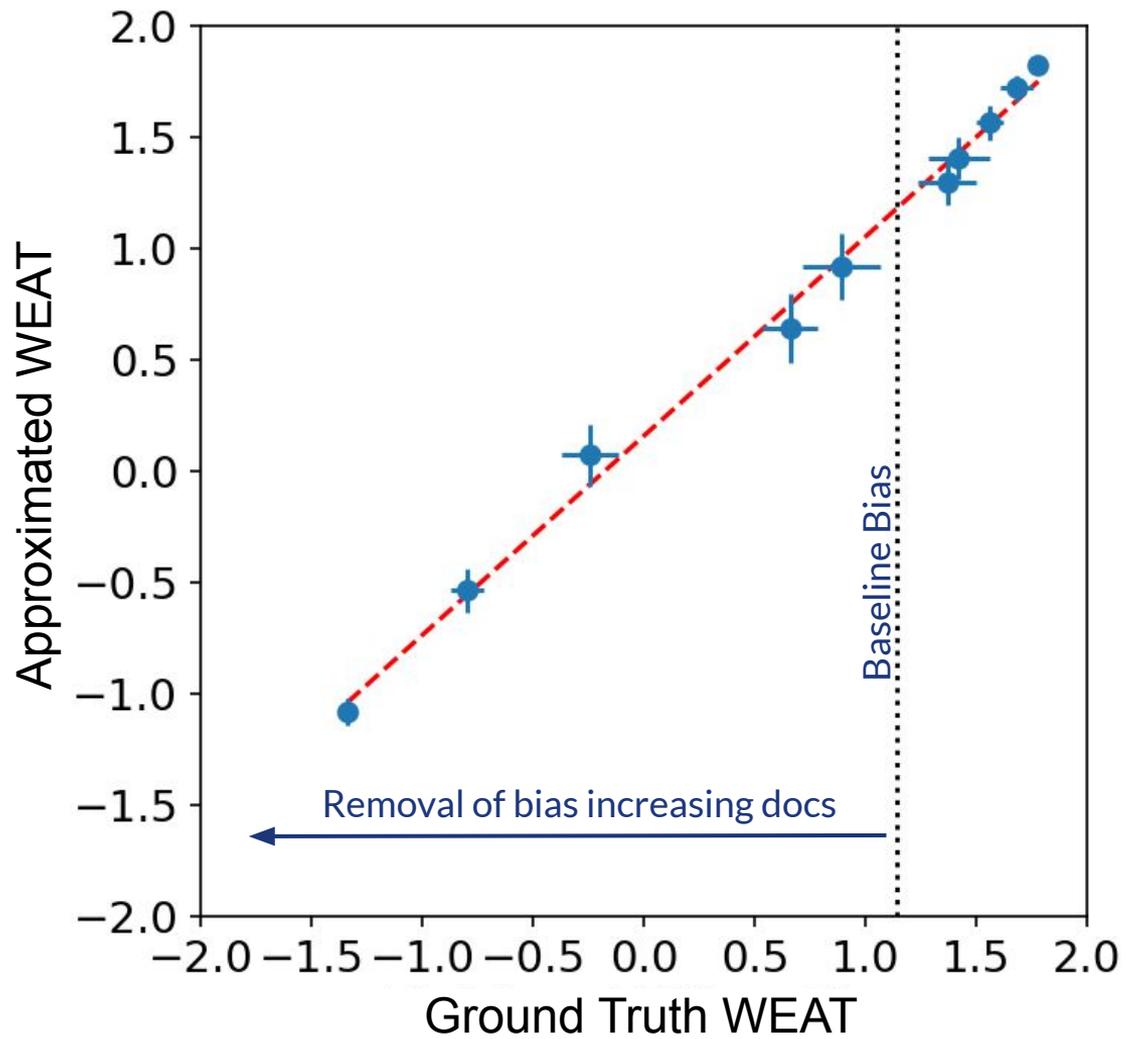
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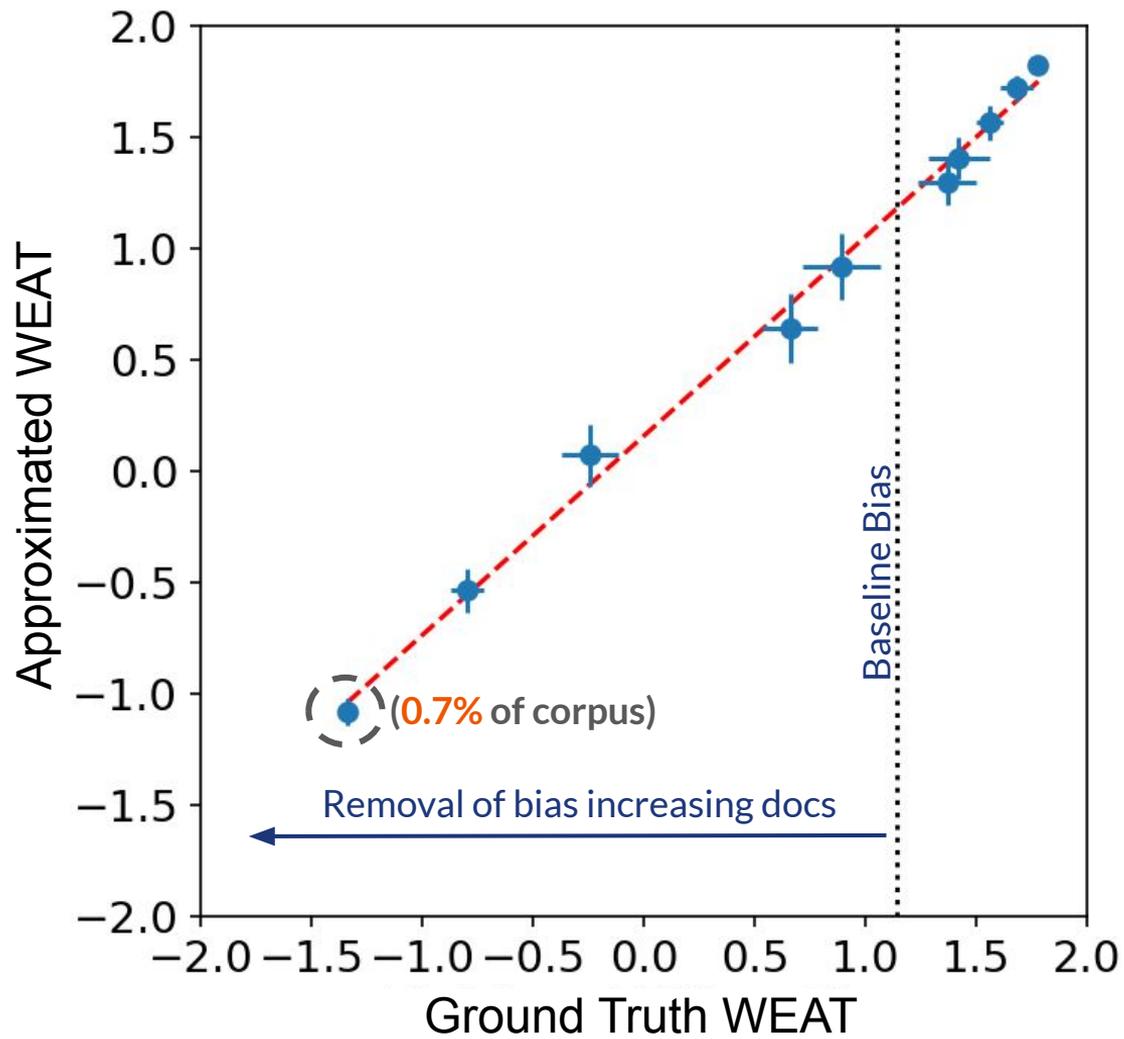




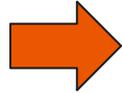








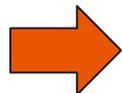
$\Delta_d B$	Bias Decreasing
-0.52	Hormone Therapy Study Finds Risk for Some
-0.50	For Women in Astronomy, a Glass Ceiling in the Sky
-0.49	Sorting Through the Confusion Over Estrogen
-0.36	Young Astronomers Scan Night Sky and Help Wanted Ads
$\Delta_d B$	Bias Increasing
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$\Delta_d B$	Bias Increasing
0.38	Kaj Aage Strand, 93, Astronomer At the U.S. Naval Observatory
0.32	Gunman in Iowa Wrote of Plans In Five Letters
0.29	ENGINEER WARNED ABOUT DIRE IMPACT OF LIFTOFF DAMAGE
0.29	Fred Gillett, 64; Studied Infrared Astronomy
0.27	Robert Harrington, 50, Astronomer in Capital



Document Impact Generalizes

WEAT₁ (Science v.s. Arts Gender Bias)

	remove bias increasing docs	baseline (no removals)	remove bias decreasing docs
GloVe	-1.27	1.14	1.7
word2vec	0.11	1.35	1.6

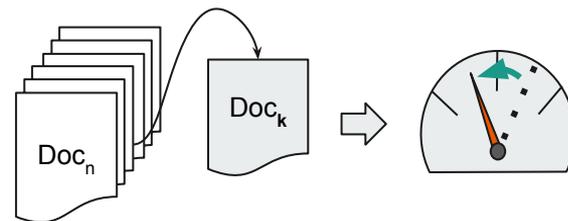
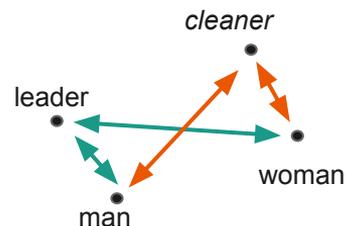
Removal of documents also **affects word2vec**, and other metrics!

Limitations & Future Work

- Consider **multiple biases** at simultaneously
- Use metrics that depend on **more words**
- Consider bias in **downstream tasks** where embeddings are used
- Does this carry over to **BERT**?

Recap

- Bias can be quantified; correlates with known human biases
- We can identify the documents that most impact bias, and approximate impact
- These documents are qualitatively meaningful, and impact generalizes



$\Delta_d B$	Bias Decreasing
-0.52	Hormone Therapy Study Finds Risk for Some
-0.50	For Women in Astronomy, a Glass Ceiling in the Sky
-0.49	Sorting Through the Confusion Over Estrogen
-0.36	Young Astronomers Scan Night Sky and Help Wanted Ads

Thank you!

Poster # 146

mebrunet@cs.toronto.edu

arXiv: 1810.03611



Marc



Colleen



Ashton



Rich



References

- T. Bolukbasi, K.-W. Chang, J. Zou, V. Saligrama, and A. Kalai. Man is to computer programmer as woman is to homemaker? debiasing word embeddings. In 30th Conference on Neural Information Processing Systems (NIPS), 2016.
- A. Caliskan, J. J. Bryson, and A. Narayanan. Semantics derived automatically from language corpora contain human-like biases. *Science*, 356(6334):183–186, 2017.
- P. W. Koh and P. Liang. Understanding Black-box Predictions via Influence Functions. In Proceedings of the 34th International Conference on Machine Learning, volume 70 of Proceedings of Machine Learning Research, pages 1885–1894, 2017.

Measuring Bias



“...results raise the possibility that **all** implicit human **biases** are **reflected in** the statistical properties of **language.**”

Aylin Caliskan, Joanna J. Bryson, Arvind Narayanan (Science 2017)

Impact on Word2Vec



Removal of Documents Identified by our Method

	Decrease (0.7%)	Baseline	Increase (0.7%)
GloVe	-1.27	1.14	1.7
word2vec	0.11	1.35	1.6

Word Embeddings

Compact vector representation
(like a dictionary for machines)

Learned from **LARGE** corpora.



Used in many NLP tasks:

- Sentiment Analysis
- Text summarization
- Machine Translation

dic·tion·ar·y

/ˈdɪkʃənəri/ 

noun

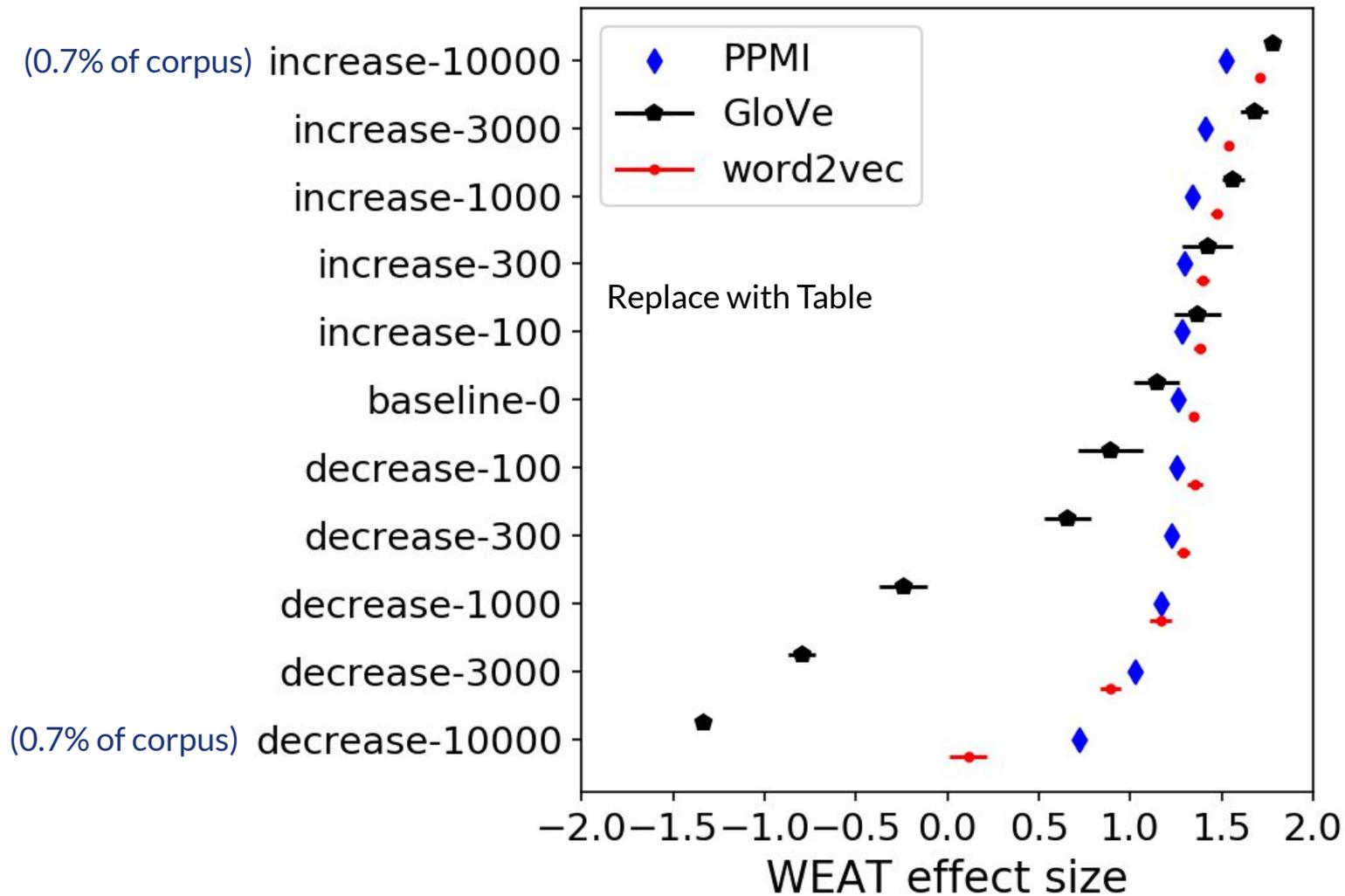
a book or electronic resource that lists the words of a language (typically in alphabetical order) and gives their meaning, or gives the equivalent words in a different language, often also providing information about pronunciation, origin, and usage.

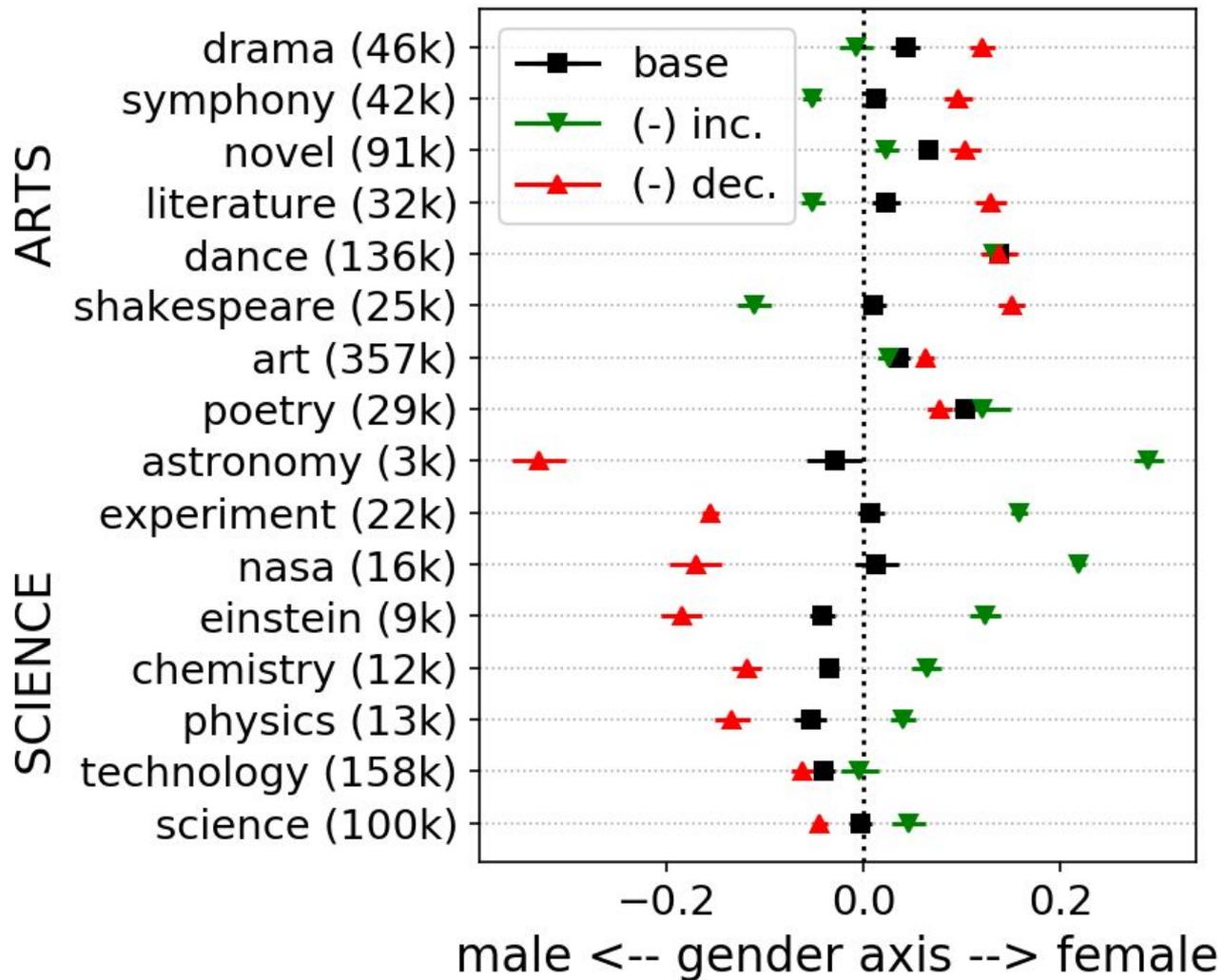
"I'll look up 'love' in the dictionary"

synonyms: [lexicon](#), [wordbook](#), [glossary](#), [vocabulary list](#), [vocabulary](#), [word list](#), [wordfinder](#)

"half of the words in his text were not in the dictionary"

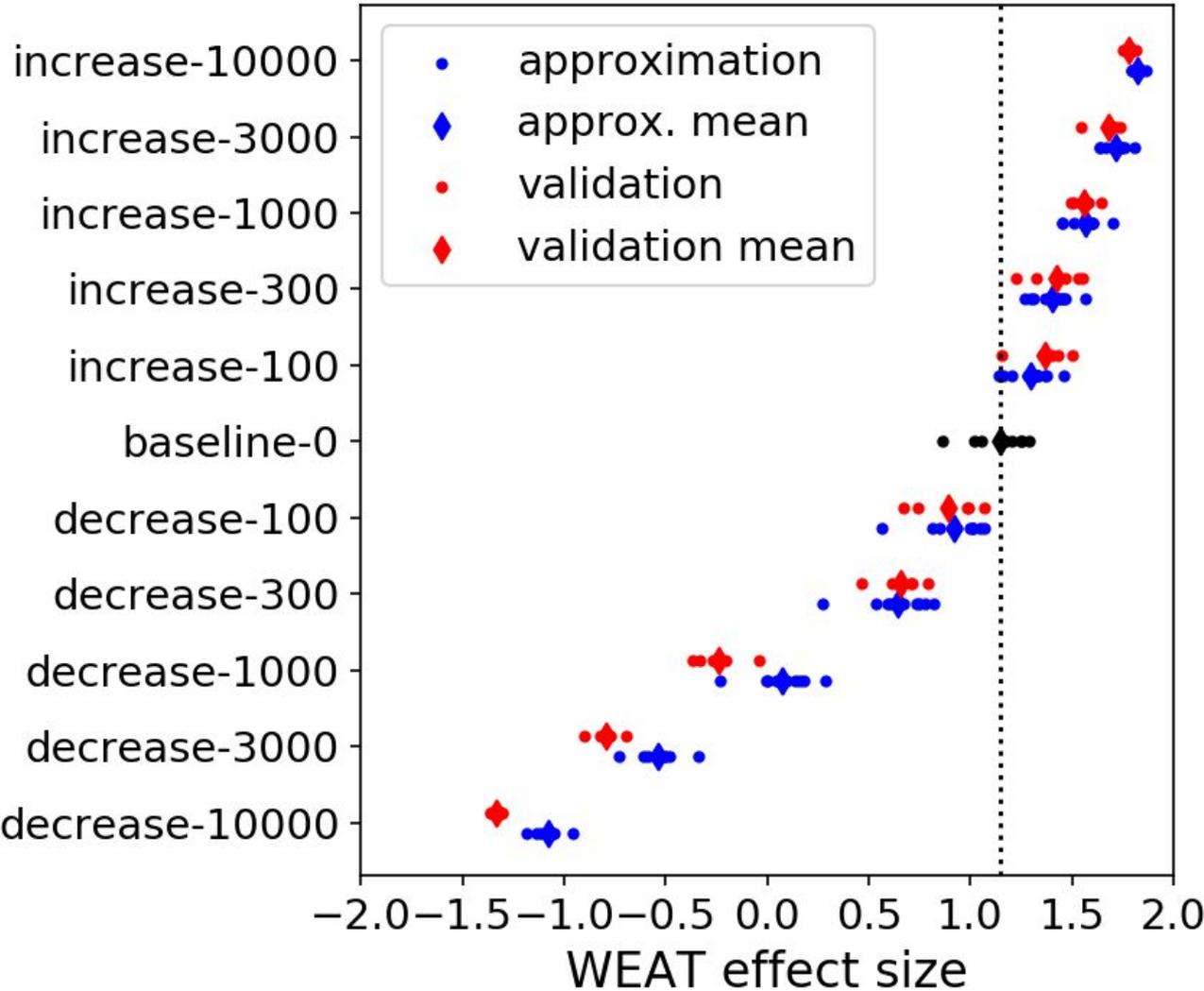
```
{  
  "dictionary": [1.23, -0.52, 1.01, -2.14 ... ],  
  "dictionally": [1.33, -0.48, 0.98, -2.33 ... ],  
  "dictionaries": [1.04, -0.63, 0.87, -2.23 ... ],  
  ...  
}
```





(0.7% of corpus)

(0.7% of corpus)



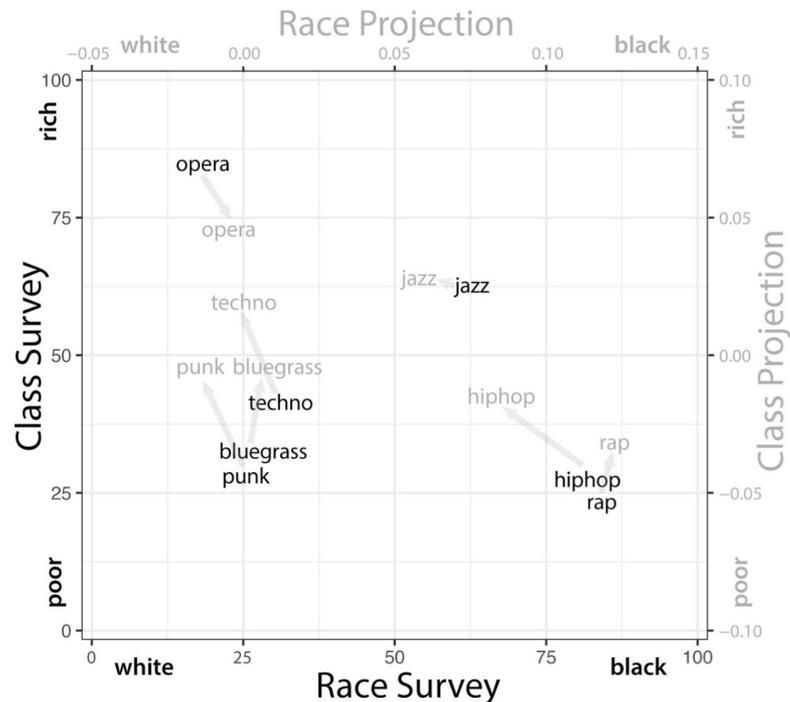
S	science	science, technology, physics, chemistry, einstein, nasa, experiment, astronomy
T	arts	poetry, art, shakespeare, dance, literature, novel, symphony, drama
A	male	male, man, boy, brother, he, him, his, son
B	female	female, woman, girl, sister, she, her, hers, daughter

Psychology, Bias, and Embeddings

One study examined a dozen well-known human biases: **all present**

Others examined the **geometry** of

- Class
- Race
- Gender



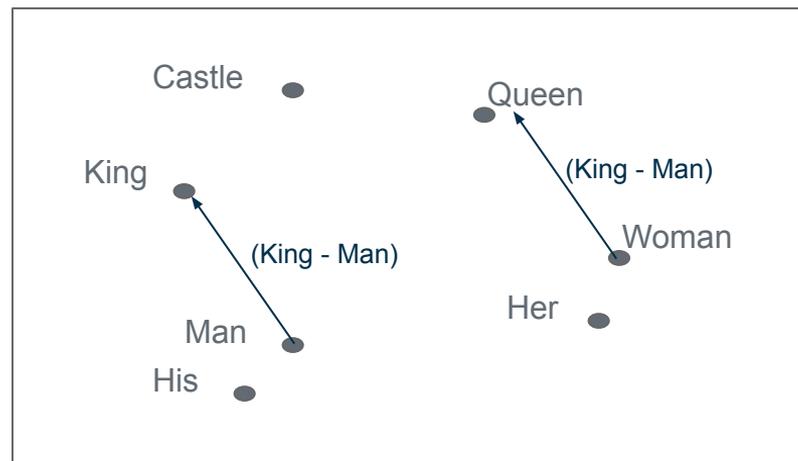
Word Embeddings

What are they?

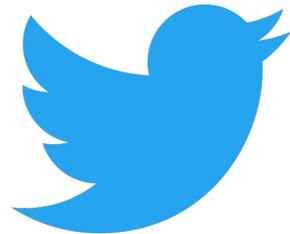
- A compact vector representation for words
- Learned from a very large corpus of text
- Preserves syntactic and semantic meaning through vector arithmetic (**very useful**)

Applications:

- Sentiment analysis
- Document classification / summarization
- Translation
- Temporal semantic trajectories



A Motivating Example



“She is actually a good leader. He is just pretty.”
#NoPlanetB



Presumptuous Translation

Translate

Turn on instant translation



Armenian English French Detect language ▾



English Armenian French ▾

Translate

She is actually a good leader. ✕
He is just pretty.



49/5000

Presumptuous Translation

Translate

Turn on instant translation



Armenian English French Detect language ▾



English Armenian French ▾

Translate

She is actually a good leader. ✕
He is just pretty.



49/5000

Նա իրականում լավ առաջնորդ է:

Նա պարզապես գեղեցիկ է:



Presumptuous Translation

Translate

Turn on instant translation



Armenian English French Detect language



English Armenian French

Translate

Նա իրականում լավ առաջնորդ է:
Նա պարզապես գեղեցիկ է:

51/5000

He is really a good leader.
She's just beautiful.

Translate

Turn on instant translation



Armenian English French Detect language ▾



English Armenian French ▾

Translate

He is a nurse.
She is an engineer.



34/5000

Նա բուժքույր է:
Նա ինժեներ է:



Translate

Turn on instant translation



Armenian English French Detect language ▾



English Armenian French ▾

Translate

Նա բուժքույր է:
Նա ինժեներ է:



29/5000

She is a nurse.
He is an engineer.



Why does this happen?

Translate

Turn on instant translation



Armenian English French Detect language



English Armenian French

Translate

He is a nurse.
She is an engineer.



34/5000

Նա բուժքույր է:
Նա ինժեներ է:



Translate

Turn on instant translation



Armenian English French Detect language



English Armenian French

Translate

Նա բուժքույր է:
Նա ինժեներ է:



29/5000

She is a nurse.
He is an engineer.





Word Co-Occurrences

	engineer	nurse	leader	pretty	(all)
Ratio of he:she co-occurrences	6.25	0.550	9.25	3.07	3.53

The New York Times Annotated Corpus (1987-2007, approx. 1B words, context window: 8)

GloVe: Global Vectors for Word Representations

$$J(X, w, u, b, c) = \sum_{i=1}^V \sum_{j=1}^V f(X_{ij})(w_i^T u_j + b_i + c_j - \log X_{ij})^2$$

X: co-occurrence Matrix

$\{w_i\}$: set of word vectors

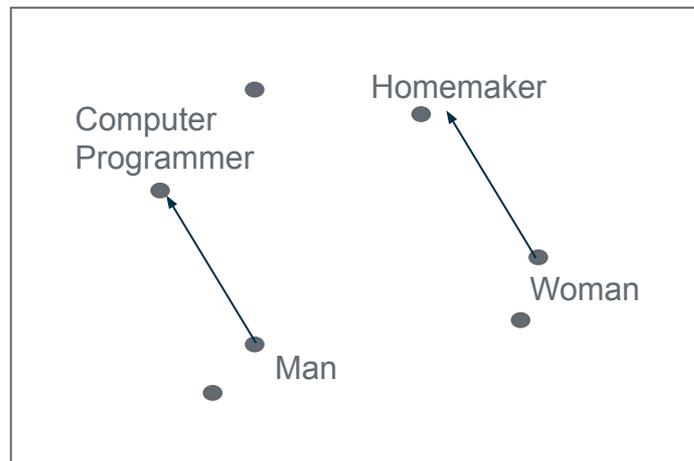
$\{u_j\}, b, c$: other model parameters

$$f(x) = \begin{cases} (x/x_{max})^\alpha & \text{if } x < x_{max} \\ 1 & \text{otherwise} \end{cases}$$

Jeffrey Pennington, Richard Socher, and Christopher D. Manning. 2014.

Bad Analogies

- 😊 King : Man :: Queen : Woman
- 😊 Paris : France :: London : England
- 😞 Man : Computer_Programmer :: Woman : Homemaker



Tolga Bolukbasi, Kai-Wei Chang, James Zou, Venkatesh Saligrama, Adam Kalai (NeurIPS 2016)

WEAT

Target Word Sets:

S = {physics, chemistry...} \approx *Science*

T = {poetry, literature...} \approx *Arts*

Attribute Word Sets:

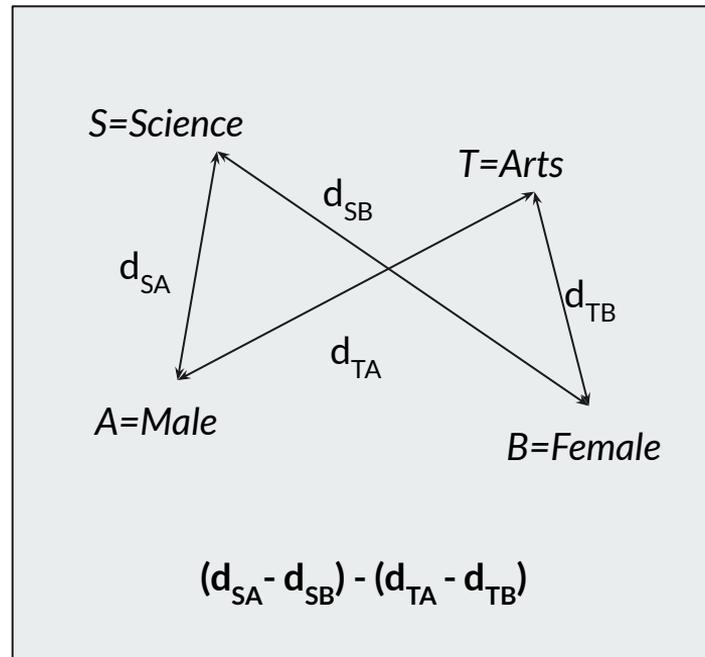
A = {he, him, man...} \approx *Male*

B = {she, her, woman} \approx *Female*

Measures relative
association between
four concepts

$$f(w, A, B) = \text{mean}_{a \in A} \cos(\vec{w}, \vec{a}) - \text{mean}_{b \in B} \cos(\vec{w}, \vec{b})$$

$$\text{Effect Size} = \frac{\text{mean}_{s \in S} f(s, A, B) - \text{mean}_{t \in T} f(t, A, B)}{\text{std-dev}_{w \in S \cup T} f(w, A, B)}$$



Applying IF to GloVe

GloVe Loss :

$$J(X, w, u, b, c) = \sum_{i=1}^V \sum_{j=1}^V f(X_{ij})(w_i^T u_j + b_i + c_j - \log X_{ij})^2$$

Our “datapoints” are NOT documents, but rather the entries of X.

So one document removal: $\tilde{X} = X - X^{(k)}$, perturbs multiple “datapoints”.

$$\text{IF Approx: } \tilde{\theta} \approx \theta^* - \frac{1}{n} H_{\theta^*}^{-1} \sum_{k \in \delta} [\nabla_{\theta} L(\tilde{z}_k, \theta^*) - \nabla_{\theta} L(z_k, \theta^*)]$$

Applying IF to GloVe

$$\tilde{w}_i \approx w_i^* - \frac{1}{V} \underbrace{H_{w_i}^{-1}}_{\text{Computed once per WEAT word}} \left[\underbrace{\nabla_{w_i} L(\tilde{X}_i, w^*)}_{\text{Computed for every perturbation of interest}} - \underbrace{\nabla_{w_i} L(X_i, w^*)}_{\text{Computed once per WEAT word}} \right]$$

Notice that for all i where $\tilde{X}_i = X_i$, $\tilde{w}_i = w_i^*$

Influence Functions (IF)

$$R(z, \theta) = \frac{1}{n} \sum_{i=1}^n L(z_i, \theta) \quad \theta^* = \operatorname{argmin}_{\theta} R(z, \theta)$$

$$\tilde{\theta} \approx \theta^* - \underbrace{\frac{1}{n} H_{\theta^*}^{-1}}_{\text{Inverse Hessian}} \sum_{k \in \delta} \underbrace{[\nabla_{\theta} L(\tilde{z}_k, \theta^*) - \nabla_{\theta} L(z_k, \theta^*)]}_{\text{Difference of Gradients}}$$

Perturbed Original

δ : Set of perturbed data points