Social and Information Networks

CSCC46H, Fall 2020

Lecture 8

Prof. Ashton Anderson
ashton@cs.toronto.edu

Today

A3 out last week, due next Thursday 2pm

Logistics

Blog posts A-H due Friday, Nov 6
Blog posts I-N due Friday, Nov 13
Blog posts O-Z due Friday, Nov 20

Today

Game Theory

First: a game!

Everyone will guess a number between 0 and 100 (inclusive), and whoever's number is closest to **2/3 of the average guess** will win!

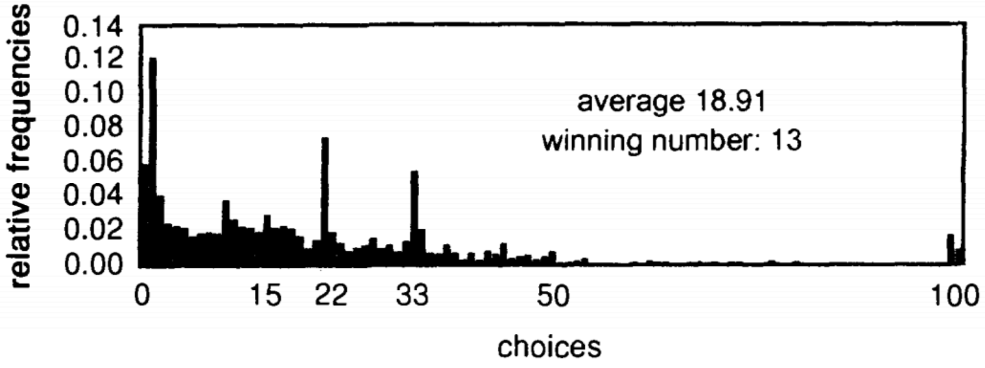
No speaking

Write down your UTORid along with a single guess

“Beauty contest” experiment in newspapers

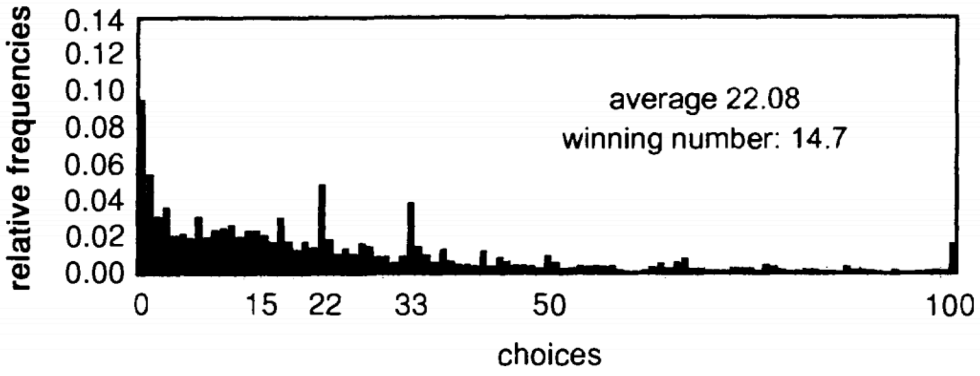
(a)

***Financial Times* experiment (1,468 subjects)**



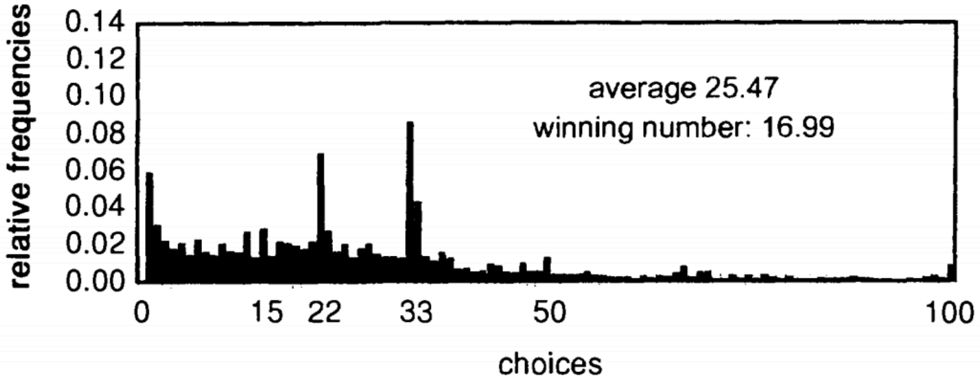
(b)

***Spektrum* experiment (2,729 subjects)**

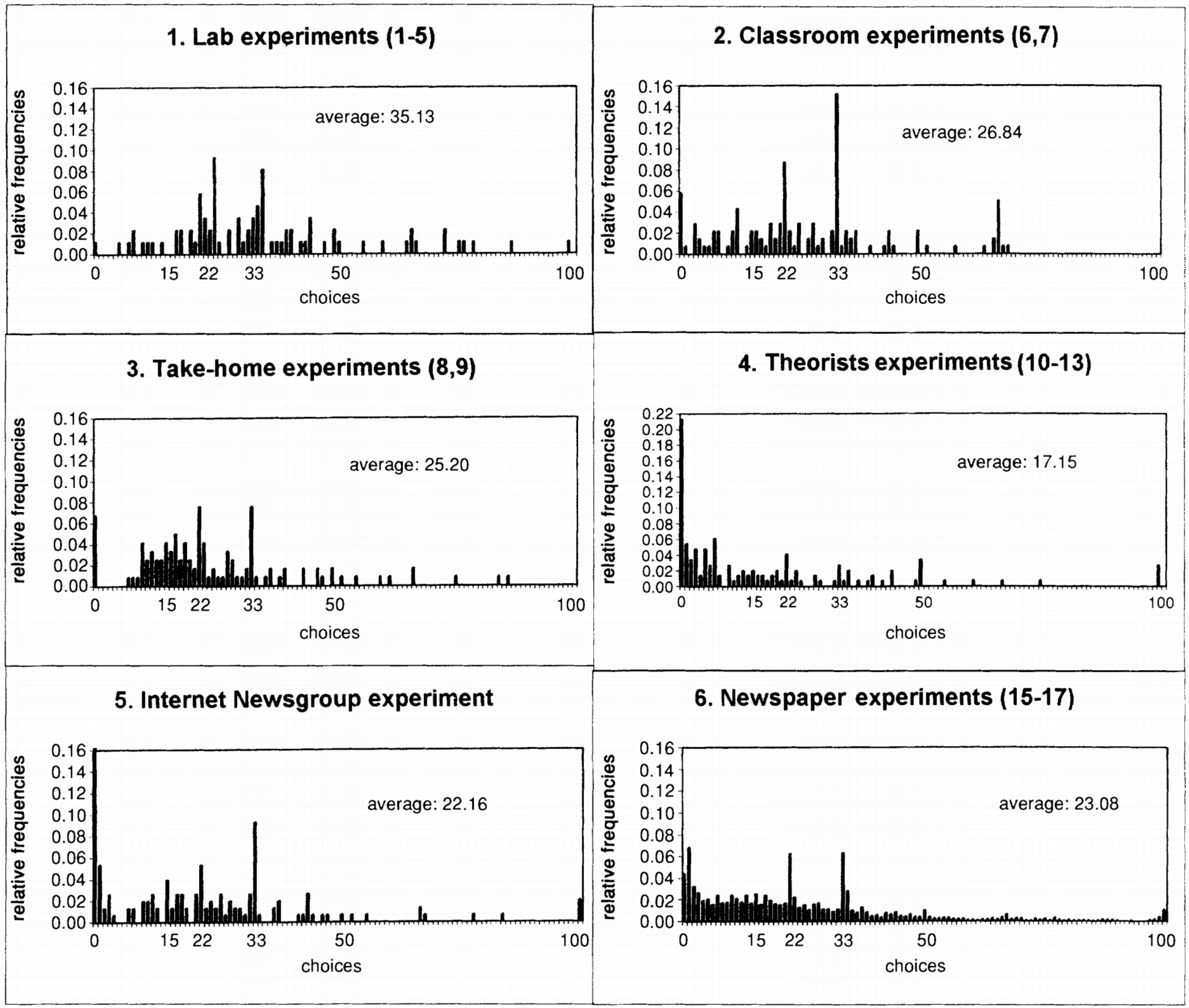


(c)

***Expansión* experiment (3,696 subjects)**



“Beauty contest” experiment in newspapers



What is “rational” play?

Assume everyone is rational (“common knowledge of rationality”)

Notice: **anything between 66.7 and 100 can never win!**

Even if everyone guessed 100, 100*2/3 = 66.6, so 66.6 is a better guess than anything above it

What is “rational” play?

What now?

66.6 is the new 100!

By the same reasoning, if everyone is rational, no one will guess above 66.6

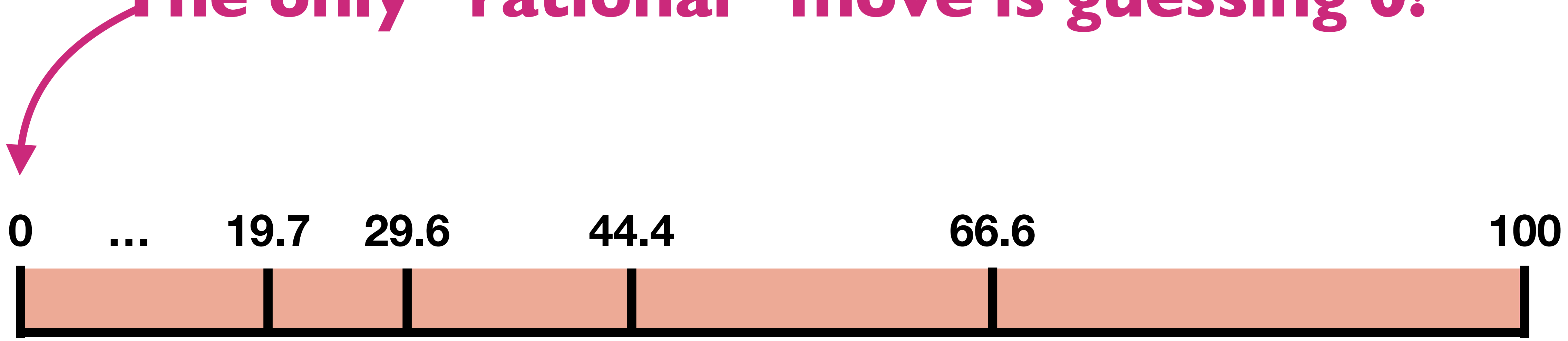
If that’s true, then a rational person should never guess anything between 44.4 and 66.6

What is “rational” play?

Repeat!

44.4 is the new 66.6, and so on

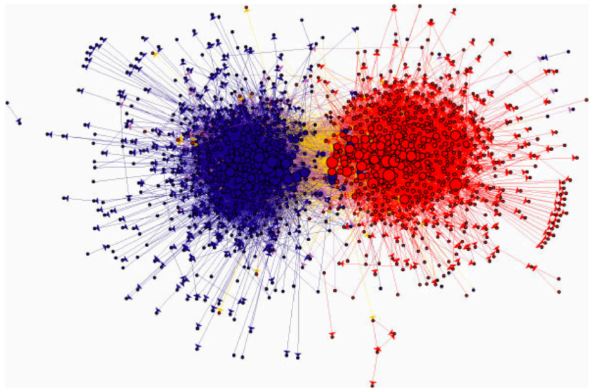
The only “rational” move is guessing 0!

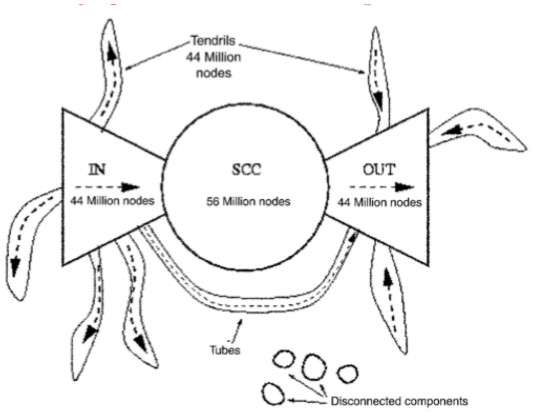


(of course, in real life not everyone is rational)

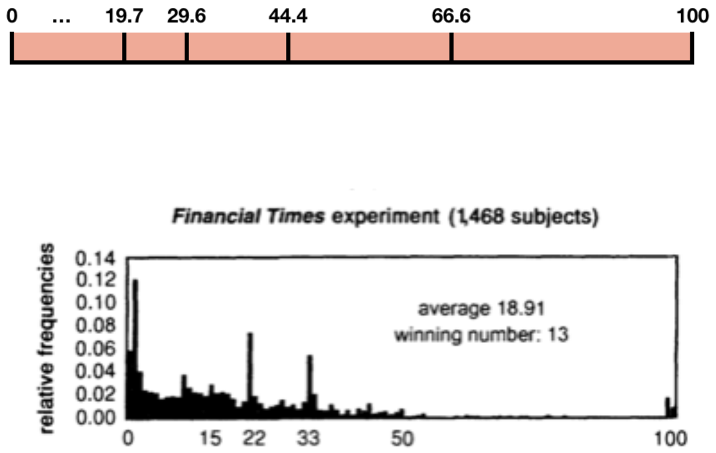
Today: Game theory

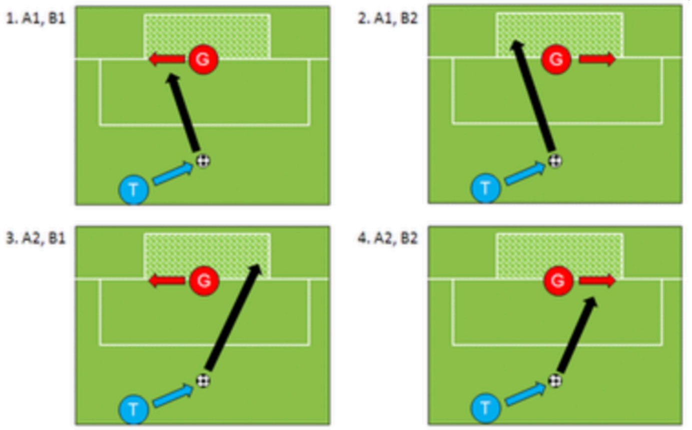
Networks: interconnected **structure**

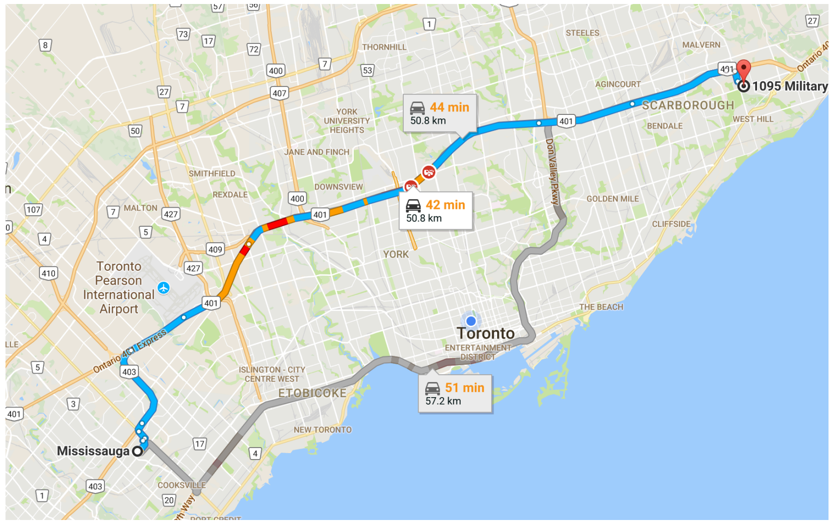




Game theory: interconnected **behaviour**







Exam or Presentation?

A class has two grades: individual exam and a two-person presentation
- Overall grade is the average of your exam and your presentation
- Can't fully prepare for both (sound familiar?)

Exam:
- If you study for the exam you'll do well (92%)
- If you don't study then you'll do less well (80%)

[And same for your partner!]

Presentation:
- If you both prepare for the presentation you'll do extremely well (100%)
- If just one person prepares then medium (92%), if neither then bad (84%)

What should you do?

Exam or Presentation?

We can summarize the situation in a 2x2 table
Your choices are the rows, and your partner's choices are the columns
Each box gives the grades: first you, then your partner

Both work on presentation: Avg(100,80)=90
One works on presentation: Avg(92,80)=86, other studies for exam: Avg(92,92)=92
Both study for exam: Avg(84,92)=88

Your score depends not only on your choice but your partner's choice too!

$$
\begin{array}{ll|c|c|}
 & & \multicolumn{2}{c}{\text{Your Partner}} \\
 & & \mathit{Presentation} & \mathit{Exam} \\
\hline
\text{You} & \mathit{Presentation} & 90, 90 & 86, 92 \\
\hline
 & \mathit{Exam} & 92, 86 & 88, 88 \\
\hline
\end{array}
$$

Exam-Presentation Game

What should you do?

If you knew your partner would study for the **exam**, what should you do?

You'd choose **exam (88 > 86)**

If you knew your partner would work on the **presentation**, what should you do?

You'd choose **exam (92 > 90)**

No matter what, you should choose exam!

		Your Partner	
		Presentation	*Exam*
You	*Presentation*	90, 90	86, 92
	Exam	92, 86	88, 88

Exam-Presentation Game

The situation is totally symmetric for your partner, they should choose the **exam** no matter what too

But you'd both be better off preparing for the presentation!

		Your Partner	
		Presentation	*Exam*
You	*Presentation*	90, 90	86, 92
	Exam	92, 86	88, 88

Basic Definitions

Players: you and your partner

Strategies: prepare presentation or study for final

Payoff: grade as a function of everyone's strategy

Payoff matrix: see below

This is a ***game*** (as in game theory)

Played once, and players select strategies simultaneously and without consulting one another

		Your Partner	
		Presentation	*Exam*
You	*Presentation*	90, 90	86, 92
	Exam	92, 86	88, 88

Basic Definitions

A game G is a tuple (P,S,O):

P = set of Players

S = a set of strategies for every player

O = for every outcome (where every player is picking one strategy), a payoff for each player

Payoff matrix summarizes all of these (each dimension is a player, every row/column/etc is a strategy for one player, every cell expresses payoffs for each player)

Underlying Assumptions

Payoffs summarize **everything** a player cares about

Every player knows everything about the structure of the game: who the players are, the strategies available to everyone, payoffs for each player and strategy

Every player is **rational**: wants to maximize payoff and succeeds in doing so

		Your Partner	
		Presentation	*Exam*
You	*Presentation*	90, 90	86, 92
	Exam	92, 86	88, 88

Underlying Assumptions

Weird conclusions? Assumptions are probably to blame!

		Your Partner	
		Presentation	*Exam*
You	*Presentation*	90, 90	86, 92
	Exam	92, 86	88, 88

The Prisoner’s Dilemma

Two bank robbery suspects are held in separate chambers

Not enough evidence to convict them, but they resisted arrest

The Prisoner’s Dilemma

Two bank robbery suspects are held in separate chambers. Not enough evidence to convict them, but they did resist arrest

Police take both aside **separately**, and tell each one:

- If you confess, and your partner doesn’t confess:
    - **You will be released**
    - **Your partner will be sent to prison for 10 years**
- If you both confess, then we don’t need either of you to testify against the other, and:
    - You will both be convicted of the robbery
    - **Both serve 4 years in prison**
- Finally, if neither of you confesses, then we can’t convict either of you of the robbery:
    - **Both charged with resisting arrest only (1 year in prison)**
- Your partner is being offered the same deal. **Do you want to confess?**”

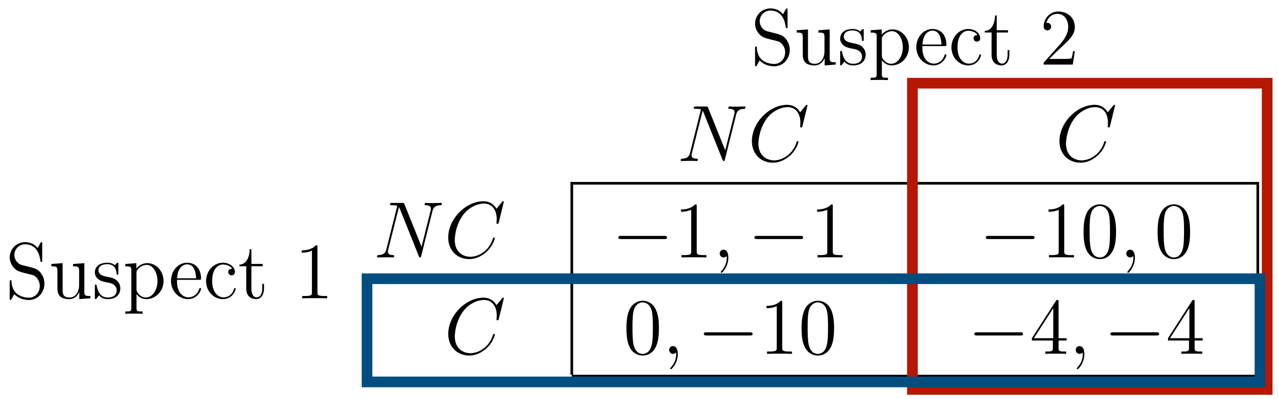
The Prisoner’s Dilemma

We can represent this situation in a simple matrix:
Suspect 1’s choices are the rows, and Suspect 2’s choices are the columns (**C**onfess and **N**ot-**C**onfess)
Each box gives the outcomes: first Suspect 1, then Suspect 2

$$\begin{array}{cc|c|c|} & & \multicolumn{2}{c}{\text{Suspect 2}} \\ & & NC & C \\ \cline{3-4} \text{Suspect 1} & NC & -1,-1 & -10,0 \\ \cline{3-4} & C & 0,-10 & -4,-4 \\ \cline{3-4} \end{array}$$

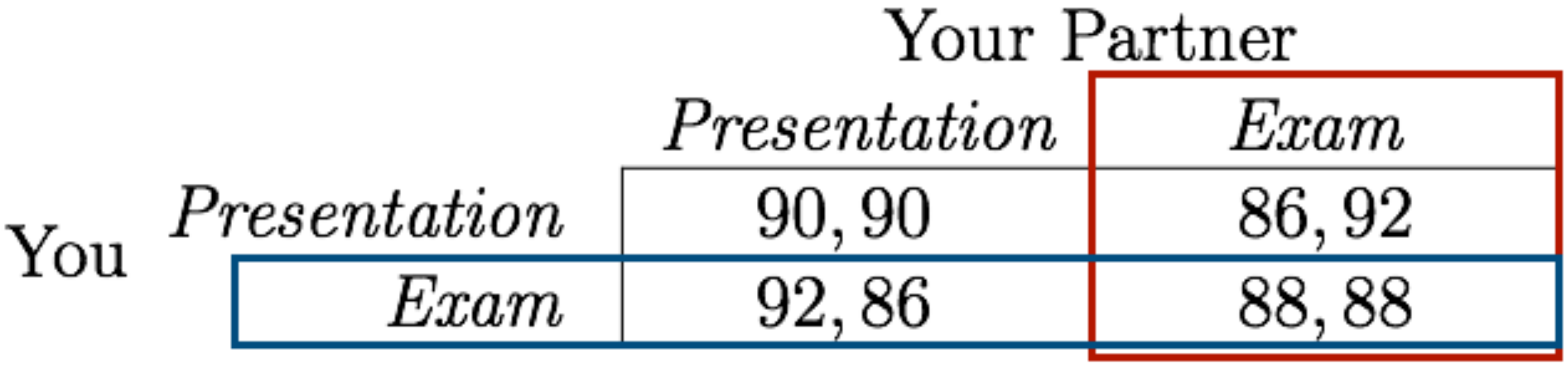
The Prisoner's Dilemma

Similar situation! Confessing is best for both suspects



		Suspect 2	
		NC	*C*
Suspect 1	*NC*	−1, −1	−10, 0
	C	0, −10	−4, −4

Compare with exam vs. presentation game:



		Your Partner	
		Presentation	*Exam*
You	*Presentation*	90, 90	86, 92
	Exam	92, 86	88, 88

Fundamental Concepts: Strict Dominant Strategy

A strategy that is strictly better than all other options, regardless of what other players do

Exam is a **strictly dominant strategy** for both players

Sadly, (90,90) is not achievable with rational play

Even if you could commit to preparing for the presentation, your partner would still be better off studying for the final

		Your Partner	
		Presentation	*Exam*
You	*Presentation*	90, 90	86, 92
	Exam	92, 86	88, 88

Prisoner’s Dilemma in the Real World

Drug doping in professional sports (dope vs. don’t dope)

Arms races between countries (build arms vs. don’t)

Countries respecting climate change treaties (Do or don’t restrict CO2 emissions)

Overfishing (do or don’t overfish the seas)

Advertising (advertise or don’t)

		Suspect 2	
		NC	C
Suspect 1	NC	$-1, -1$	$-10, 0$
	C	$0, -10$	$-4, -4$

Practice Question

Recall the game Rock-Paper-Scissors (paper beats rock, scissors beat paper, rock beats scissors)
Representing win/draw/loss as +1/0/-1, express Rock-Paper-Scissors as a game theory game

Practice Question

Recall the game Rock-Paper-Scissors (paper beats rock, scissors beat paper, rock beats scissors)

Representing win/draw/loss as +1/0/-1, express Rock-Paper-Scissors as a game theory game

Player 2

Player 1

P1\P2	**Rock**	**Paper**	**Scissors**
Rock	0,0	-1,+1	+1,-1
Paper	+1,-1	0,0	-1,+1
Scissors	-1,+1	+1,-1	0,0

Fundamental Concepts: Best Response

Let’s define some more of the fundamental concepts we just used

Best response is just what it sounds like: if player 2 plays **T**, then the best thing I can do is play **S**

$$\begin{array}{cc|c|c|} & & \multicolumn{2}{c}{\text{Suspect 2}} \\ & & NC & C \\ \hline \text{Suspect 1} & NC & -1,-1 & -10,0 \\ \hline & C & 0,-10 & -4,-4 \\ \hline \end{array}$$

S1’s best response to NC is: ?
S1’s best response to C is: ?

Fundamental Concepts: Best Response

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Best response is just what it sounds like: if player 2 plays **T**, then the best thing I can do is play **S**

$$\begin{array}{ccc|c|c|} & & \multicolumn{2}{c}{\text{Suspect 2}} \\ & & \multicolumn{1}{c}{NC} & \multicolumn{1}{c}{C} \\ \cline{3-4} \text{Suspect 1} & NC & -1, -1 & -10, 0 \\ \cline{3-4} & C & 0, -10 & -4, -4 \\ \cline{3-4} \end{array}$$

S1’s best response to NC is: C
S1’s best response to C is: C

Fundamental Concepts: Best Response

Let's define some more of the fundamental concepts we just used

Strategy **S** by P_1 is a **best response** to strategy **T** by P_2 if the payoff from **S** as at least as good as anyone other strategy against **T**

$P_1(S,T) \geq P_1(S',T)$ for all other S' by P_1

It's a **strict best response** if:

$P_1(S,T) > P_1(S',T)$ for all other S' by P_1

$$\begin{array}{cc|c|c|} & & \multicolumn{2}{c}{\text{Suspect 2}} \\ & & NC & C \\ \hline \text{Suspect 1} & NC & -1,-1 & -10,0 \\ \hline & C & 0,-10 & -4,-4 \\ \hline \end{array}$$

S1's best response to NC is: C
S1's best response to C is: C

Fundamental Concepts: Best Response

P1\P2	**A**	**B**	**C**	**D**	**E**
A	3, 5	-2, 1	4,3	1,6	9,2
B	2,2	1,10	3,6	4,2	5,3
C	8,-1	-2,6	-3,1	9,2	1,3

What is P1's best response to each of P2's strategies?

Fundamental Concepts: Dominant Strategy

A **dominant strategy** for P_1 is a strategy that is a best response every strategy by P_2

A **strict dominant strategy** for P_1 is a strategy that is a strict best response every strategy by P_2

		Suspect 2	
		NC	C
Suspect 1	NC	$-1, -1$	$-10, 0$
	C	$0, -10$	$-4, -4$

(Note: In Prisoner's Dilemma, P_1 has a strict dominant strategy, so we expect P1 to play it. There can be several dominant strategies, and it'd be unclear which one to expect)

Fundamental Concepts: Dominant Strategy

P1\P2	A	B	C	D	E
A	3, 5	-2, 1	4,3	1,6	9,2
B	2,2	1,8	3,6	4,9	5,3
C	8,-1	-2,2	-3,1	9,4	1,3

Does either player have a dominant strategy?

Dominant Strategies Don’t Always Exist

Prisoner’s Dilemma was relatively easy to analyze because every player has a strictly dominant strategy

However, dominant strategies don’t always exist!

P1\P2	**A**	**B**	**C**
A	3, 5	-2, 1	4,3
B	2,2	1,10	3,6
C	8,-1	-2,6	-3,1

Marketing Game

Consider a marketing scenario: two firms, Firm 1 and Firm 2

Firm 1 is more popular and gets 80% of profits when they compete
They can each either make an upscale product or a low-priced one
60% of the population prefers a low-priced product

Firm 2

Firm 1

What are the strategies? Payoffs?

Marketing Game

Consider a marketing scenario: two firms, Firm 1 and Firm 2

Firm 1 is more popular and gets 80% of profits when they compete

Two strategies each: make an upscale product or a low-priced one?

60% of population prefers a low-priced product

Does Firm 1 have a dominant strategy? Does Firm 2?

		Firm 2	
		Low-Priced	*Upscale*
Firm 1	*Low-Priced*	.48, .12	.60, .40
	Upscale	.40, .60	.32, .08

What happens?

Marketing Game

Notice Firm 1 has a strictly dominant strategy: go low-priced

Firm 2 does not have a dominant strategy

But since Firm 1 has a strictly dominant strategy, expect to play it. Firm 2's best response to Low-Priced is to play *Upscale*

Although we're reasoning in two steps, remember that the game itself is still plays the same way: both firms play their strategies simultaneously

Intuitive prediction: Firm 1 ignores Firm 2, Firm 2 steers clear of directly competing with Firm 1

		Firm 2	
		Low-Priced	*Upscale*
Firm 1	*Low-Priced*	.48, .12	.60, .40
	Upscale	.40, .60	.32, .08

What about no strictly dominant strategies?

What happens when neither player in a two-player game has a strictly dominant strategy?

Need another way to predict what will happen

A more intricate marketing game:

Players: Firm 1, Firm 2

Strategies: Approach client A, B, C

Payoff matrix:

		Firm 2		
		A	B	C
	A	4, 4	0, 2	0, 2
Firm 1	B	0, 0	1, 1	0, 2
	C	0, 0	0, 2	1, 1

A Three-Client Marketing Game

Neither firm has a dominant strategy

For Firm 1:

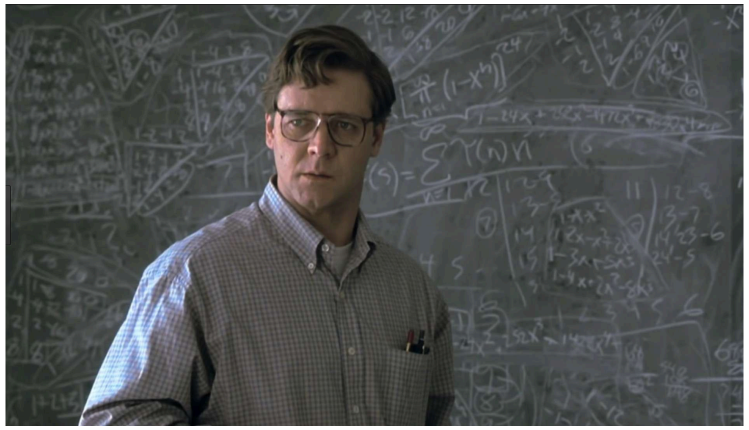
- **A** is a strict best response to strategy **A** by Firm 2
- **B** is a strict best response to **B**
- **C** is a strict best response to **C**

For Firm 2:

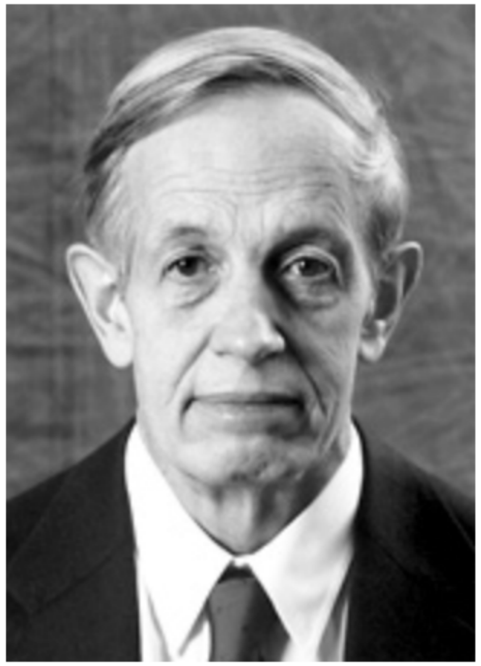
- **A** is a strict best response to strategy **A** by Firm 1,
- **C** is a strict best response to **B**,
- **B** is a strict best response to **C**

		Firm 2		
		A	B	C
Firm 1	A	$4, 4$	$0, 2$	$0, 2$
	B	$0, 0$	$1, 1$	$0, 2$
	C	$0, 0$	$0, 2$	$1, 1$

Nash Equilibrium



In 1950, John Nash proposed a **simple** and **powerful** principle for reasoning about behaviour in general games (and won the Nobel Prize for it in 1994)



Even when there are no dominant strategies, **we should expect players to use strategies that are best responses to each other**

A pair of strategies (**S**,**T**) is a Nash equilibrium if **S** is a best response to **T** and **T** is a best response to **S**

Nash Equilibrium

Why?

First consider a pair of strategies that **don't** constitute a Nash equilibrium

If both players expected (B,B) as an outcome, would they be happy?

		Firm 2		
		A	B	C
	A	$4, 4$	$0, 2$	$0, 2$
Firm 1	B	$0, 0$	$\boxed{1, 1}$	$0, 2$
	C	$0, 0$	$0, 2$	$1, 1$

Nash Equilibrium

Why?

First consider a pair of strategies that **don’t** constitute a Nash equilibrium

If both firms expected (B,B) as an outcome, would they be happy?

No! Firm 2 would rather play C in response to B.

		Firm 2		
		A	B	C
Firm 1	A	$4, 4$	$0, 2$	$0, 2$
	B	$0, 0$	$1, 1$	$0, 2$
	C	$0, 0$	$0, 2$	$1, 1$

Nash Equilibrium

Find the Nash equilibrium:

Player 2

		L	R
Player 1	**U**	1,2	2,3
	D	2,1	1,2

Nash Equilibrium

Find the Nash equilibrium:

		Player 2	
		L	**R**
Player 1	**U**	1,2	$\boxed{2,3}$
	D	2,1	1,2

Nash Equilibrium

Find the Nash equilibrium:

		Player 2	
		L	**R**
Player 1	**U**	2,1	1,2
	D	4,2	3,1

Nash Equilibrium

Find the Nash equilibrium:

Player 2

		L	R
Player 1	**U**	1,1	0,0
	D	0,0	1,1

Nash Equilibrium

Find the Nash equilibrium:

		Player 2	
		L	**R**
Player 1	**U**	$\boxed{1,1}$	0,0
	D	0,0	$\boxed{1,1}$

Multiple Equilibria

In the case of a single Nash equilibrium, it seems natural to predict that the players will play the strategies in this equilibrium (otherwise someone's not playing a best response)

A lot of games can have more than one equilibrium though

Example: coordination game
Players: you, your partner
Strategies: PowerPoint, Keynote
Payoff matrix:

$$\begin{array}{cc|c|c|} & & \multicolumn{2}{c}{\text{Your Partner}} \\ & & PowerPoint & Keynote \\ \hline \text{You} & PowerPoint & \boxed{1,1} & 0,0 \\ \hline & Keynote & 0,0 & \boxed{1,1} \\ \hline \end{array}$$

Multiple Equilibria

This is called a **Coordination game** because all the players care about is playing the same strategy

Lots of coordination games in real life: what side of the street to walk on, what side of the road to drive on, what hand to shake with

$$\begin{array}{llcc}
 & & \multicolumn{2}{c}{\text{Your Partner}} \\
 & & PowerPoint & Keynote \\
\text{You} & PowerPoint & \boxed{1,1} & 0,0 \\
 & Keynote & 0,0 & \boxed{1,1}
\end{array}$$

Multiple Equilibria

How does society deal with this?

Sometimes there is a ***focal point*** that causes the players to focus on one strategy over the others ("it's just the way we do things")

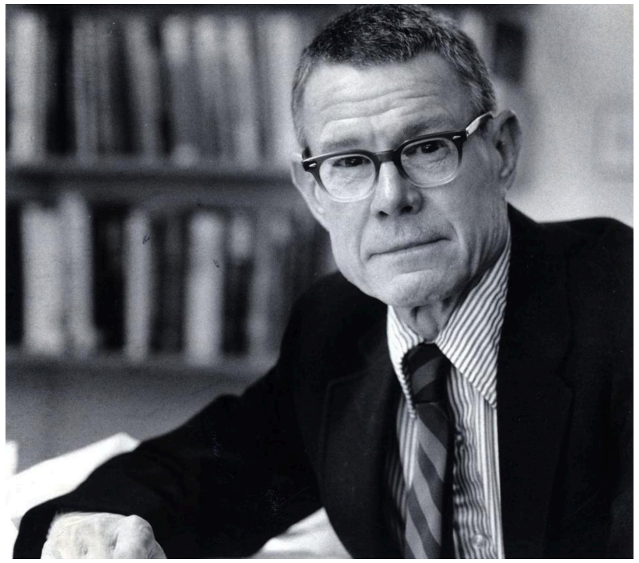
Example: what side of the road to drive on

Social norms, conventions are often ways of introducing a focal point into coordination games

Unbalanced Coordination Games

Focal point idea: use a feature **intrinsic to the game** (rather than an external social convention) to make a prediction

		Your Partner	
		PowerPoint	*Keynote*
You	*PowerPoint*	1, 1	0, 0
	Keynote	0, 0	2, 2



		Driver 2	
		L	**R**
Driver 1	**L**	100,100	-100,-100
	R	0,0	1,1

Unbalanced Coordination Games

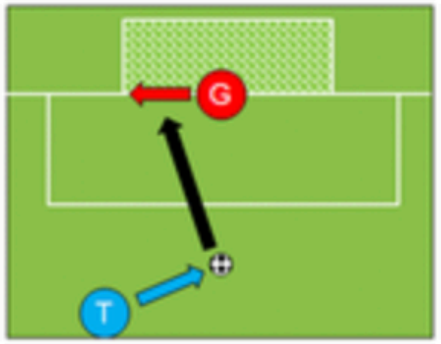
But say you and your partner disagree on the best slides software

$$\begin{array}{rr|c|c|} & & \multicolumn{2}{c}{\text{Your Partner}} \\ & & PowerPoint & Keynote \\ \hline \text{You} & PowerPoint & \boxed{1,2} & 0,0 \\ & Keynote & 0,0 & \boxed{2,1} \\ \hline \end{array}$$

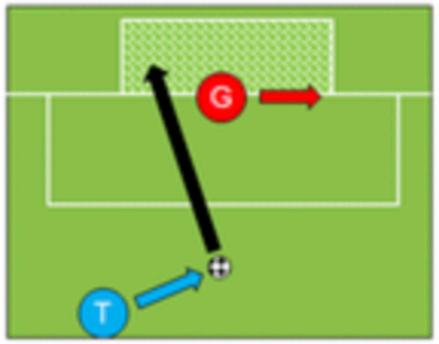
("Battles of the Sexes")

Matching Pennies

1. A1, B1

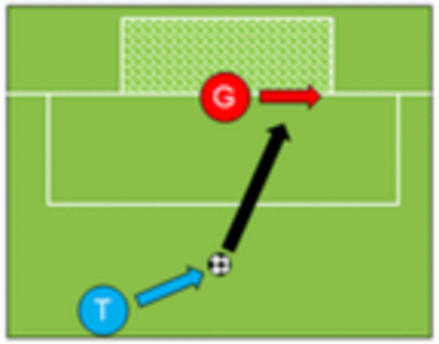


2. A1, B2



3. A2, B1

4. A2, B2



Matching Pennies

Attack-defense structure: interests are in direct conflict

“Zero-sum game”

Players: 1, 2

Strategies: Heads, Tails

Payoff matrix:

$$\begin{array}{cc|c|c|}
 & & \multicolumn{2}{c}{\text{Player 2}} \\
 & & H & T \\ \cline{3-4}
\text{Player 1} & H & -1, +1 & +1, -1 \\ \cline{3-4}
 & T & +1, -1 & -1, +1 \\ \cline{3-4}
\end{array}$$

What are Nash equilibria of this game?

There are none: no pair of strategies are best responses to each other

Mixed strategies

Solution: **introduce randomization**

Sometimes I'll do this, sometimes I'll do that (randomly)

Intuition: make it harder for my opponent to exploit me

Strategy: now corresponds to a choice of mixture probabilities between "pure" strategies.

Payoffs: Expected value under other person's mixture

Matching Pennies

		Player 2	
		H	T
Player 1	H	$-1, +1$	$+1, -1$
	T	$+1, -1$	$-1, +1$

Players: 1, 2

Strategies:

1: play H probability p

2: play H probability q

If P1 chooses p=1 corresponding to pure strategy H: payoff becomes

$$(-1)(q) + (1)(1-q) = 1 - 2q$$

If P1 chooses p=0 corresponding to pure strategy T: payoff becomes

$$(1)(q) + (-1)(1-q) = 2q - 1$$

Equilibrium in Matching Pennies

Note there pure strategies can’t be part of a Nash equilibrium, so p and q must be strictly between 0 and 1

What is Player 1’s best strategy to Player 2 choosing *q*?

Playing H gives him 1—2q, and playing T gives him 2q—1

If one was bigger than the other, he should just put all the weight on the bigger one

But no pure strategy Nash equilibrium, so 1—2q=2q—1

In any Nash equilibrium, we must have q = 1/2

Similarly for Player 1: we must have p = 1/2

		Player 2	
		H	T
Player 1	H	$-1, +1$	$+1, -1$
	T	$+1, -1$	$-1, +1$

Equilibrium in Matching Pennies

Intuitively: if Player 1 believes that Player 2 will play H strictly more than T, then she should definitely play T — in which case Player 2 should not be playing H more than half the time.

Make yourself the **least exploitable possible**

Make the opponent **indifferent** between their strategies

$$\begin{array}{cc|c|c|} & & \multicolumn{2}{c}{\text{Player 2}} \\ & & H & T \\ \cline{3-4} \text{Player 1} & H & -1,+1 & +1,-1 \\ \cline{3-4} & T & +1,-1 & -1,+1 \\ \cline{3-4} \end{array}$$

Equilibrium in Matching Pennies

Large game-theoretic study of 1400 penalty kicks

Kind of a real-life matching pennies

		Goalie	
		L	R
Kicker	L	$0.58, -0.58$	$0.95, -0.95$
	R	$0.93, -0.93$	$0.70, -0.70$

To make kicker indifferent between shooting L or R, goalie needs to select right *q*:

$$(.58)(q) + (.95)(1-q) = (.93)(q) + (.70)(1-q)$$

q = 0.42

Amazing fact: goalies dive left exactly 42% of the time!

Equilibrium in Matching Pennies

Every game has a mixed-strategy Nash equilibrium

[Nash, 1950]

Solutions to games

Dominant strategy? Sometimes.

Pure Nash Equilibria? Sometimes.

Mixed Equilibria? Always exists.

Mixed Strategies Example: Football

Players: Offense, Defense

Strategies: Run, Pass and Defend Run, Defend Pass

Payoff matrix:

		Defense	
		Defend Pass	*Defend Run*
Offense	*Pass*	$0, 0$	$10, -10$
	Run	$5, -5$	$0, 0$

Mixed Nash:
q = 2/3
p = 1/3

No Nash equilibria in this game

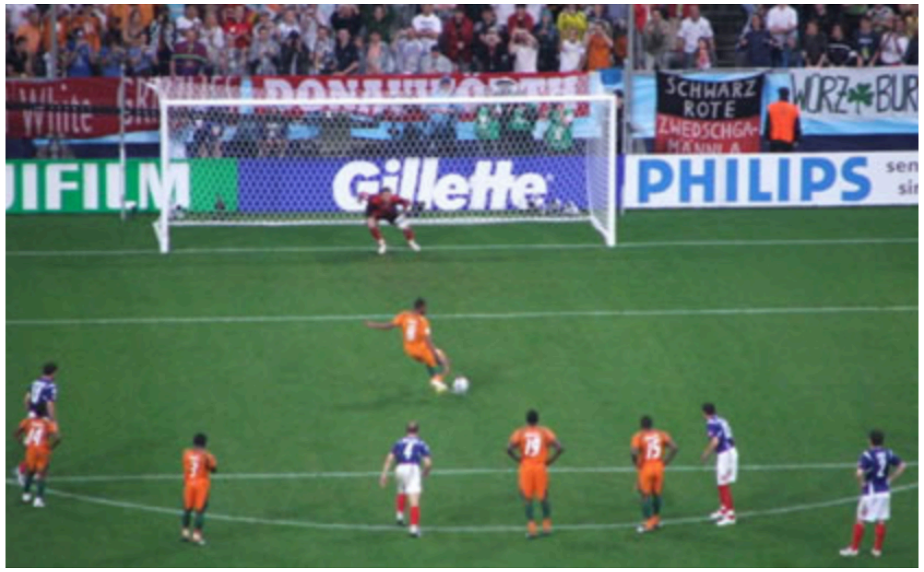
O’s expected payoff for **Pass** when D plays p: 0*(q)+10*(1-q) = 10-10q

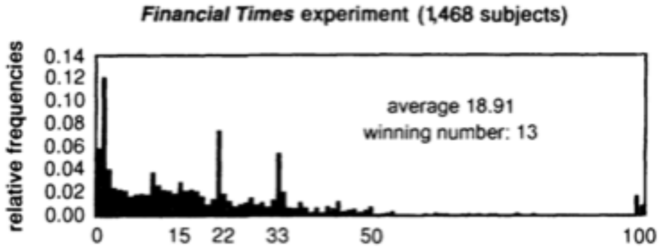
O’s expected payoff for **Run** when D plays q: 5*(q)+0*(1-q) = 5q

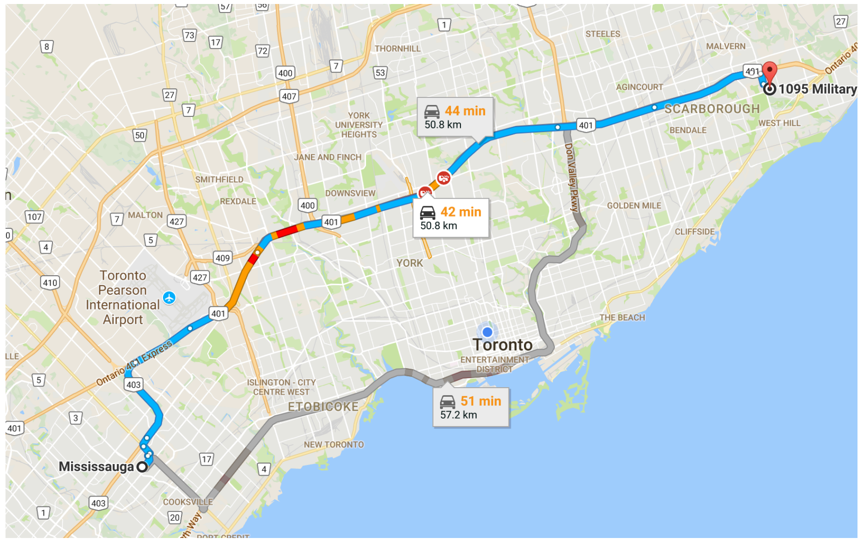
Defense makes Offense indifferent when q=2/3

Today: game theory

Mathematical framework to analyze strategic behaviour







- A game is characterized by players, strategies, and payoffs
- Captures a wide variety of strategic situations
- Best response, (strict) dominant strategies, mixed strategies, Nash equilibrium