Social and Information Networks

CSCC46H, Fall 2022
Lecture 9

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Logistics

Blog posts A–J due Friday, Nov 11
Blog posts K–R due Friday, Nov 18
Blog posts S–Z due Friday, Nov 25

Today

A3 due next week

Today

Game Theory: Congestion games
Decision-Based Diffusion
Information Diffusion

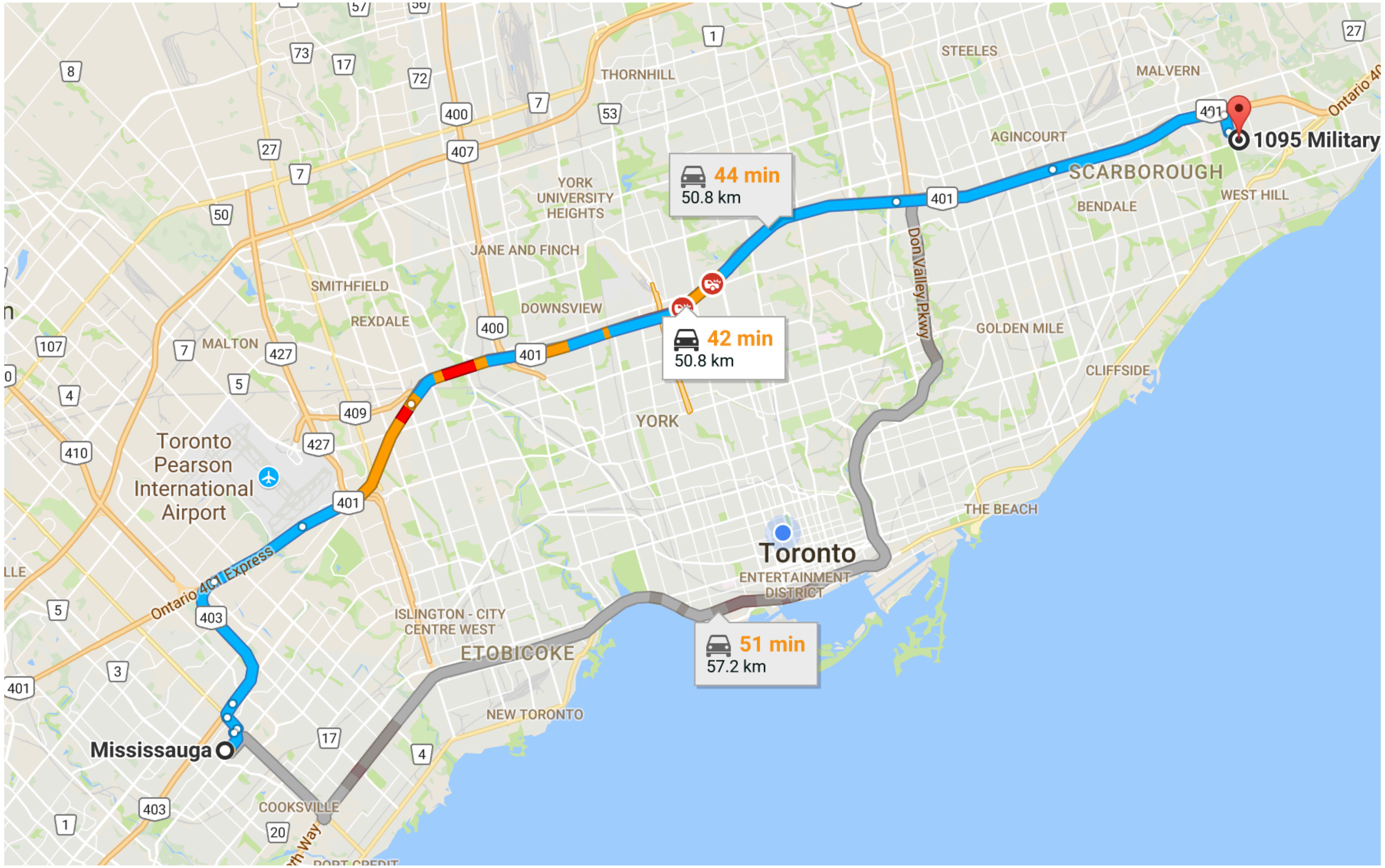
Today: Game Theory in the Wild and Influence Through Networks

If people are connected through a network, it's possible for them to influence each other's knowledge, behaviour and actions

Today: why?

- Informational
- Direct benefit
- Social conformity

Getting to UTSC: 401 or Gardiner?



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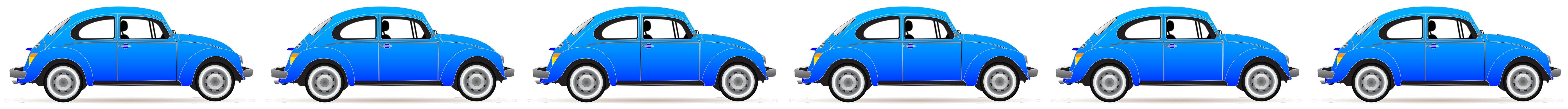
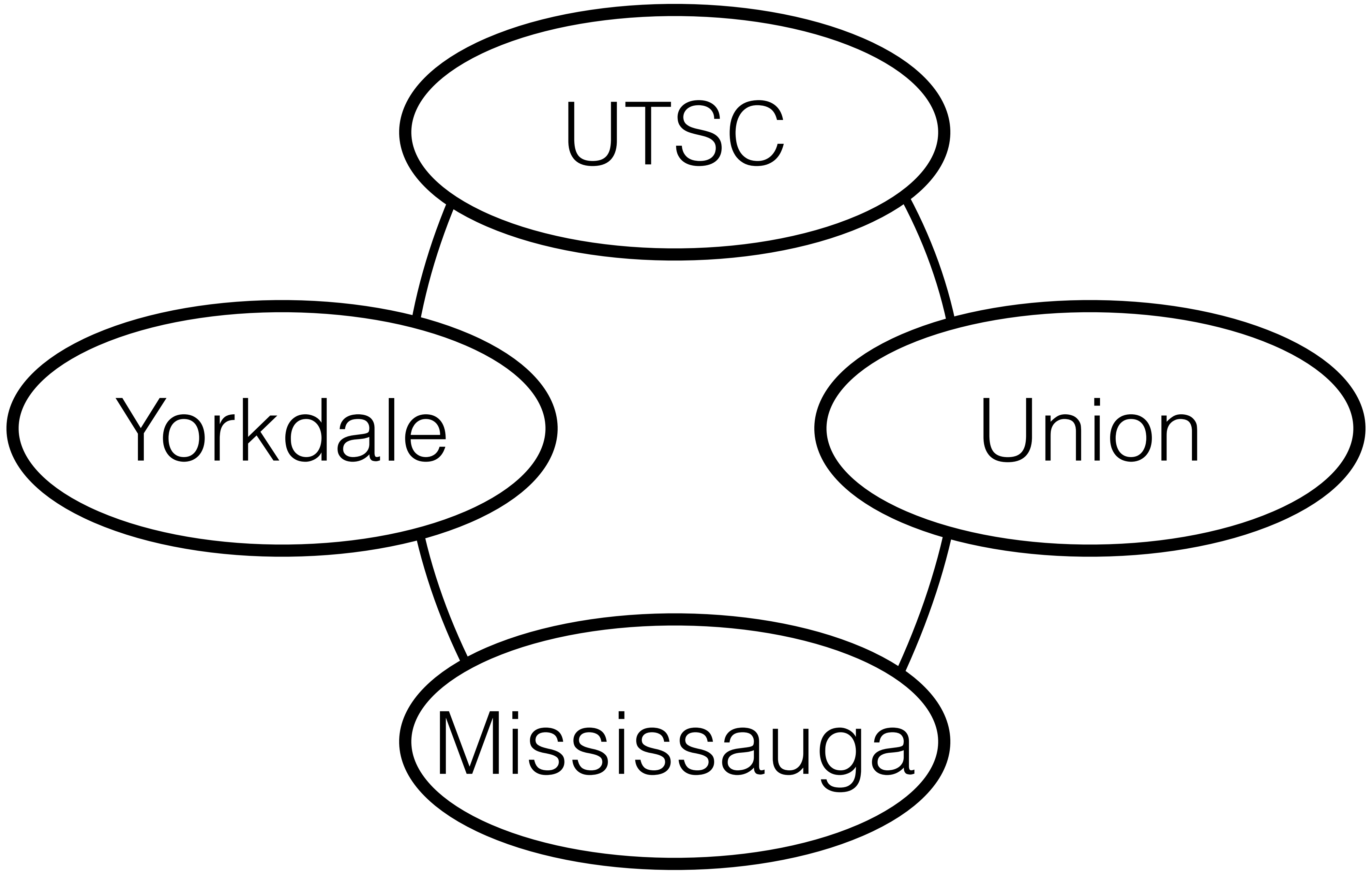


401

Gardiner



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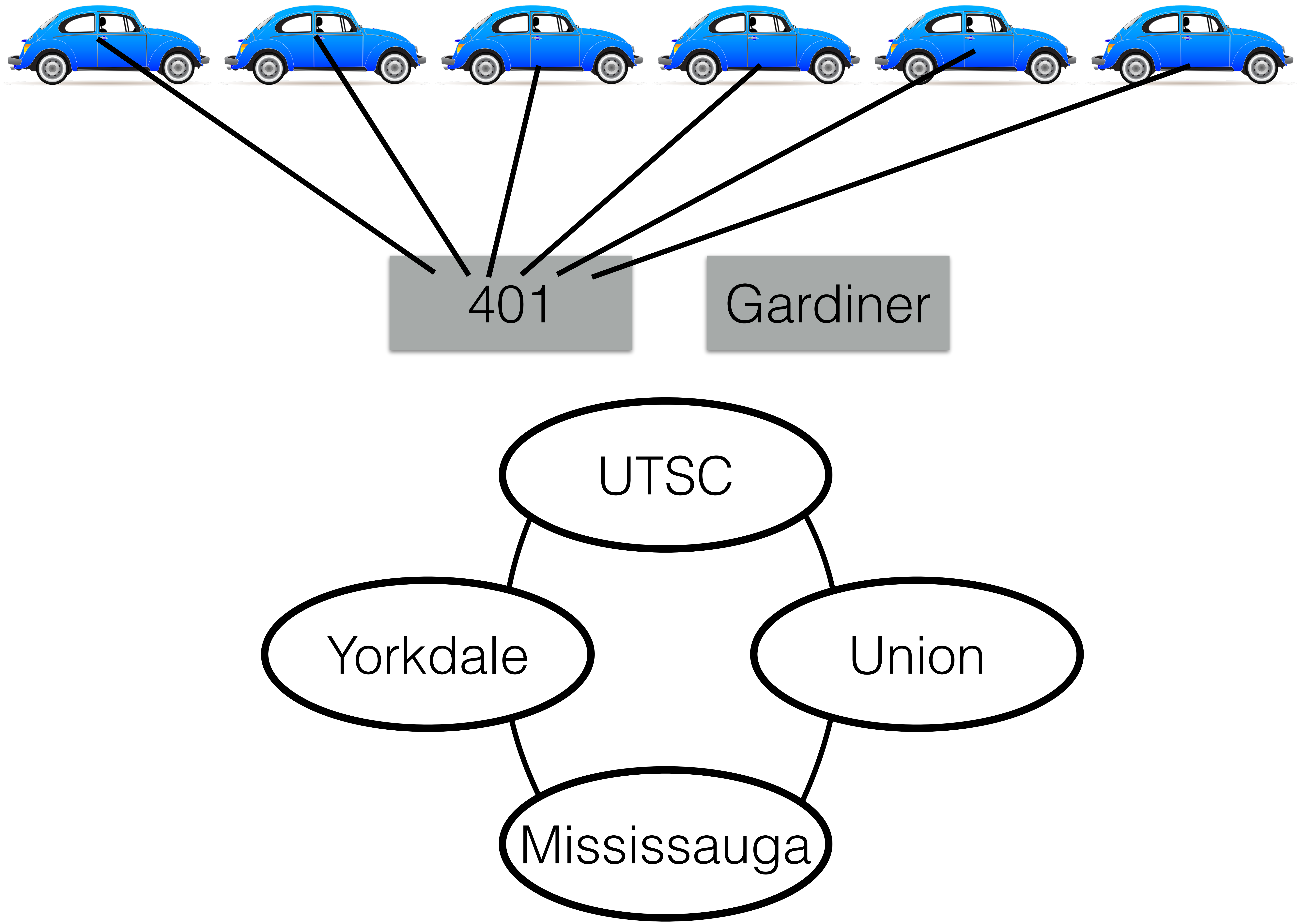

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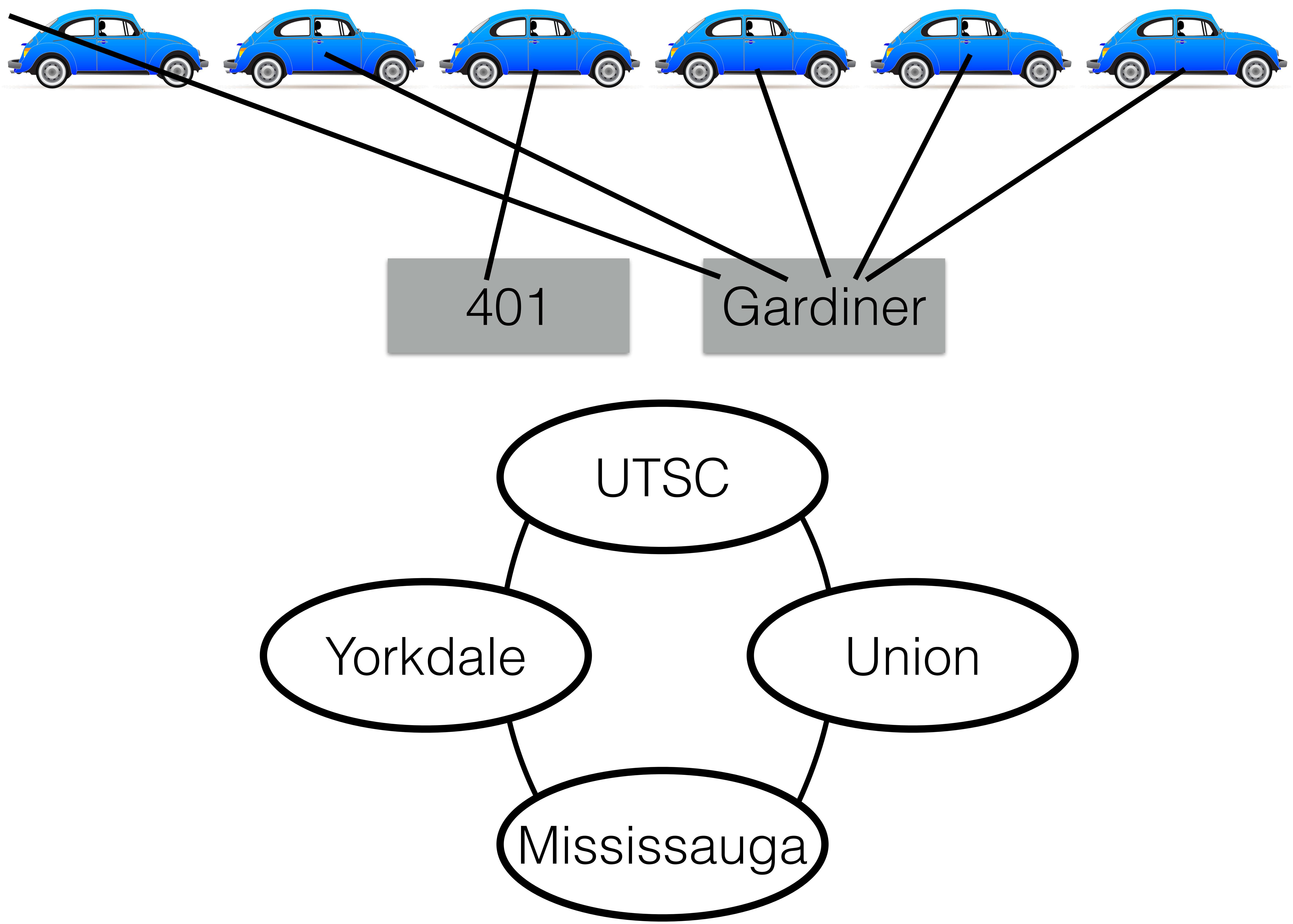

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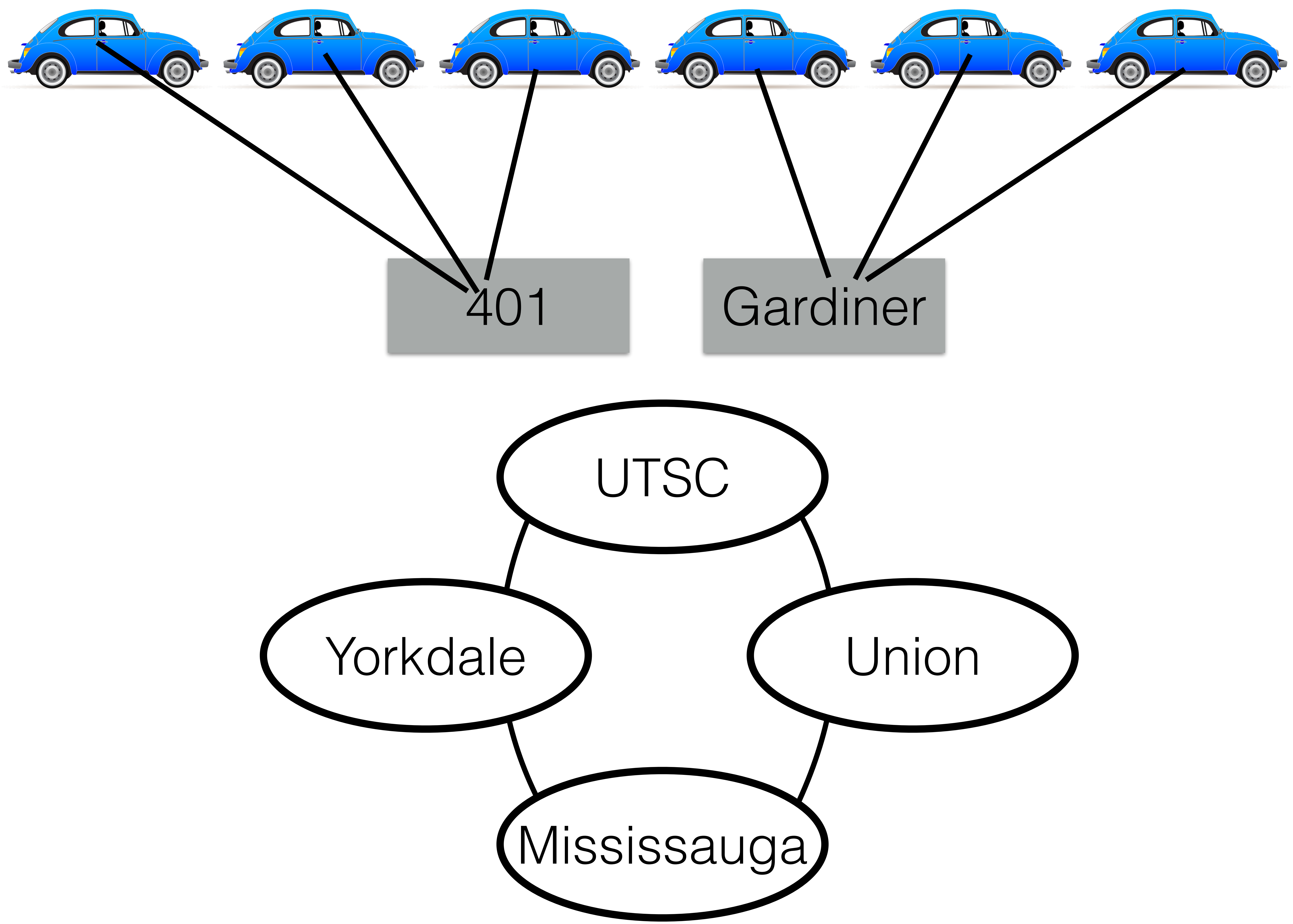
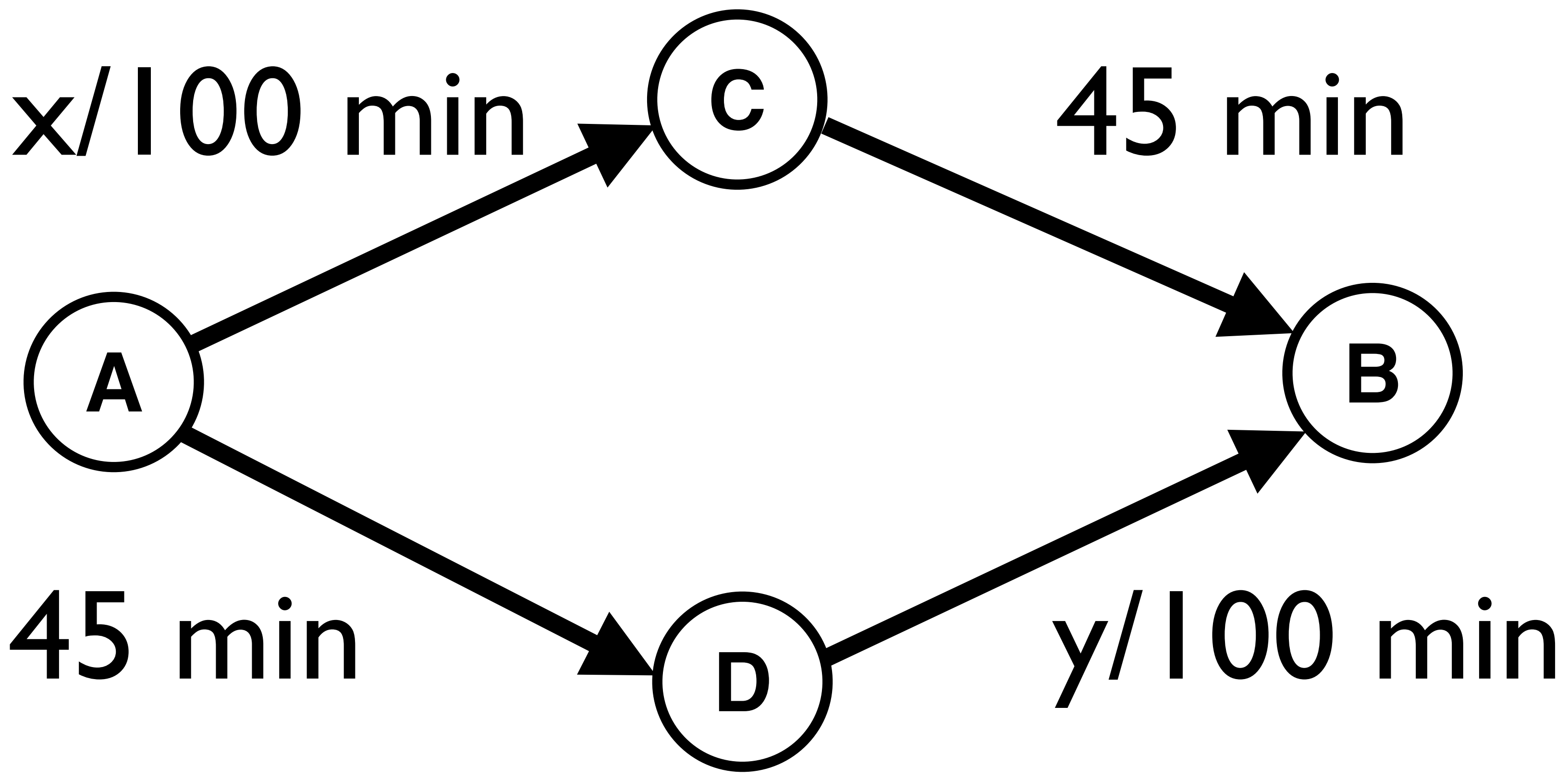


Traffic routing

Let's model this as a simple network, with two kinds of edges:

Constant edges (wide highways that don't get congested)

Traffic-dependent edges (quick routes that can get congested)



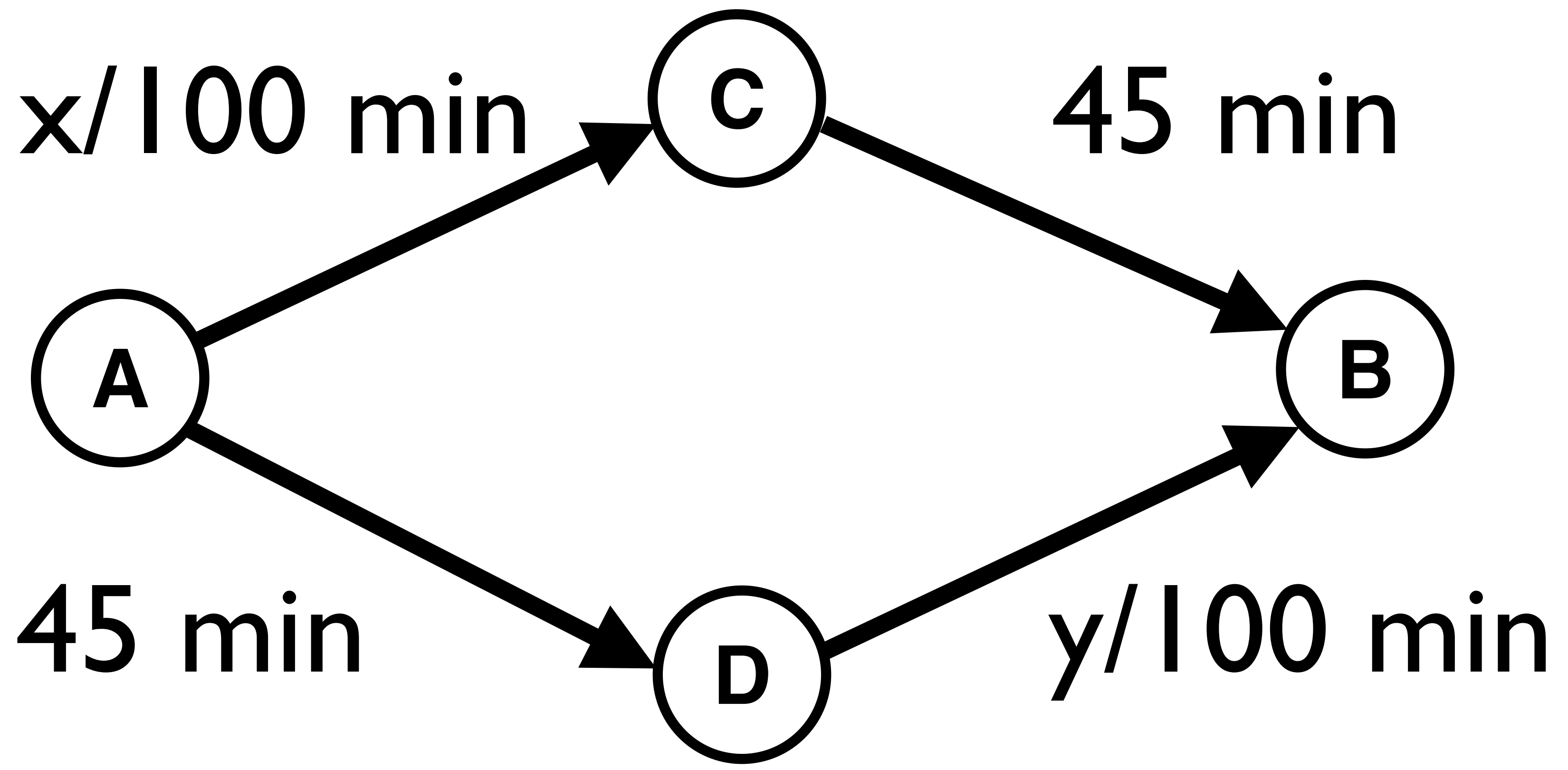
Traffic routing

Let's model this as a simple game on a network, with two kinds of edges:

Constant edges (wide highways that don't get congested)

Traffic-dependent edges (quick routes that can get congested)

There are 4000 drivers. Each one can choose A-C-B or A-D-B.

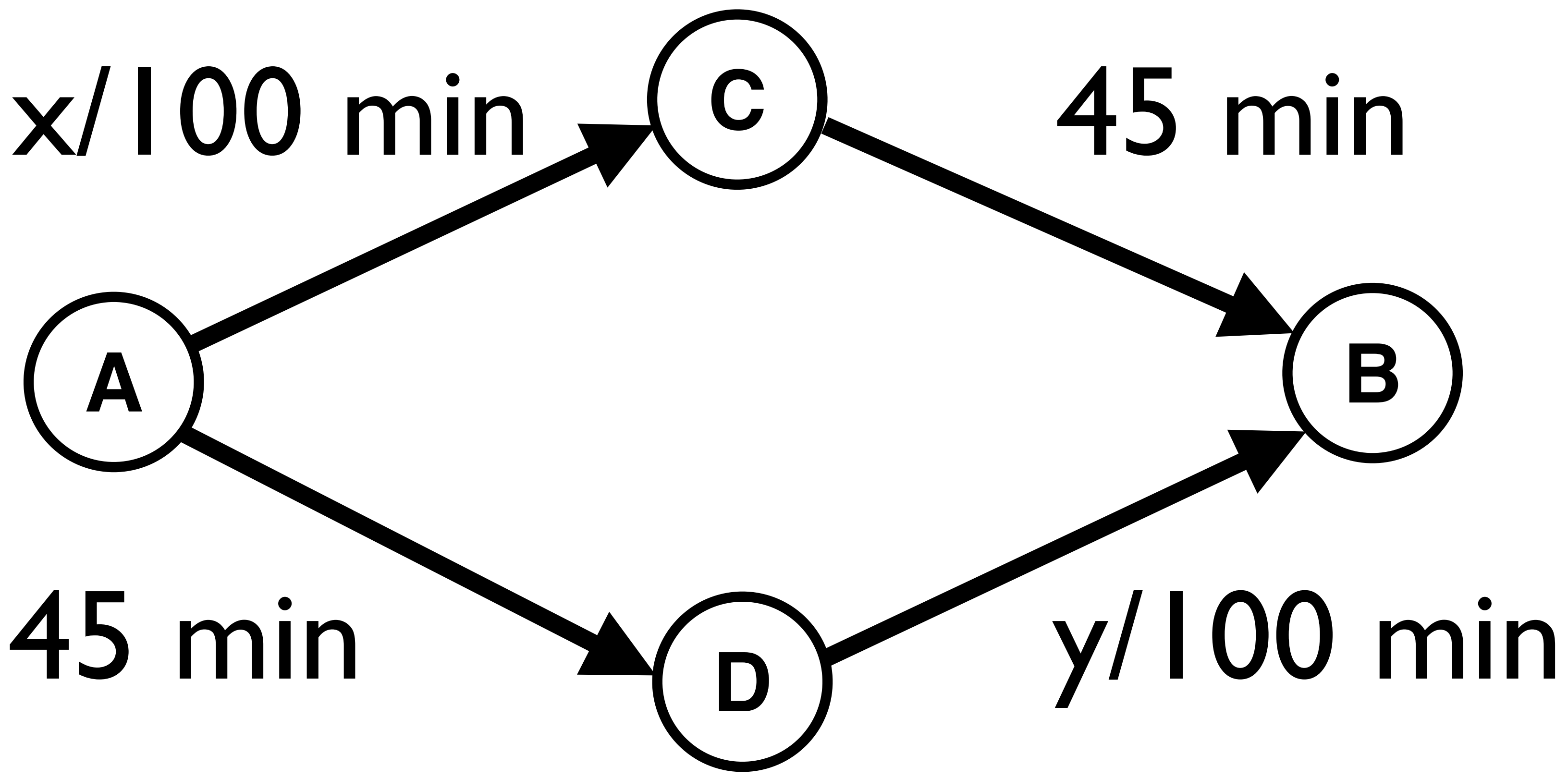


Traffic modeled as a game

Players: Drivers 1,2,3…,4000

Strategies: Two strategies each: A-C-B or A-D-B

Payoffs: ?



Traffic modeled as a game

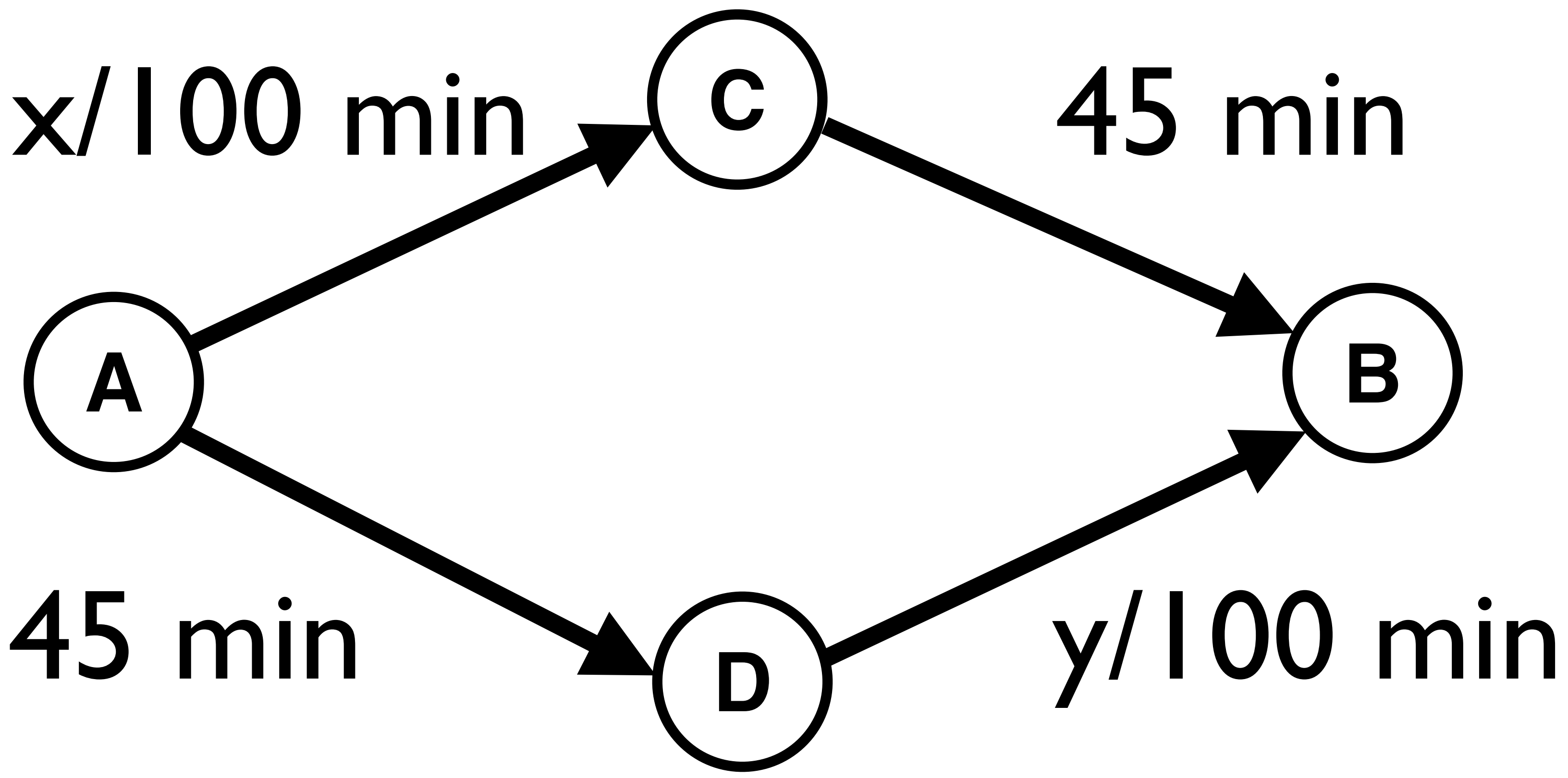
Players: Drivers 1,2,3…,4000

Strategies: Two strategies each: A-C-B or A-D-B

Payoffs: Negative drive time

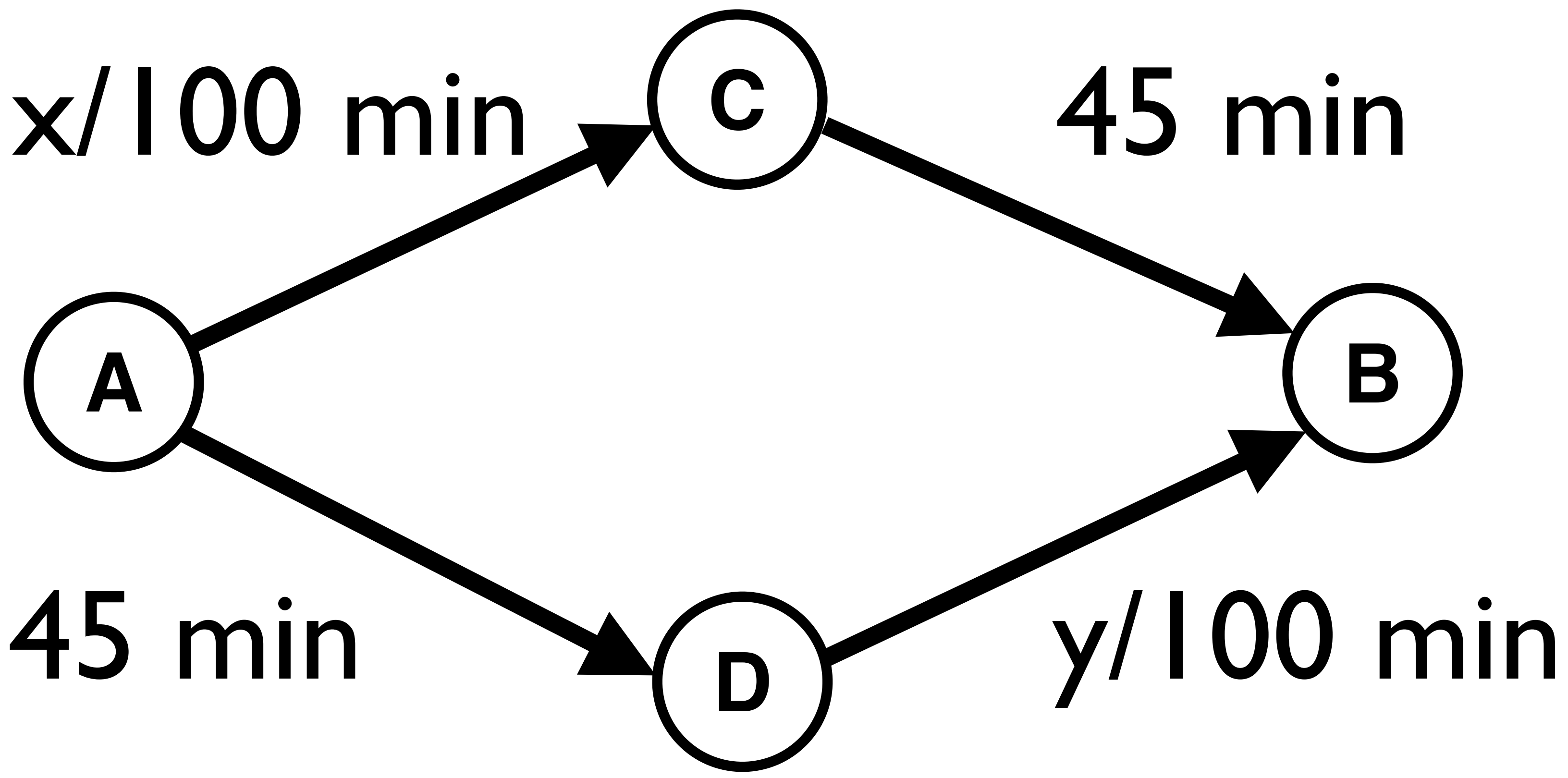
A-C-B time: - (x/100 + 45)

A-D-B time: - (45 + y/100)



Traffic Equilibrium?

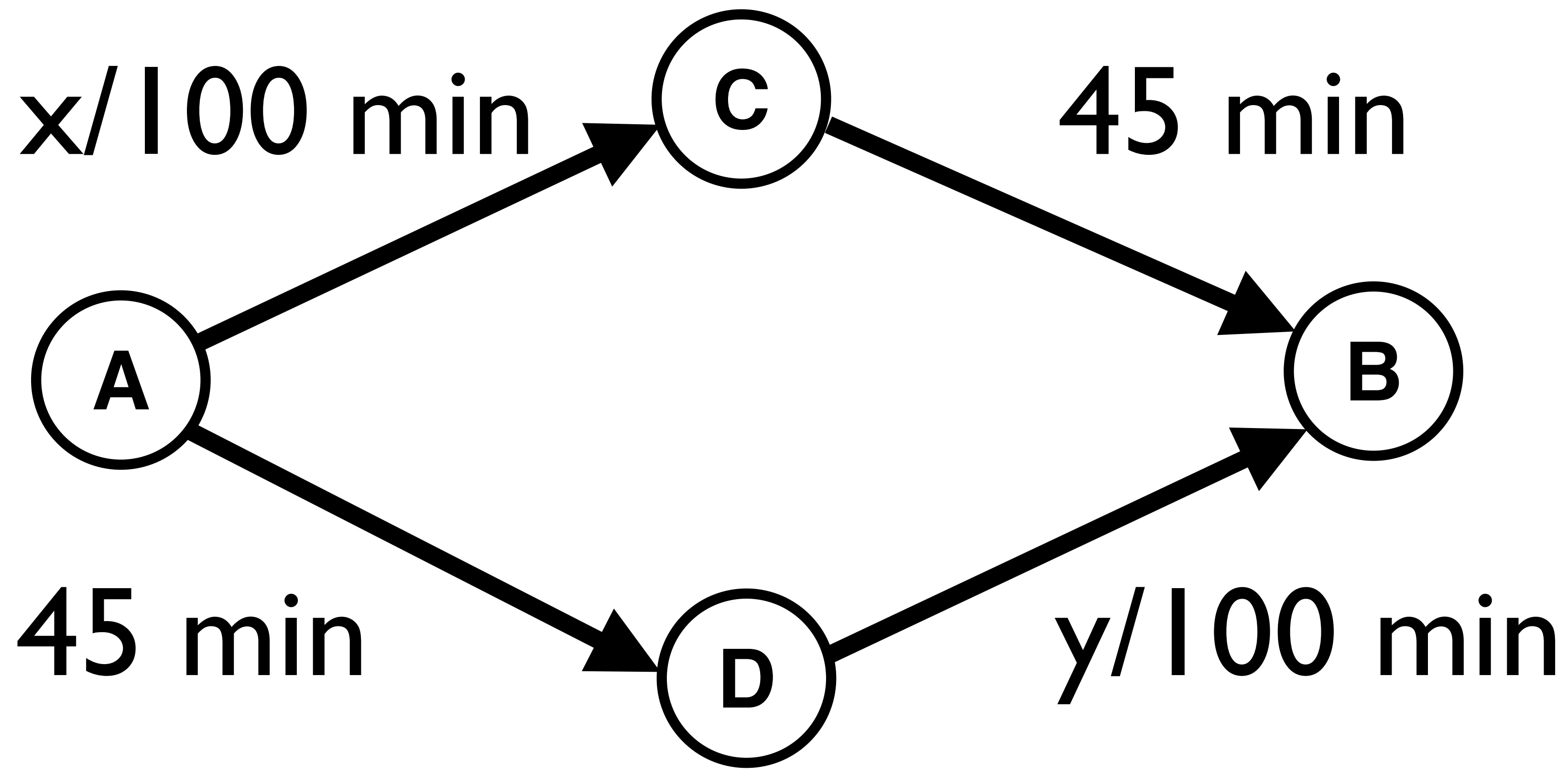
- 4000 drivers
- Two route options: A-C-B or A-D-B.
- Consider a few outcomes (strategy for each player):
  - Payoffs when 4000 choose top (ACB), 0 choose bottom (ADB):
    - Top path: 4000/100 + 45 = 85 min
    - Bottom path: 45 + 0/100 = 45 min
  - Payoffs when 0 choose top, 4000 choose bottom:
    - Top: 0/100 + 45 = 45 min
    - Bottom: 45 + 4000/100 = 85 min



Equilibrium in traffic?

- 4000 drivers
- Two route options: A-C-B or A-D-B.
- Payoffs when 2000 choose top, 2000 choose bottom:
  - Top: 2000/100 + 45 = 65 min
  - Bottom: 45 + 2000/100 = 65 min

This is an equilibrium because no one has an incentive to deviate



Equilibrium in traffic?

Payoffs when 2000 choose top, 2000 choose bottom:

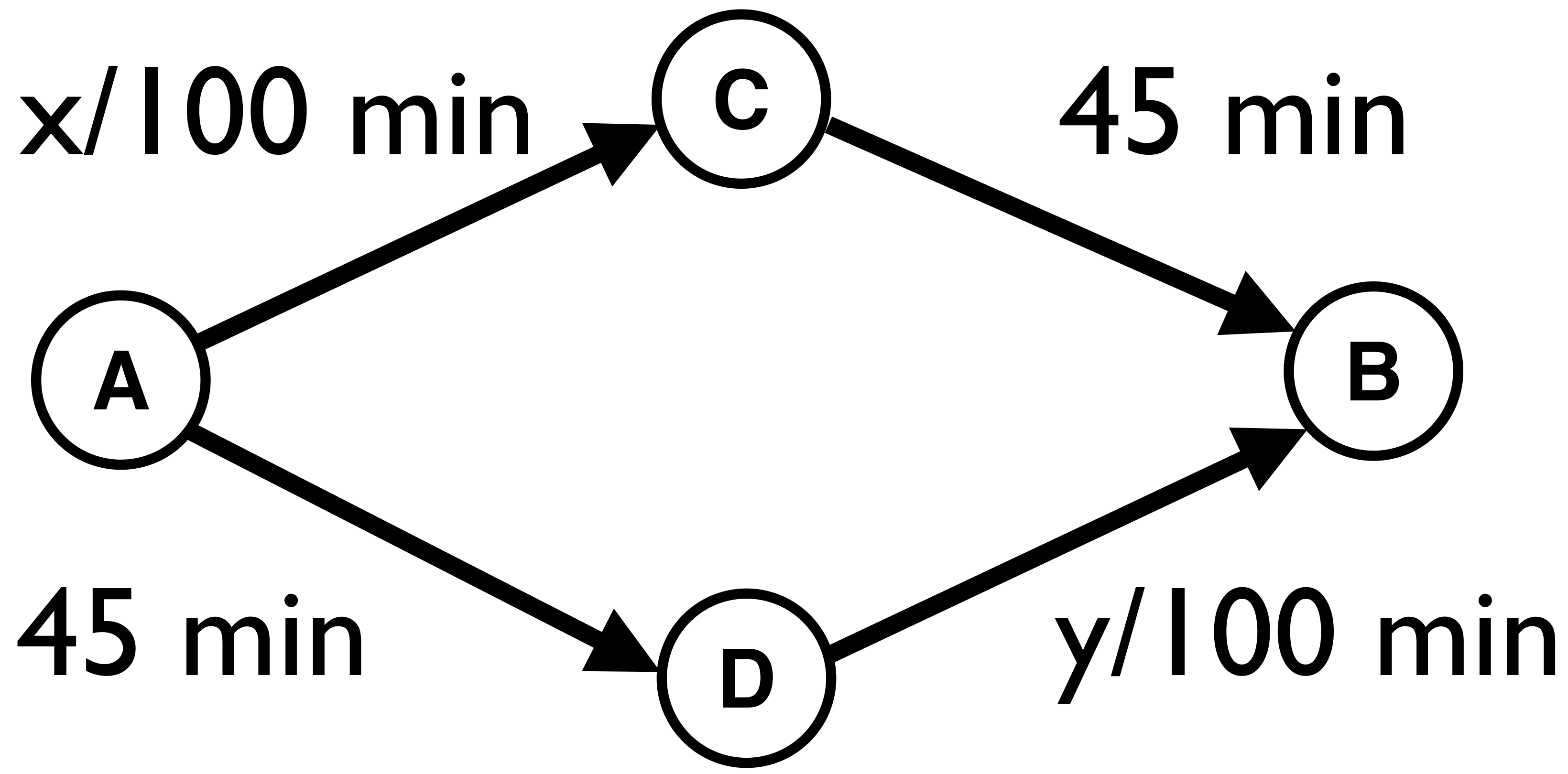
Top: 2000/100 + 45 = 65 min

Bottom: 45 + 2000/100 = 65 min

This is an equilibrium because no one has an incentive to deviate

If someone currently using A-C-B decides to switch to A-D-B:

Currently:	Top:	2000/100 + 45 = 65.00 min
Switch:	Bottom:	45 + 2001/100 = 65.01 min



Traffic modeled as a game

Players: Drivers 1,2,3…,4000

Strategies: A-C-B, A-D-B

Payoffs: Negative drive time

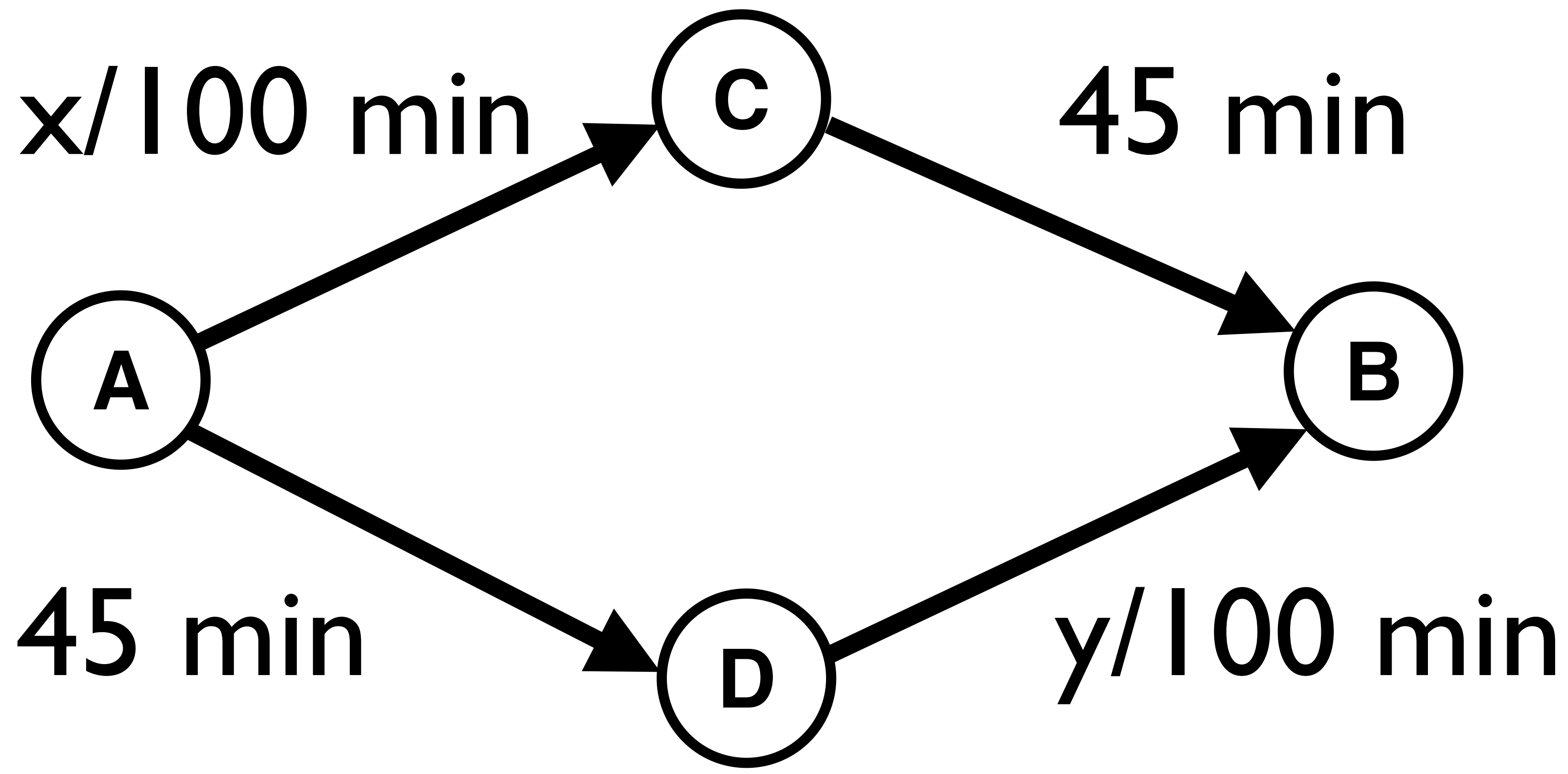
A-C-B time: -(x/100 + 45)

A-D-B time: -(45 + y/100)

You want to lower your drive time, so we take the negative drive time as the "payoff"

Notice that this actually describes many equilibria: any set of strategies "2000 choose top, 2000 choose bottom" is an equilibrium (players are interchangeable, so any set of 2000 can be using ACB and any set of 2000 can be using ADB)

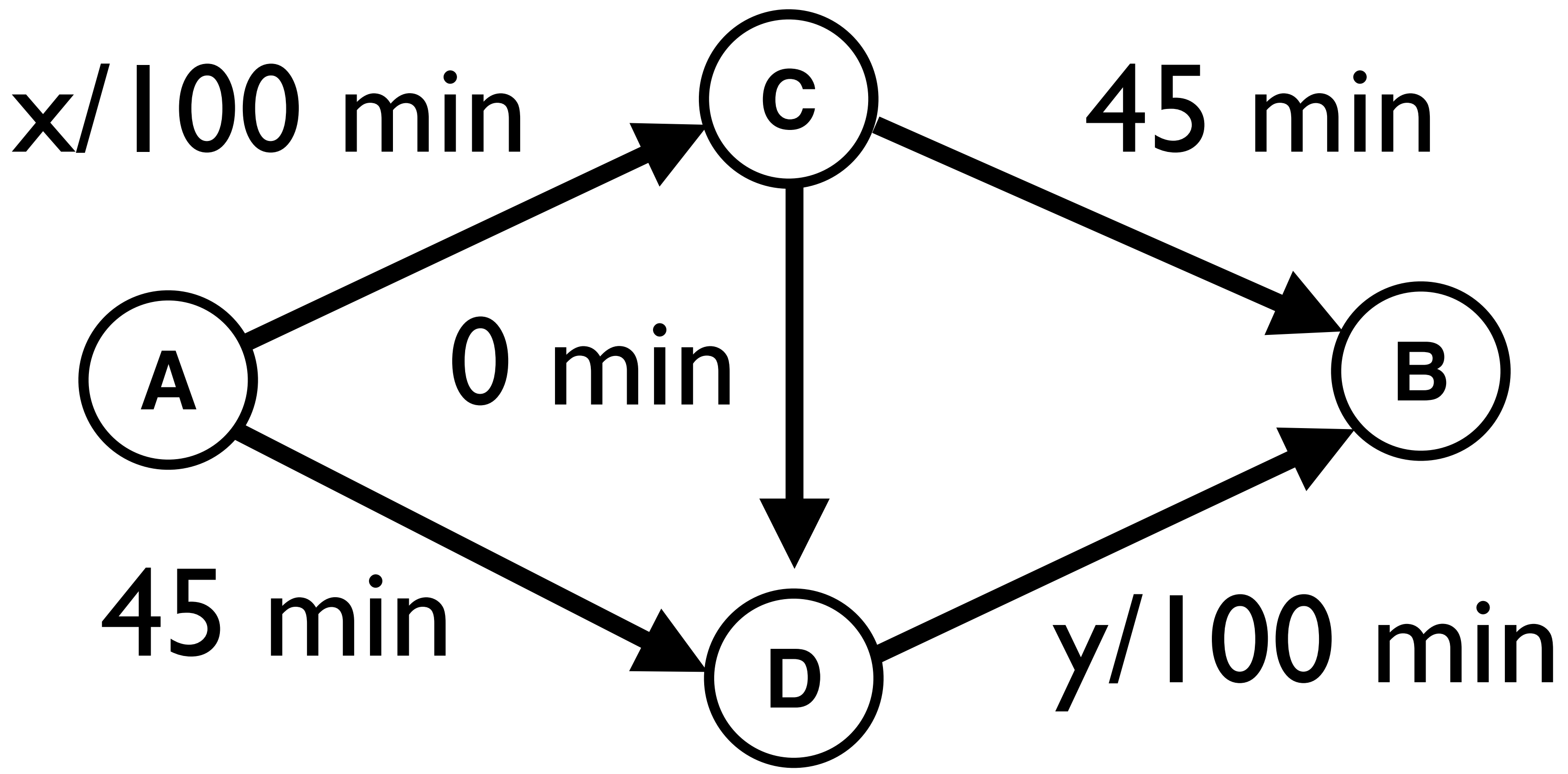
For any other set of strategies, deviation benefits someone (therefore isn't an equilibrium)



Traffic modeled as a game

Now Elon Musk adds a **teleport**!

Players can take it if they want — or not



Traffic modeled as a game

Players: Drivers 1,2,3…,4000

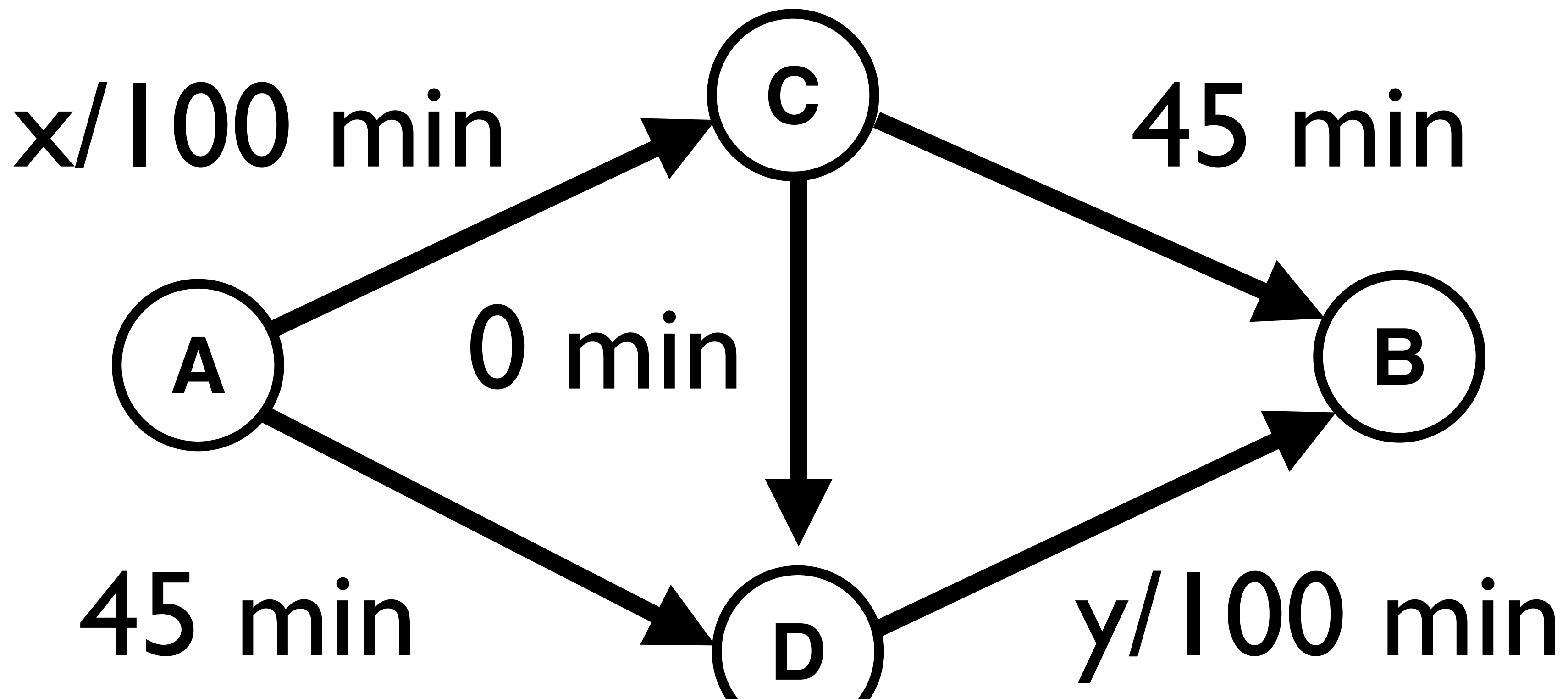
Strategies: A-C-B, A-D-B, A-C-D-B

Payoffs: Negative drive time

A-C-B time: - (x/100 + 45)

A-D-B time: - (45 + y/100)

A-C-D-B time: - (x/100 + y/100)



Would you teleport?

Say we are at the equilibrium from before: 2000 ACB, 2000 ADB, 0 ACDB

A-C-B time: - (x/100 + 45)

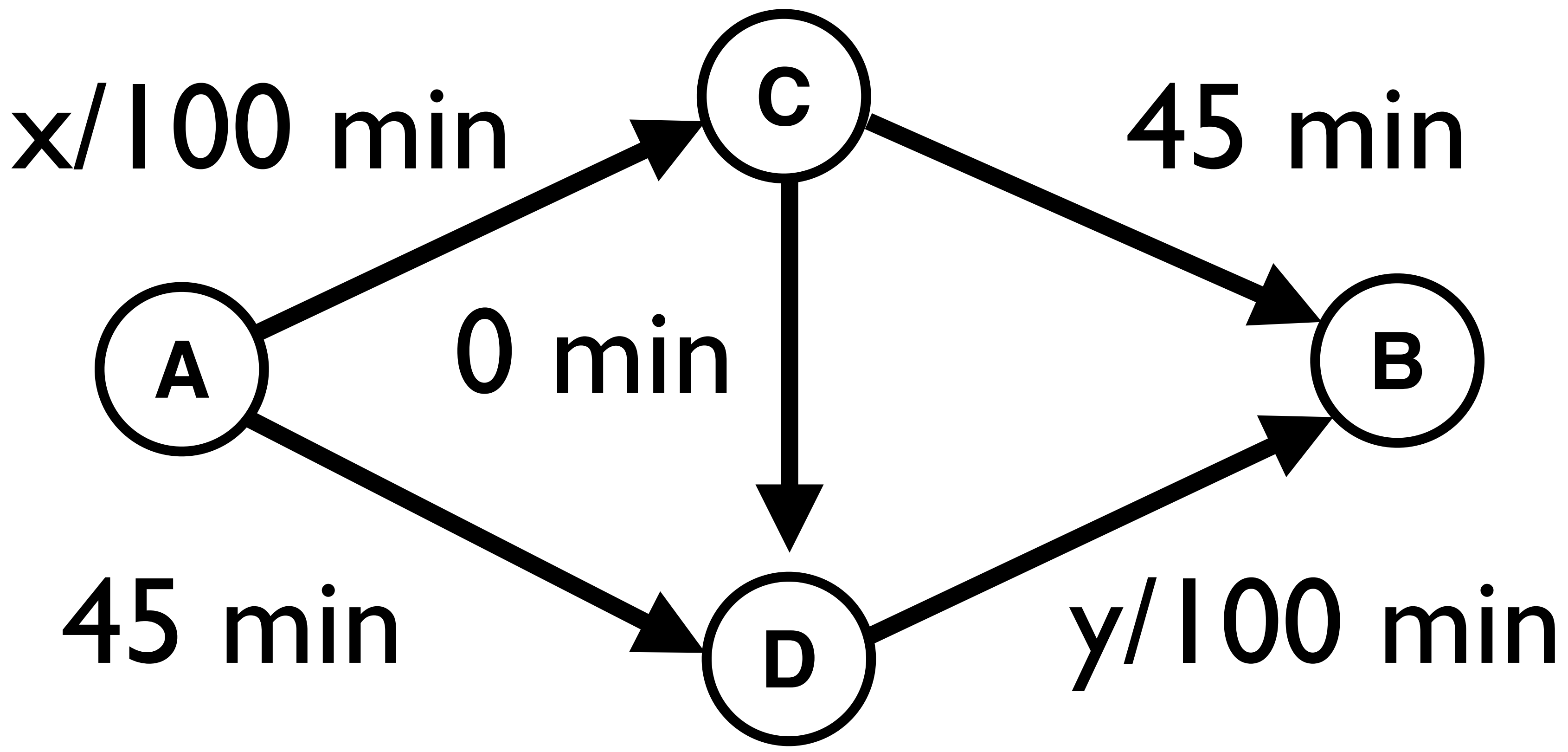
2000/100 + 45 = 65 minutes

A-D-B time: - (45 + y/100)

2000/100 + 45 = 65 minutes

A-C-D-B time: - (x/100 + y/100)

2000/100 + 2000/100 = 40 minutes



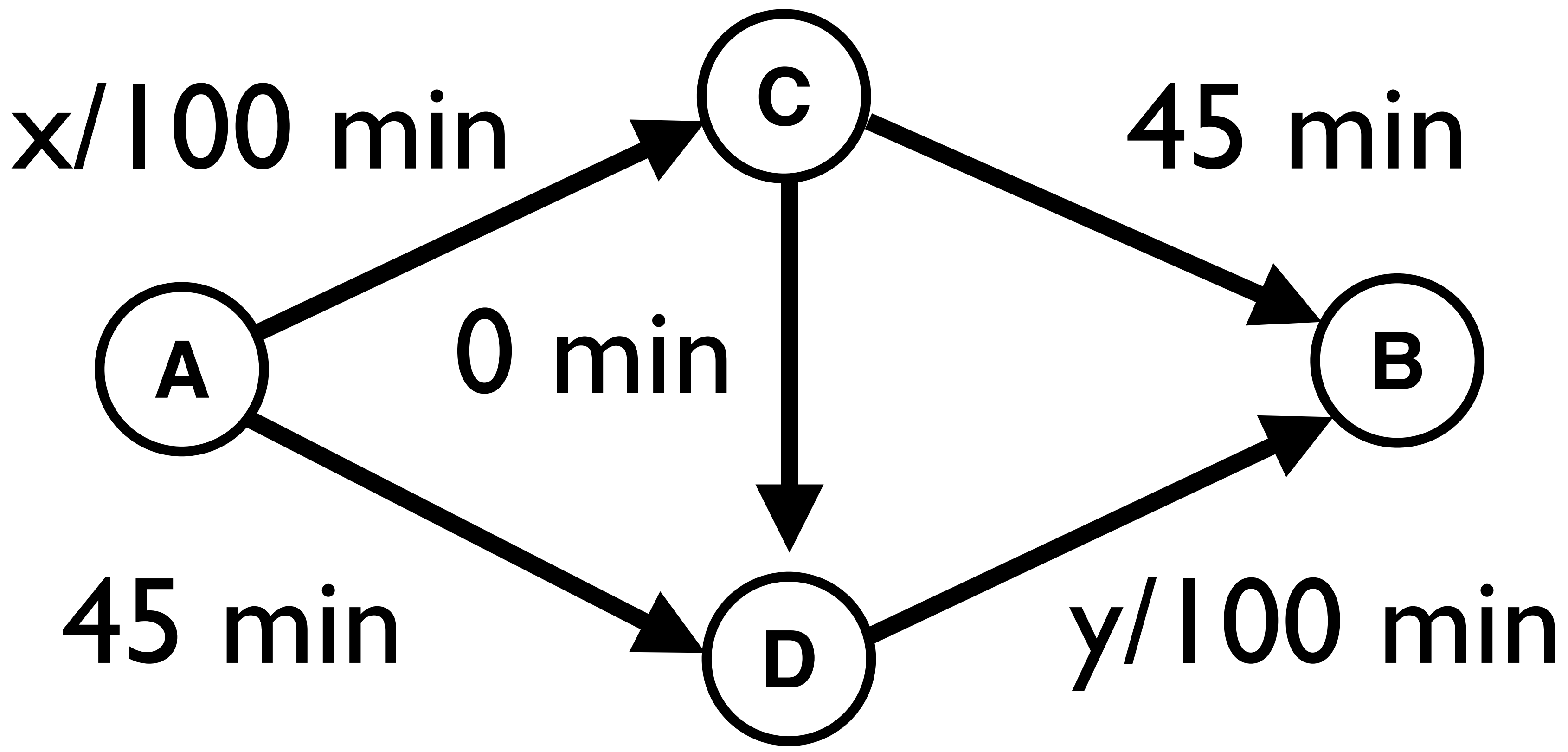
New equilibrium?

Payoffs when 0 ACB, 0 ADB, 4000 ACDB

A-C-B time: - (x/100 + 45)

A-D-B time: - (45 + y/100)

A-C-D-B time: - (x/100 + y/100)



New equilibrium?

Payoffs when 0 ACB, 0 ADB, 4000 ACDB

A-C-B time: - (x/100 + 45)

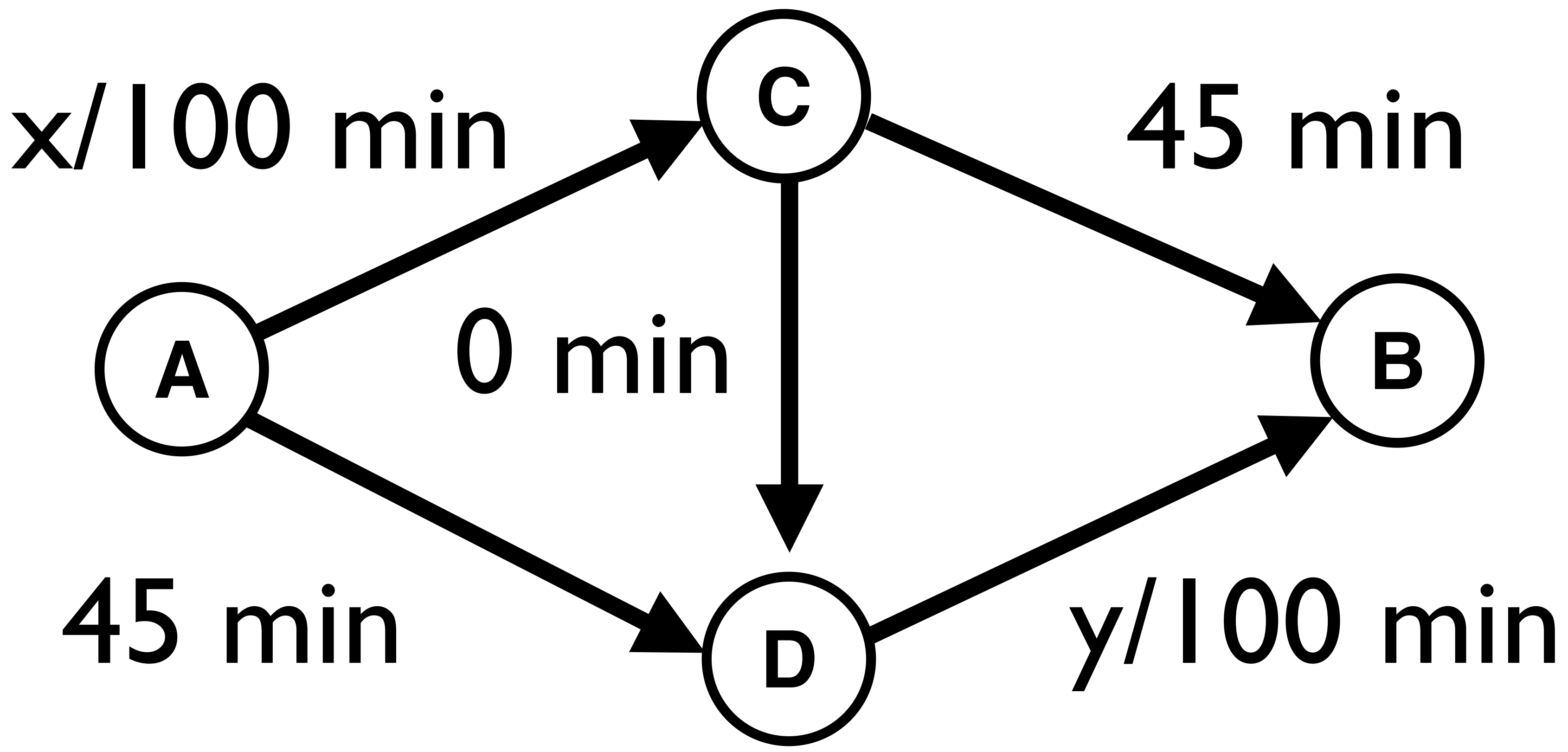
4000/100 + 45 = 85 minutes

A-D-B time: - (45 + y/100)

45 + 4000/100 = 85 minutes

A-C-D-B time: - (x/100 + y/100)

4000/100 + 4000/100 = 80 minutes



New equilibrium?

Payoffs when 0 ACB, 0 ADB, 4000 ACDB

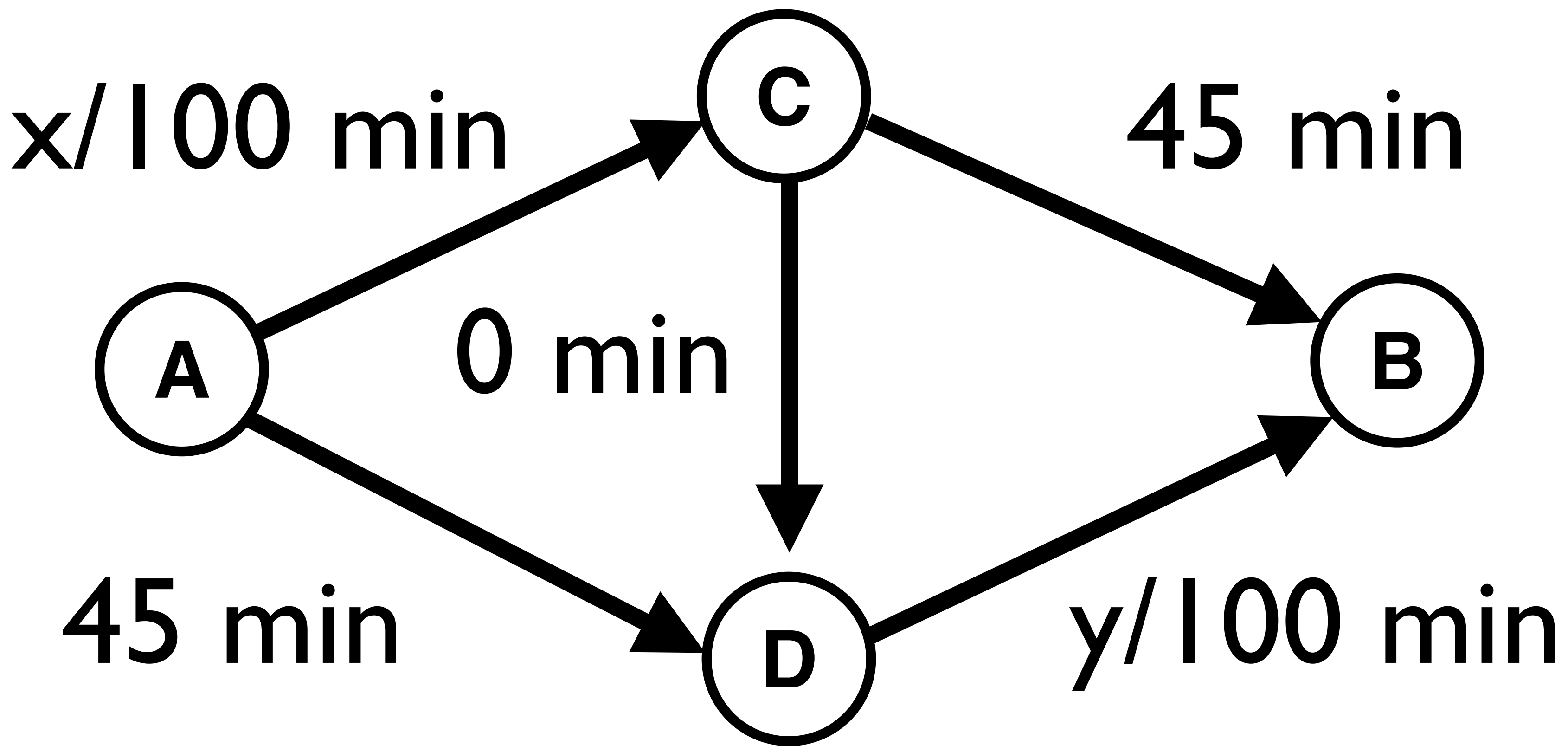
A-C-B time: - (x/100 + 45) = 4000/100 + 45 = 85 minutes

A-D-B time: - (45 + y/100) = 45 + 4000/100 = 85 minutes

A-C-D-B time: - (x/100 + y/100) = 4000/100 + 4000/100 = 80 minutes

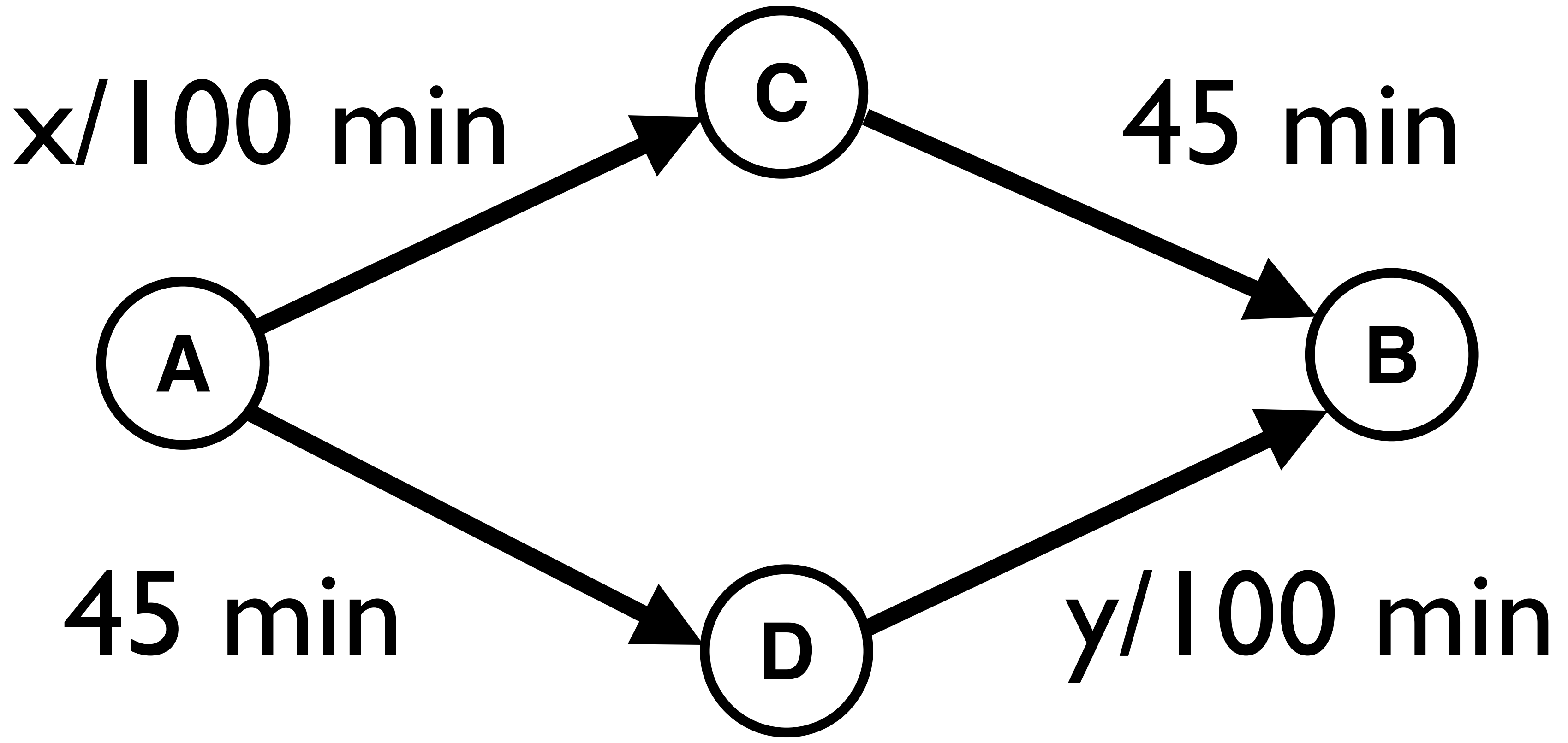
ACDB is a **strictly dominant strategy**

Everyone playing ACDB is the only equilibrium!

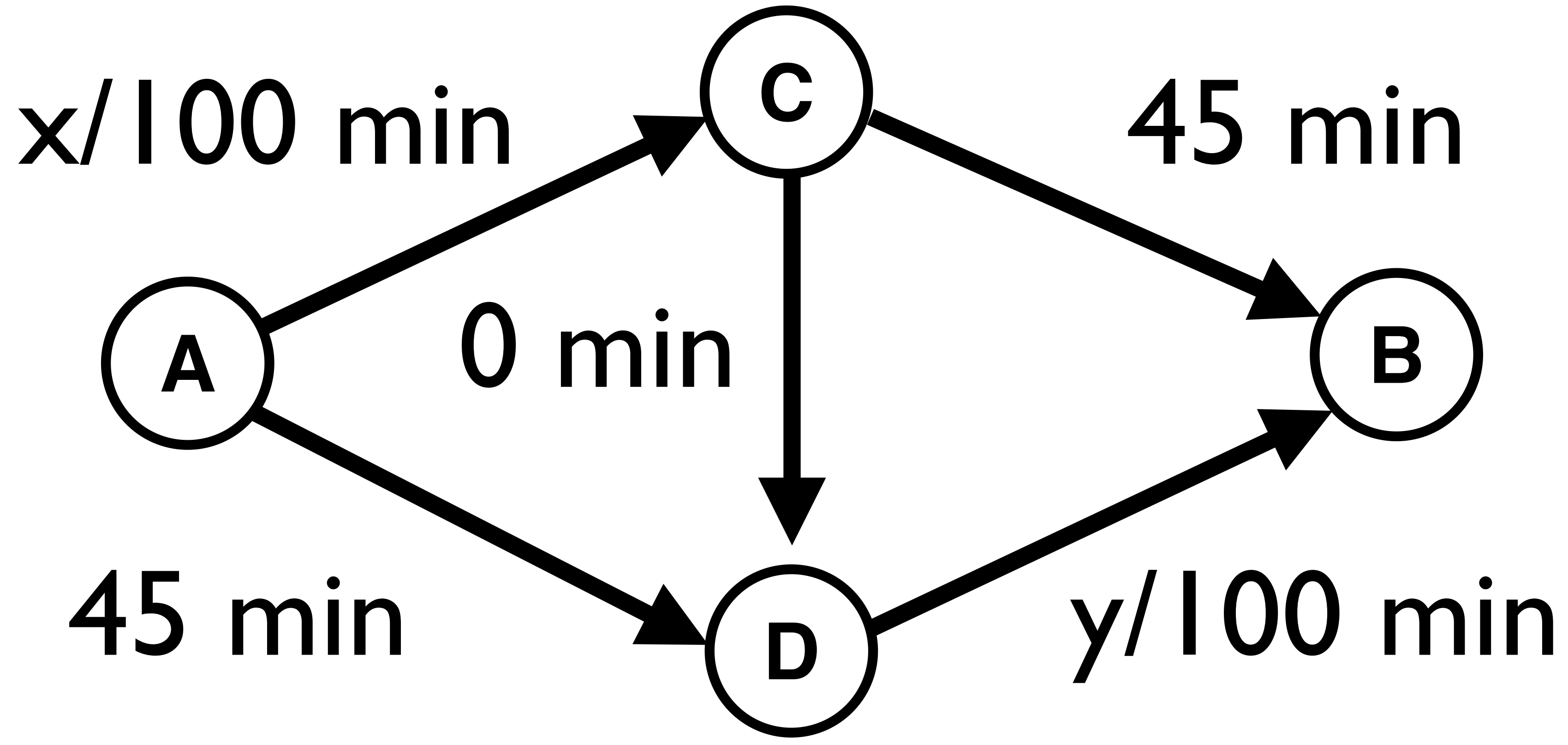


What just happened?

Equilibrium: 65 minutes for everyone



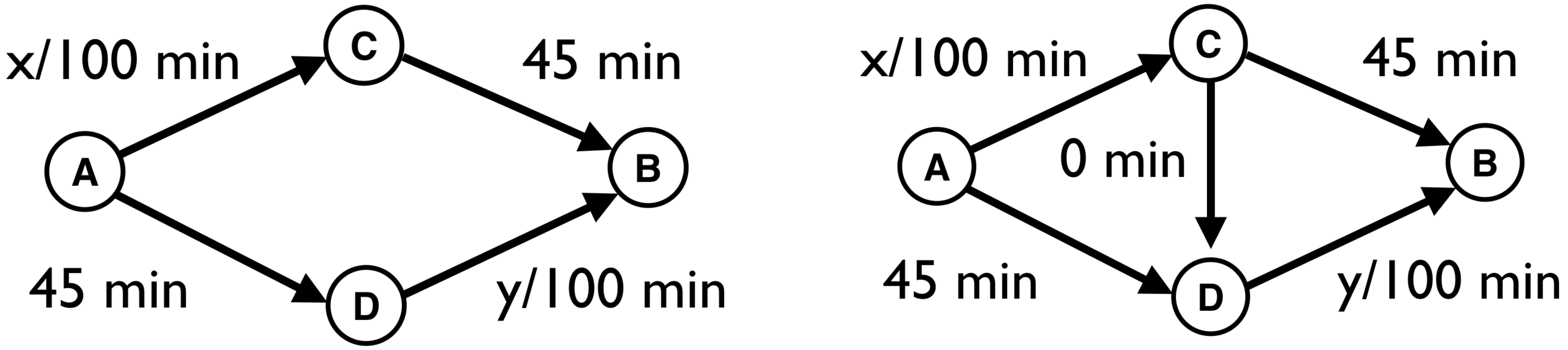
Equilibrium: 80 minutes for everyone



Same network but with an extra teleport

Braess's Paradox

Routing:

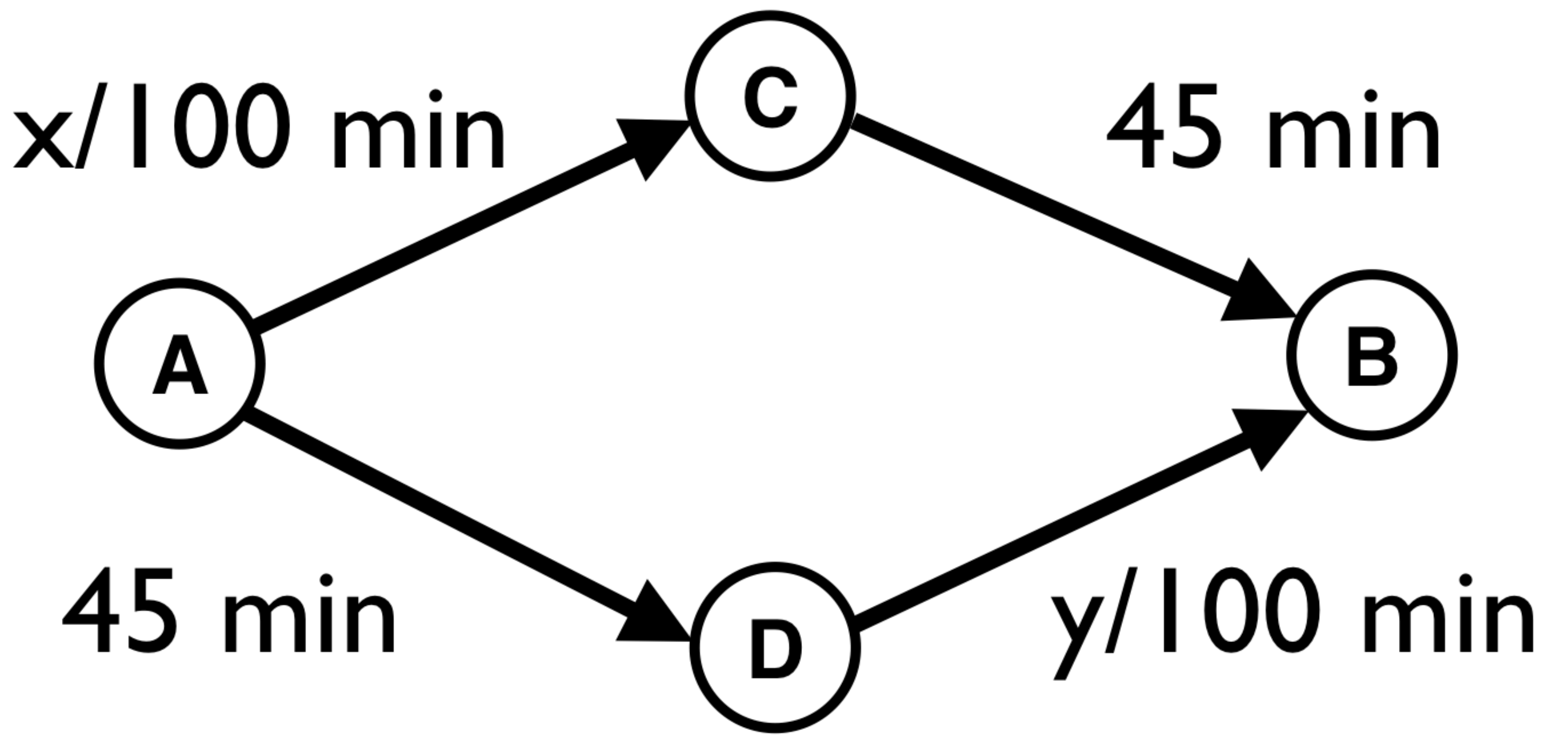


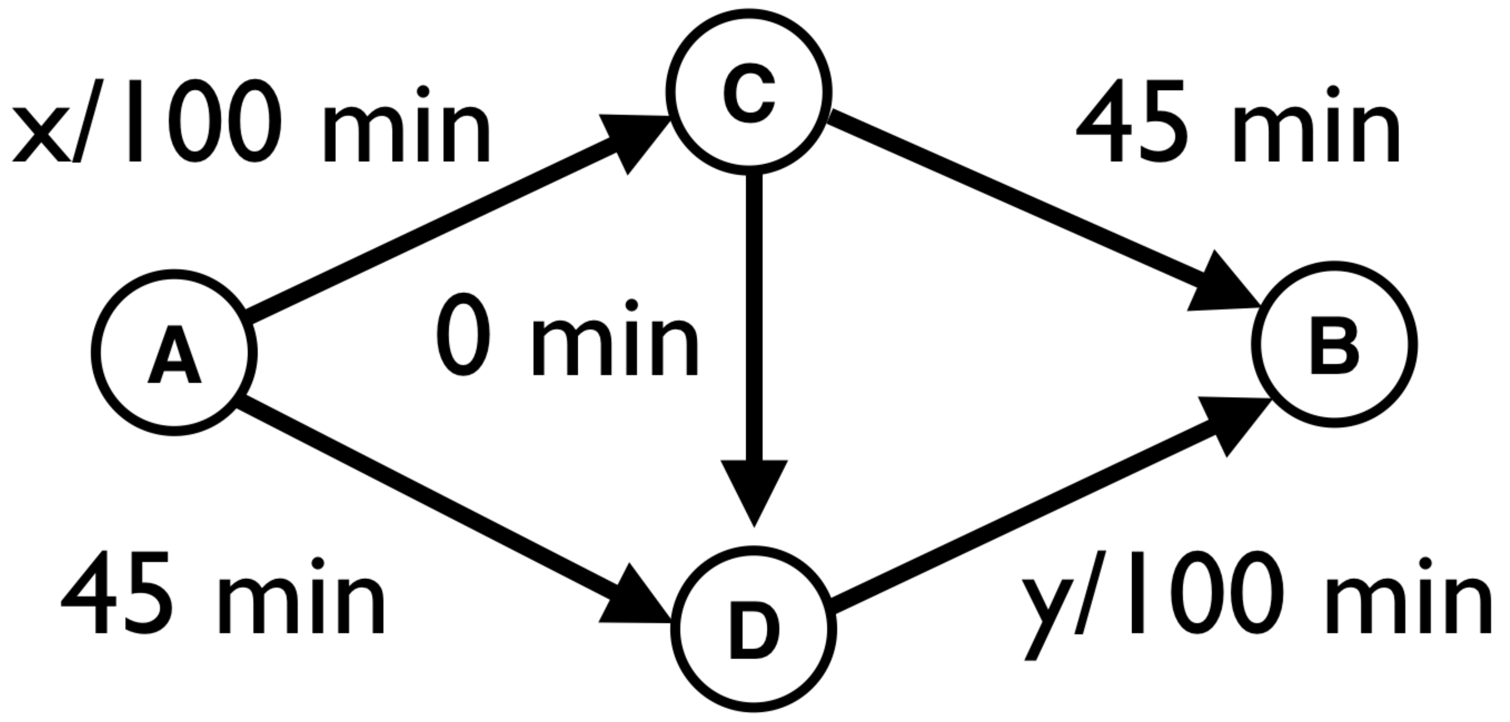
Prisoner's Dilemma:

		Suspect 2	
		NC	C
Suspect 1	NC	$-1, -1$	$-10, 0$
	C	$0, -10$	$-4, -4$

Sometimes strategies can hurt you

Routing:





Prisoner's Dilemma:

		Suspect 2	
		NC	C
Suspect 1	NC	$-1, -1$	$-10, 0$
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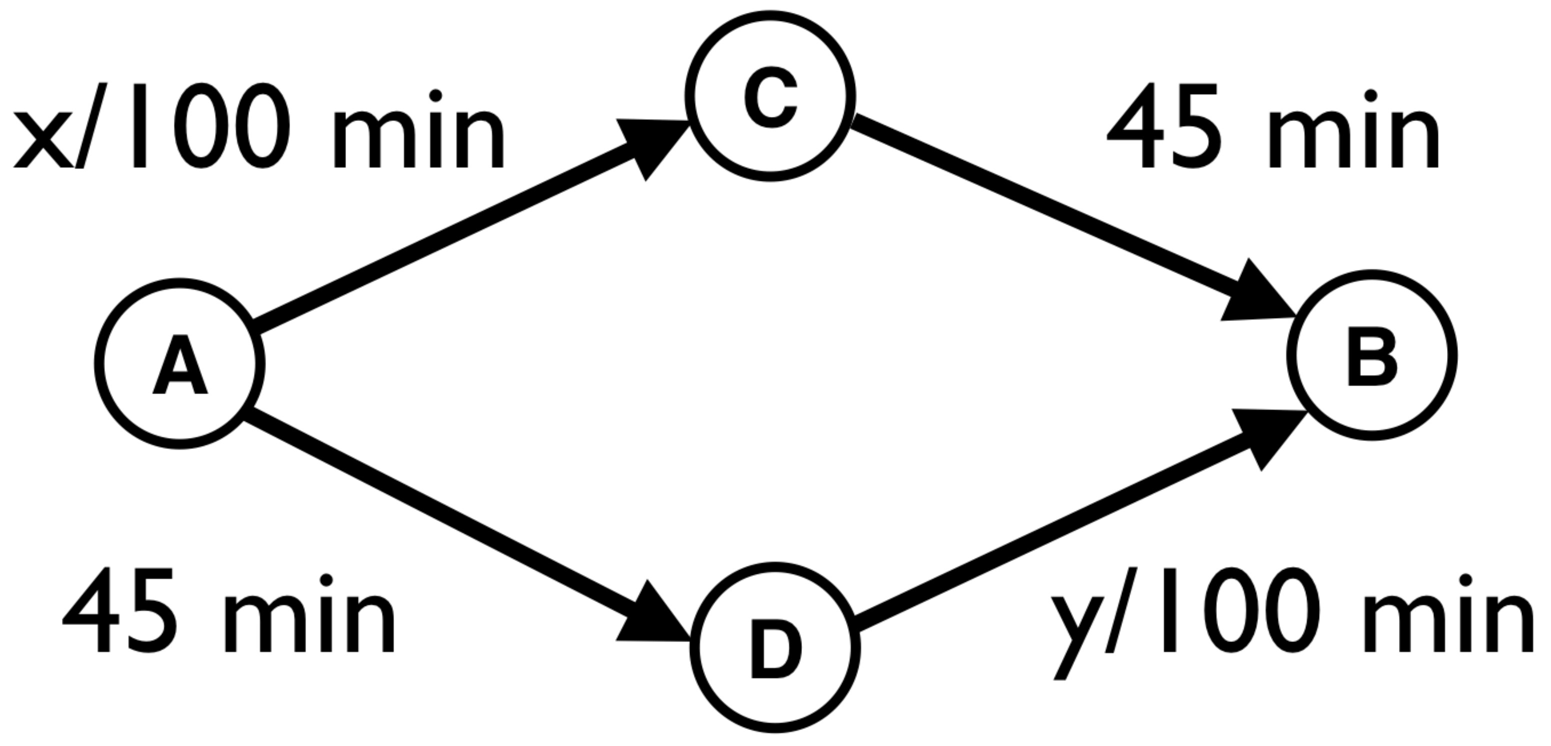
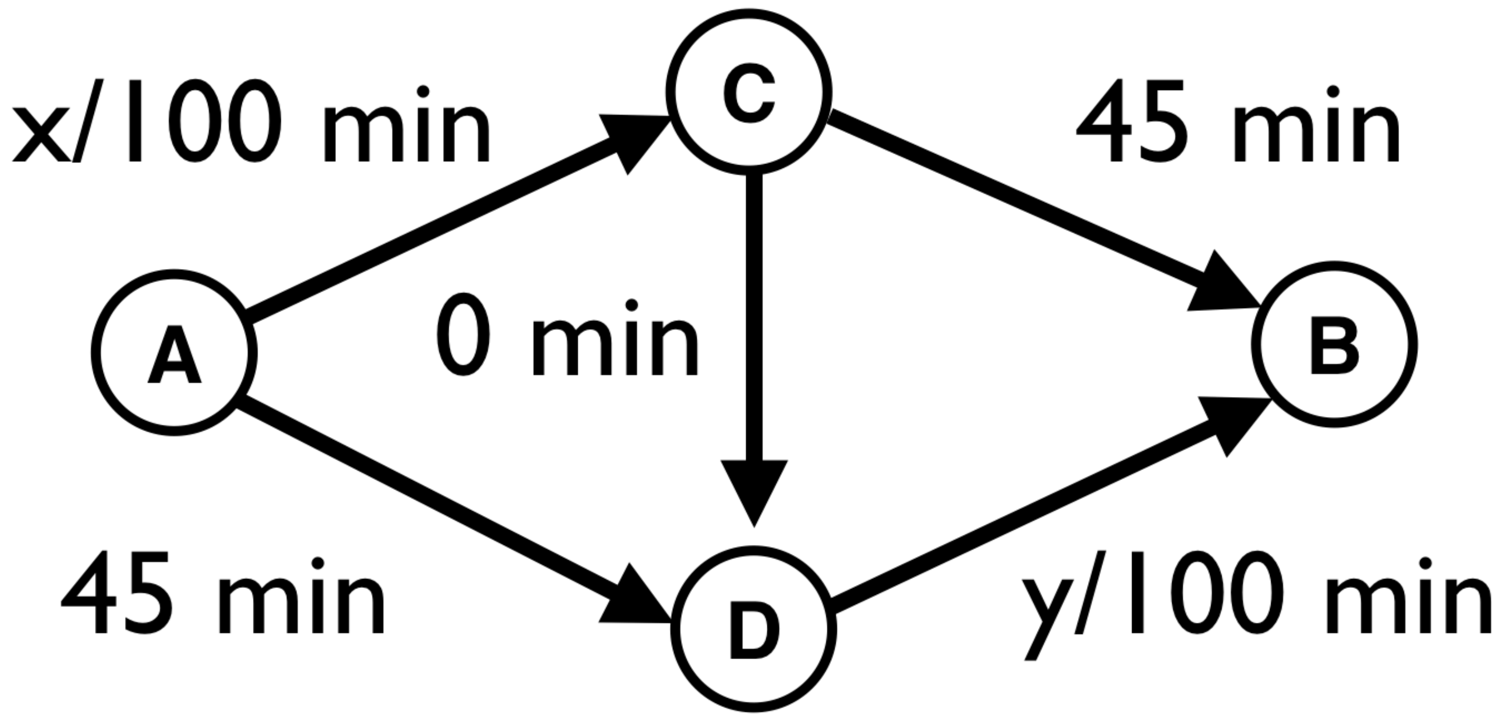
How bad can it get?

Routing:





Ratio between socially optimal and selfish routing (called the "Price of Anarchy")?

This example: 80/65 = 1.23x worse

Worst case: How bad can it get?

For selfish routing, "Price of Anarchy" = 4/3

Diffusion of Decisions

Social Decisions

Lots of decisions you make depend on what your friends are doing

Where to go?

What game to play?

What software to use?

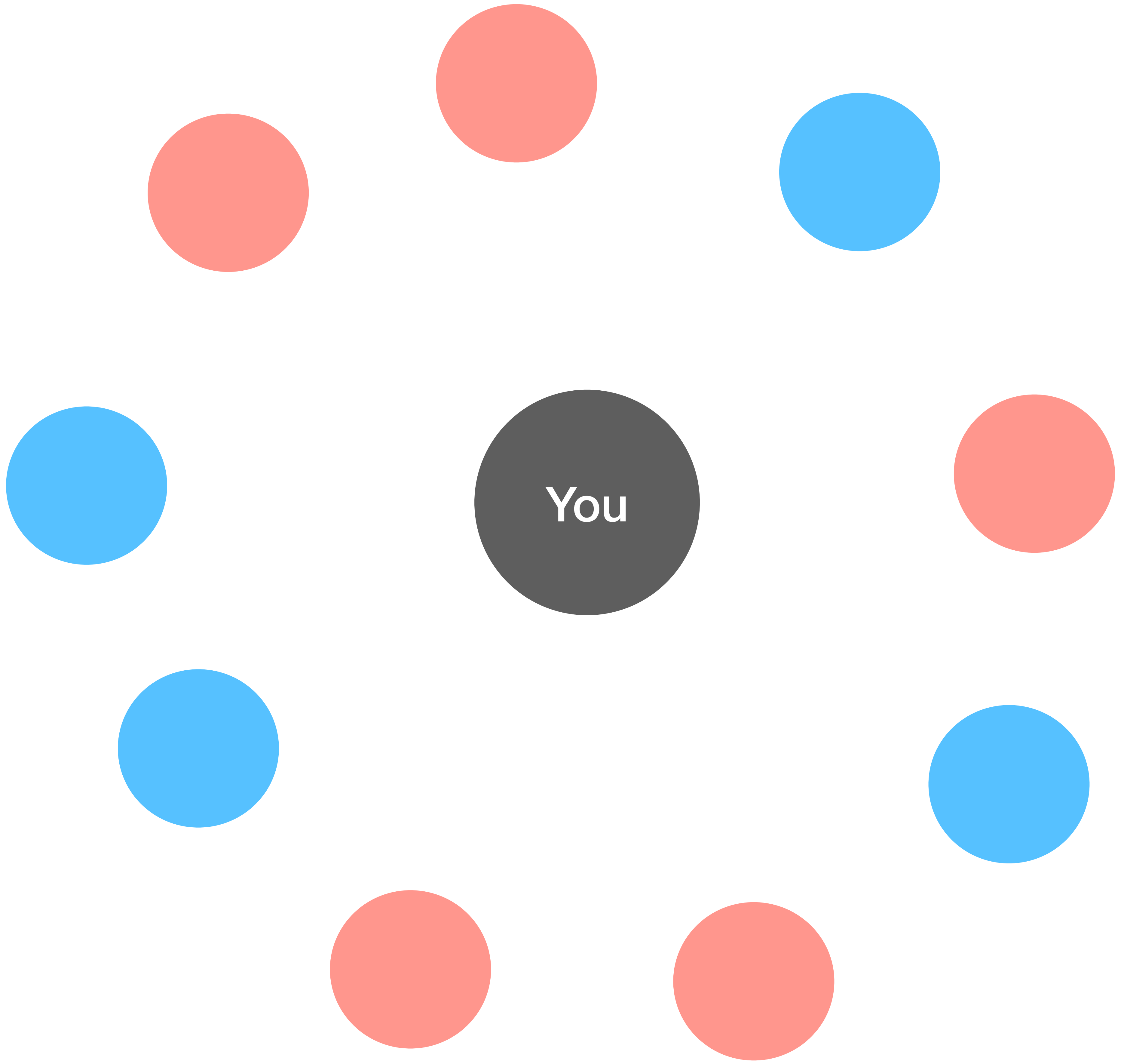
What OS to use?

Snapchat vs. Instagram

BluRay vs. HD DVD

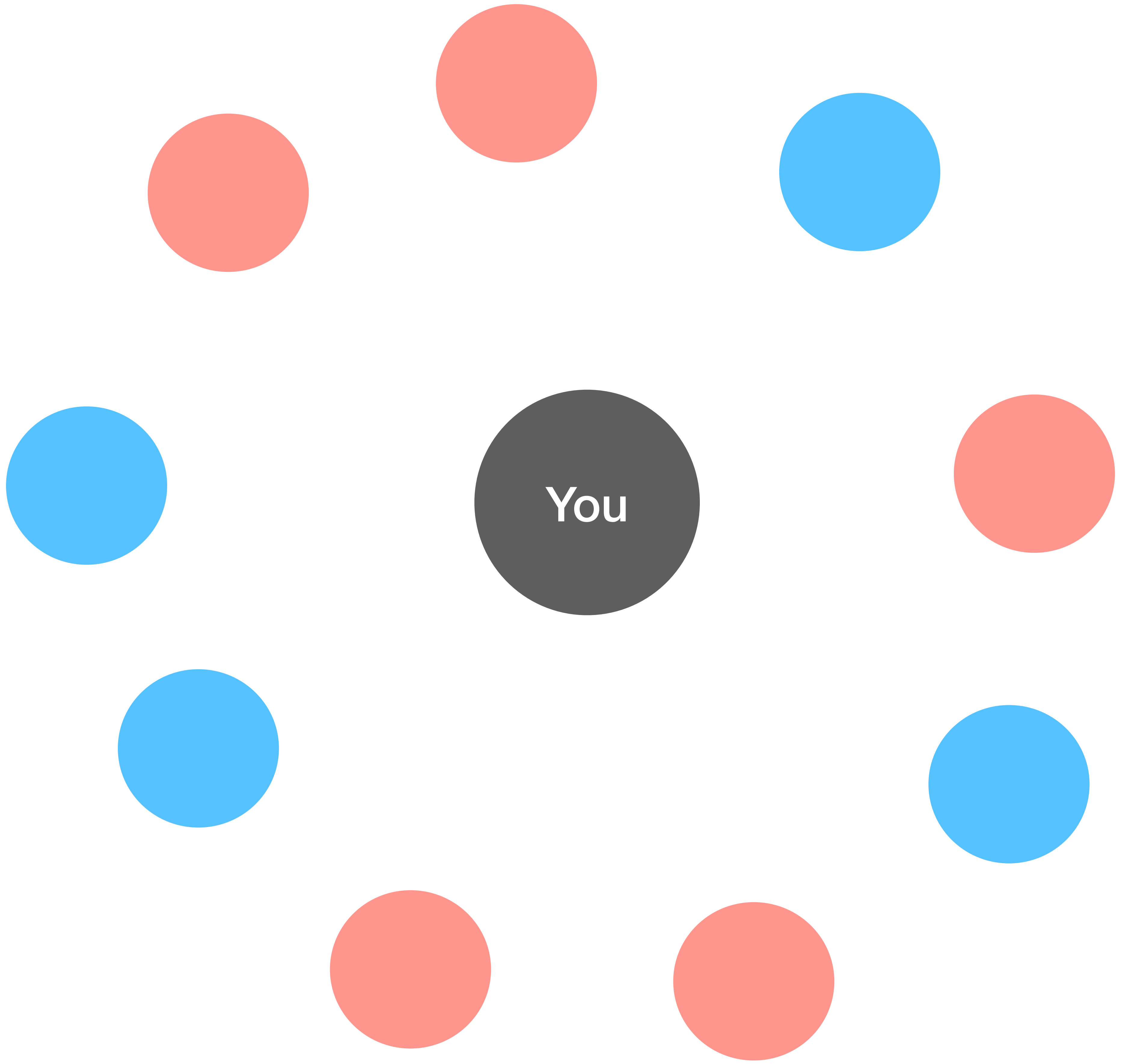
Electric Car vs. Diesel Truck

How to Reason About Social Decisions?



Given that your friends have all chosen one way or another, what should you choose?

How to Reason About Social Decisions?



"Network Effects"

Game Theoretic Model of Cascades

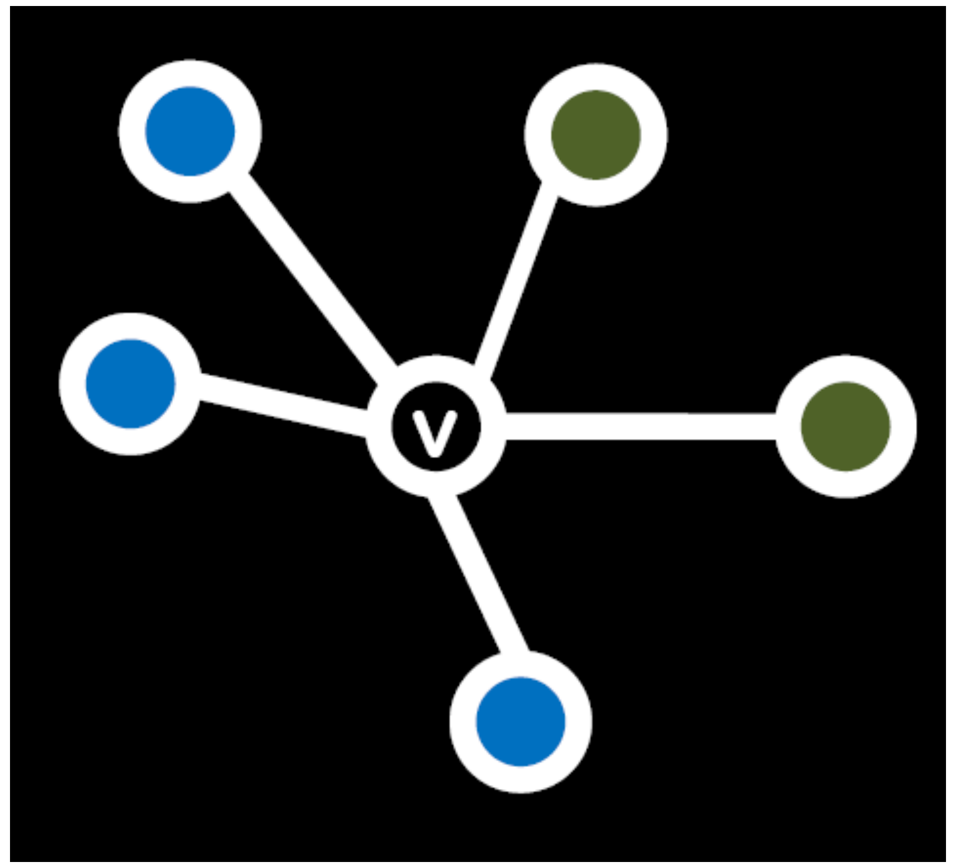
Social Networks + Game Theory can help us think about this question!

Model every friendship edge as a 2 player coordination game

2 players – each chooses technology A or B

Each person can only adopt **one** "behavior", **A** or **B**

You gain more payoff if your friend has adopted the **same** behavior as you



Local view of the network of node **v**

		w: A	w: B
v	A	a, a	$0, 0$
	B	$0, 0$	b, b

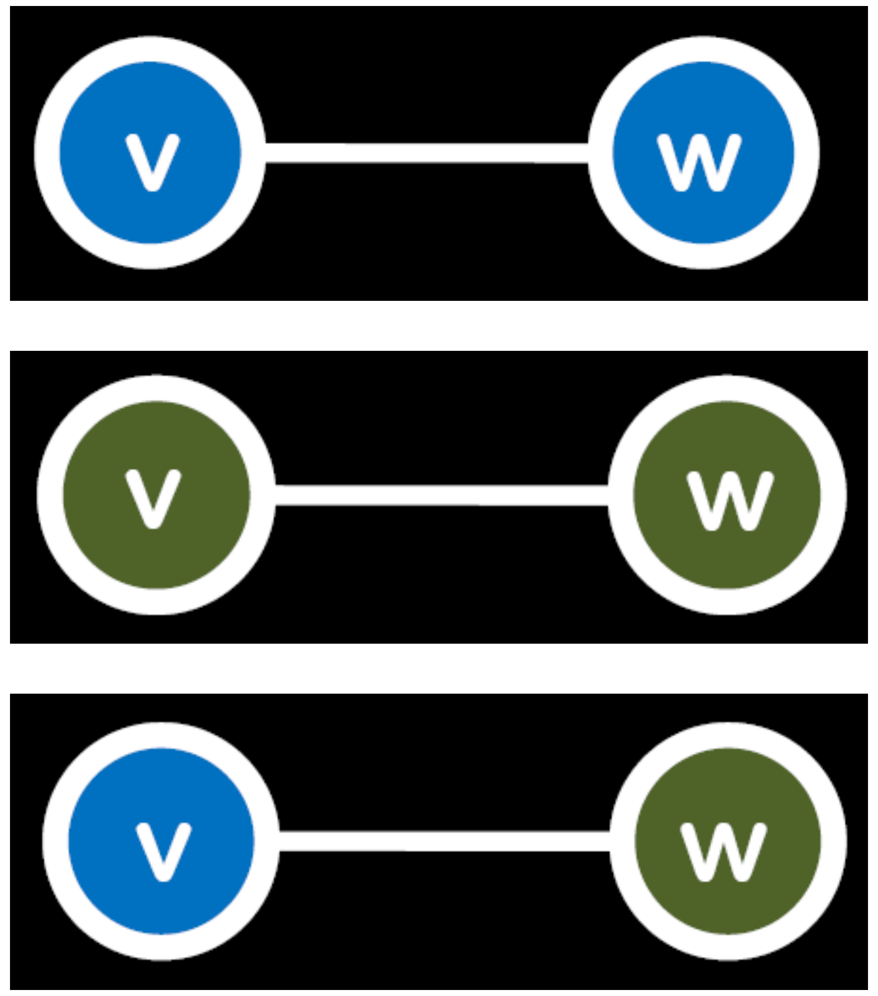
The Model for Two Nodes

Payoff matrix:

If both v and w adopt behaviour A, they each get payoff *a > 0*

If *v* and *w* adopt behaviour *B*, they each get payoff *b > 0*

If *v* and *w* adopt the opposite behaviours, they each get *0*



In some large network:

Each node *v* is playing a copy of the coordination game with each of its neighbours

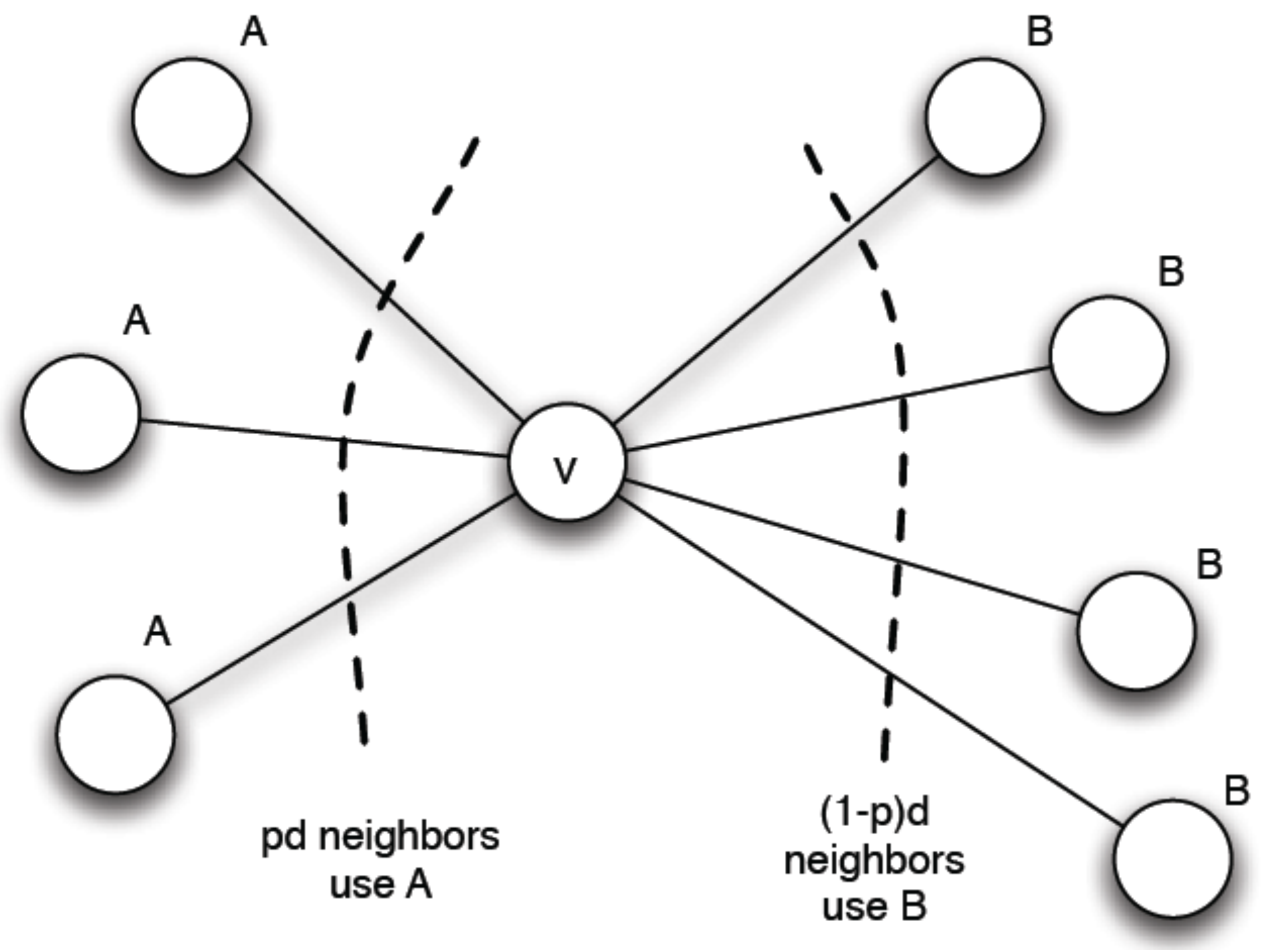
Payoff: sum of node payoffs per game

		w: A	w: B
v	A	a, a	$0, 0$
	B	$0, 0$	b, b

Calculation of Node *v*

Let **v** have ***d*** neighbours — some adopt **A** and some adopt **B**

Say fraction ***p*** of ***v***'s neighbours adopt ***A*** and **1-p** adopt **B**



$Payoff_v = a \cdot p \cdot d$ if v chooses A

$\quad\quad\quad = b \cdot (1-p) \cdot d$ if v chooses B

Thus: v chooses A if:

$a \cdot p \cdot d > b \cdot (1-p) \cdot d$

Threshold:

v chooses ***A*** if $p > \frac{b}{a+b} = q$

p… frac. v's neighbours choosing A

q… payoff threshold

Example Scenario

Scenario:

Graph where everyone starts with *B*

Small set *S* of early adopters of *A*

Hard-wire *S* – they keep using *A* no matter what payoffs tell them to do

Assume payoffs are set in such a way that nodes say:

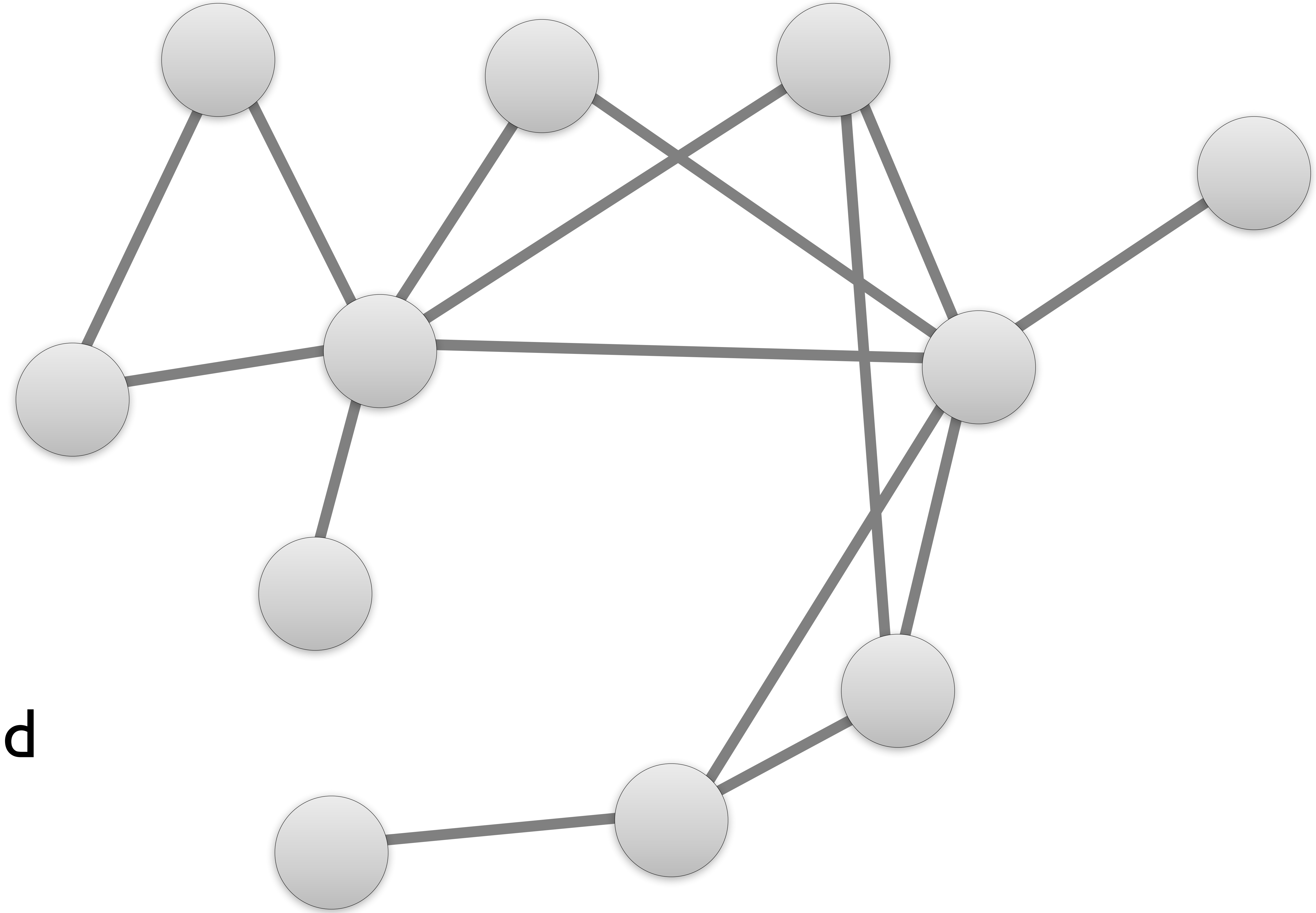
If **more than 50%** of my friends take A

I'll also take A

(this means: $a = b - \varepsilon$ and $q > 1/2$)

Example Scenario

$S = \{u, v\}$



If more than
q=50% of my
friends are red
I'll be red

Example Scenario

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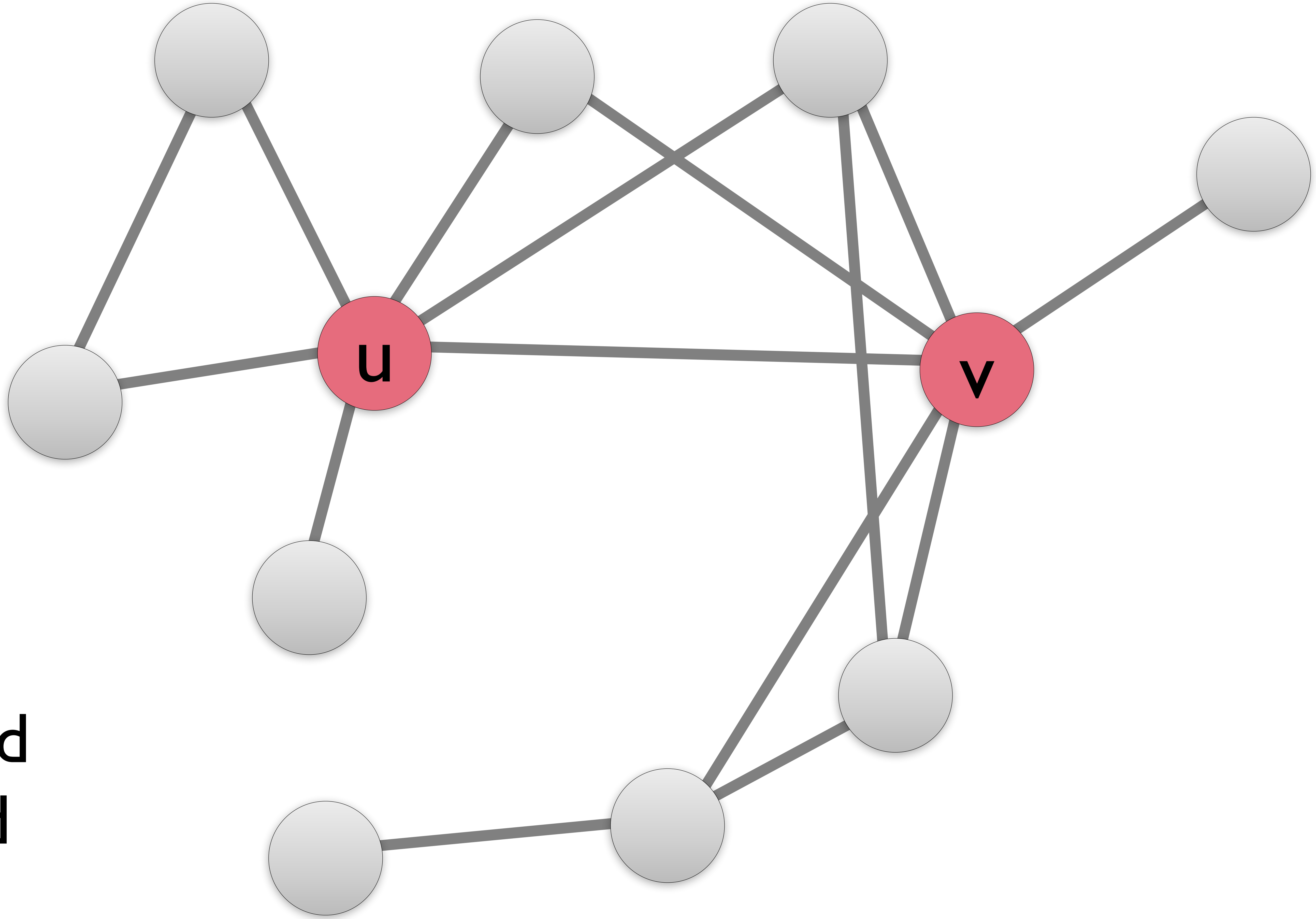

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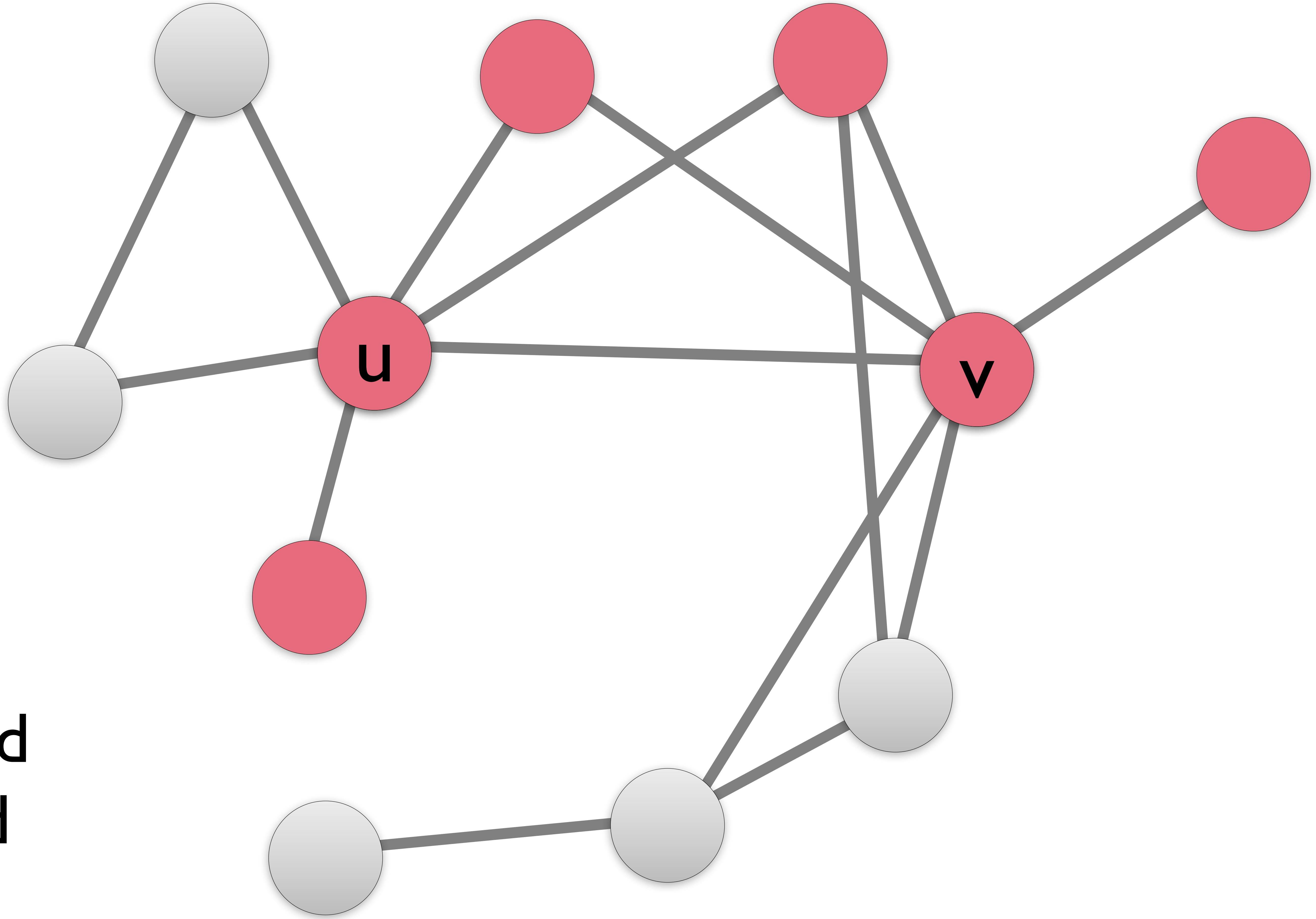

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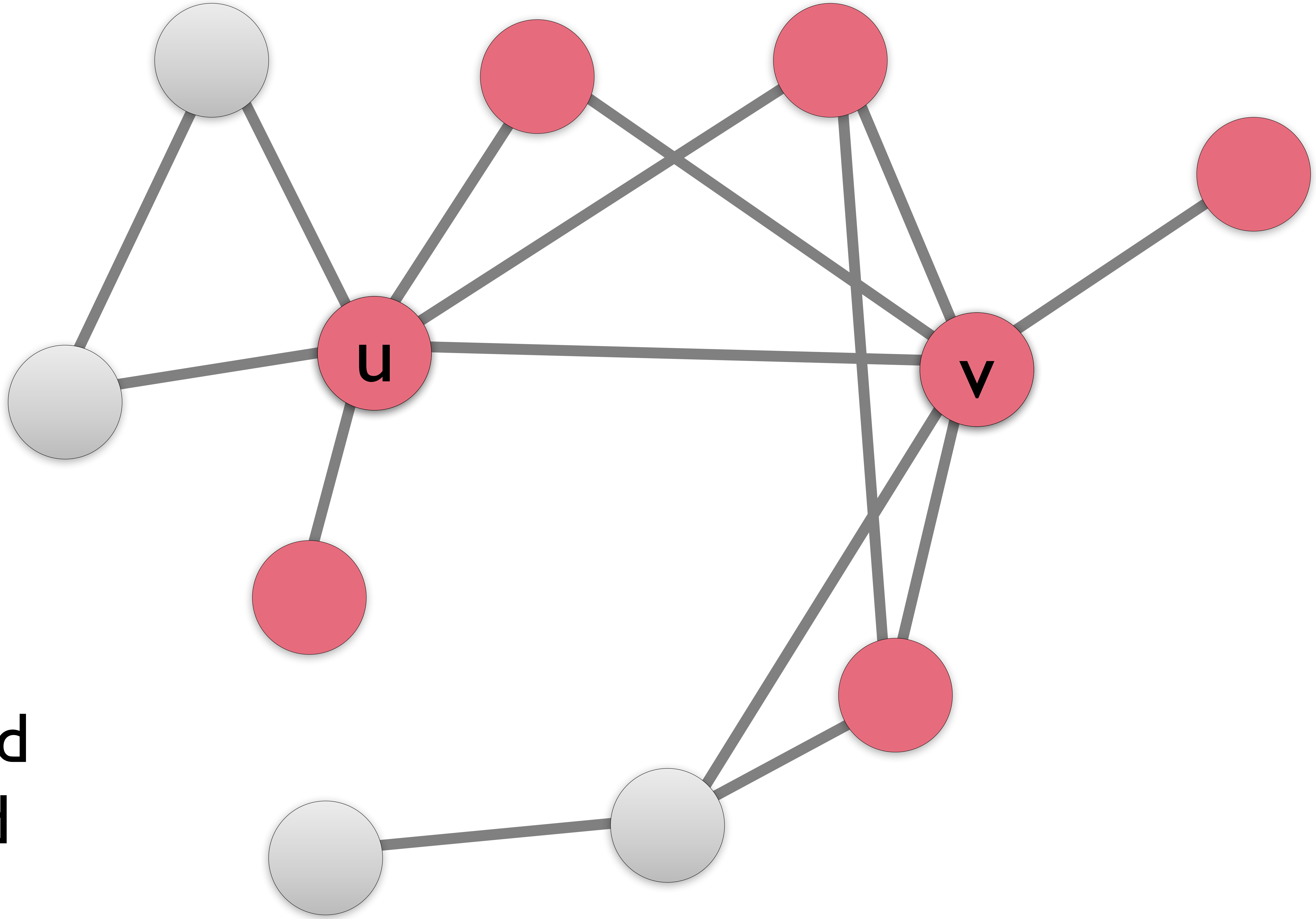

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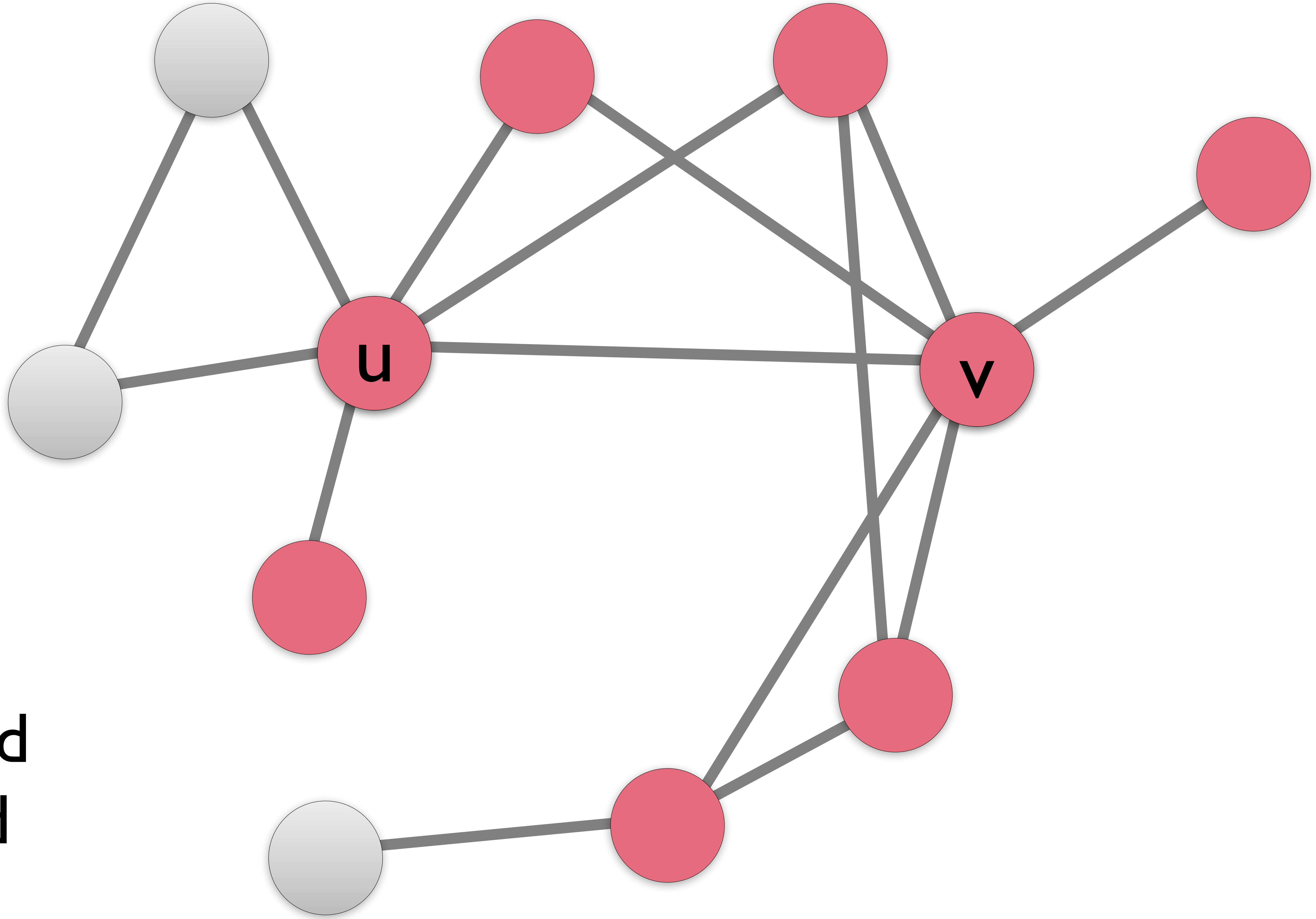


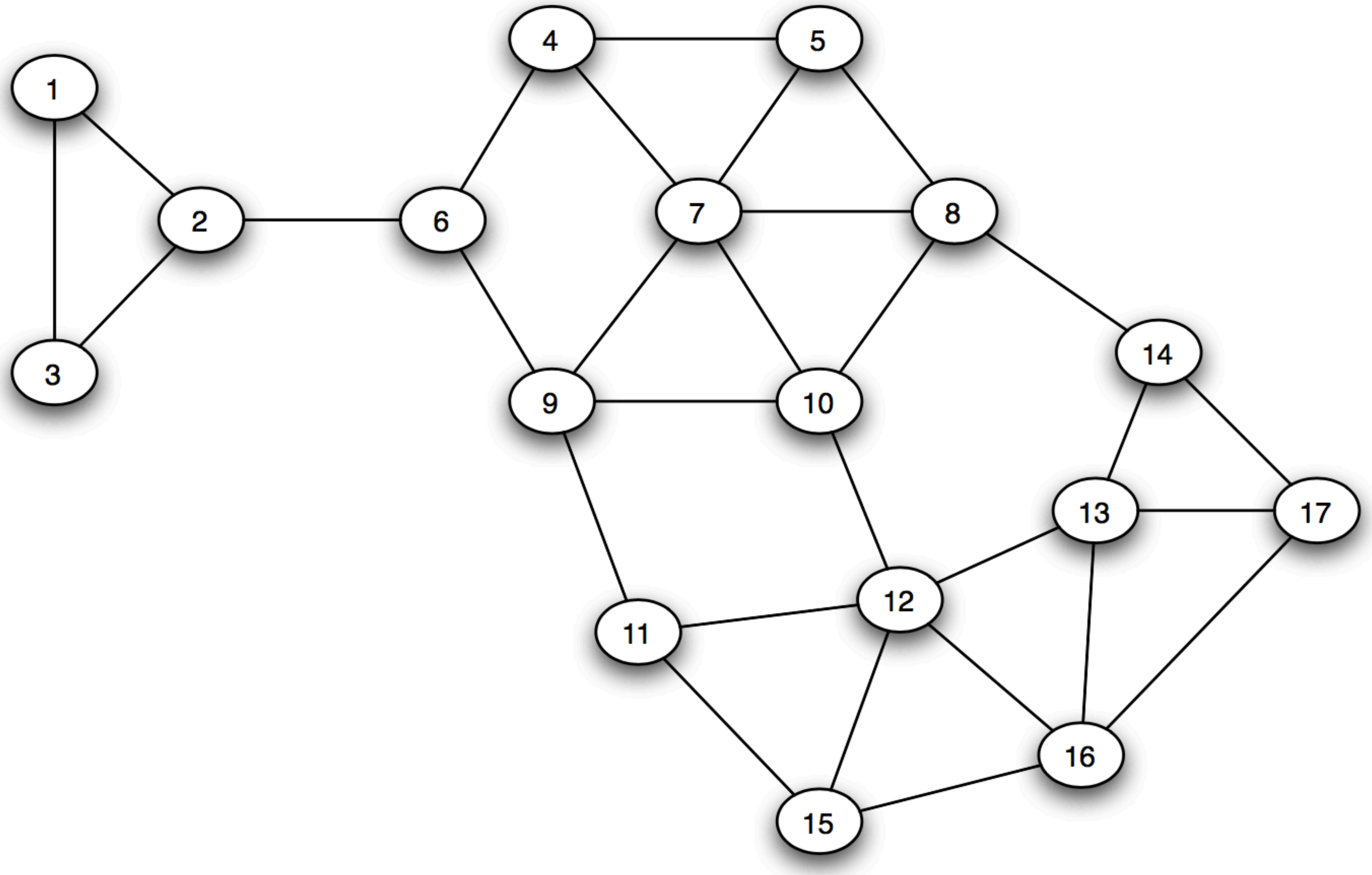
If more than q=50% of my friends are red I'll also be red

Another example with a=3 and b=2

$$p > \frac{b}{a+b} = q$$

q = 2/5

(new technology better, so q<1/2)



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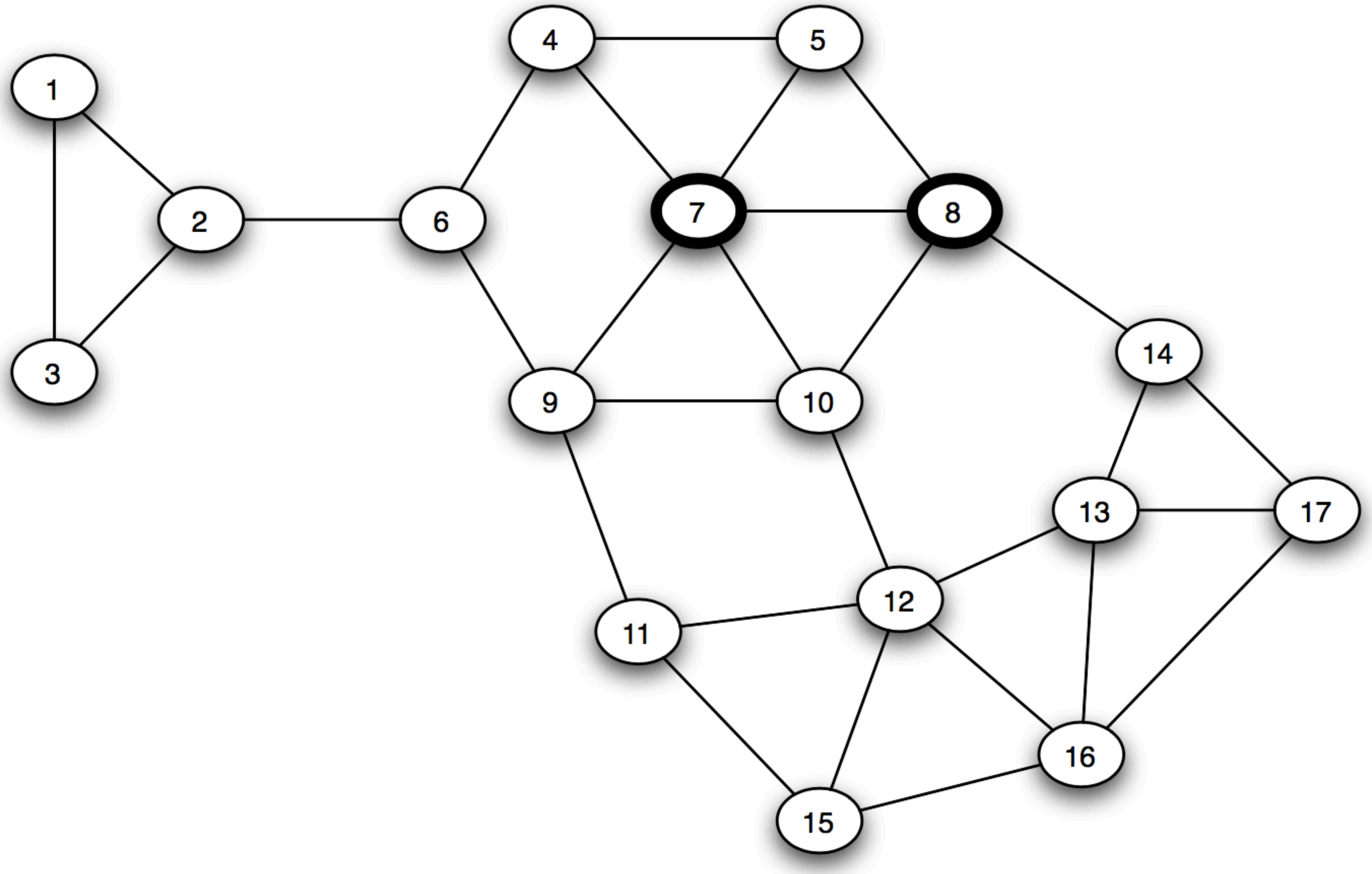

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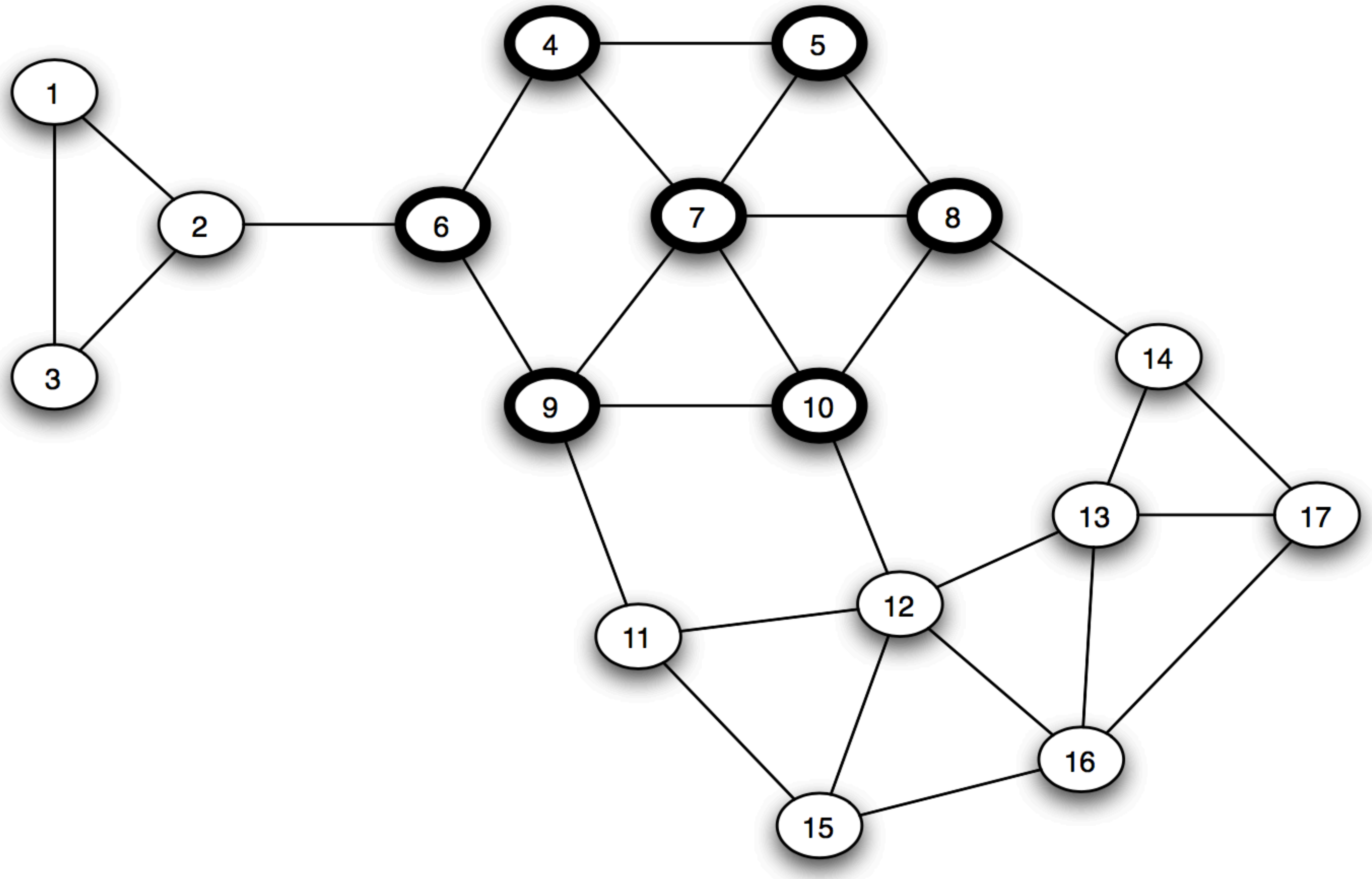


After three steps it stops

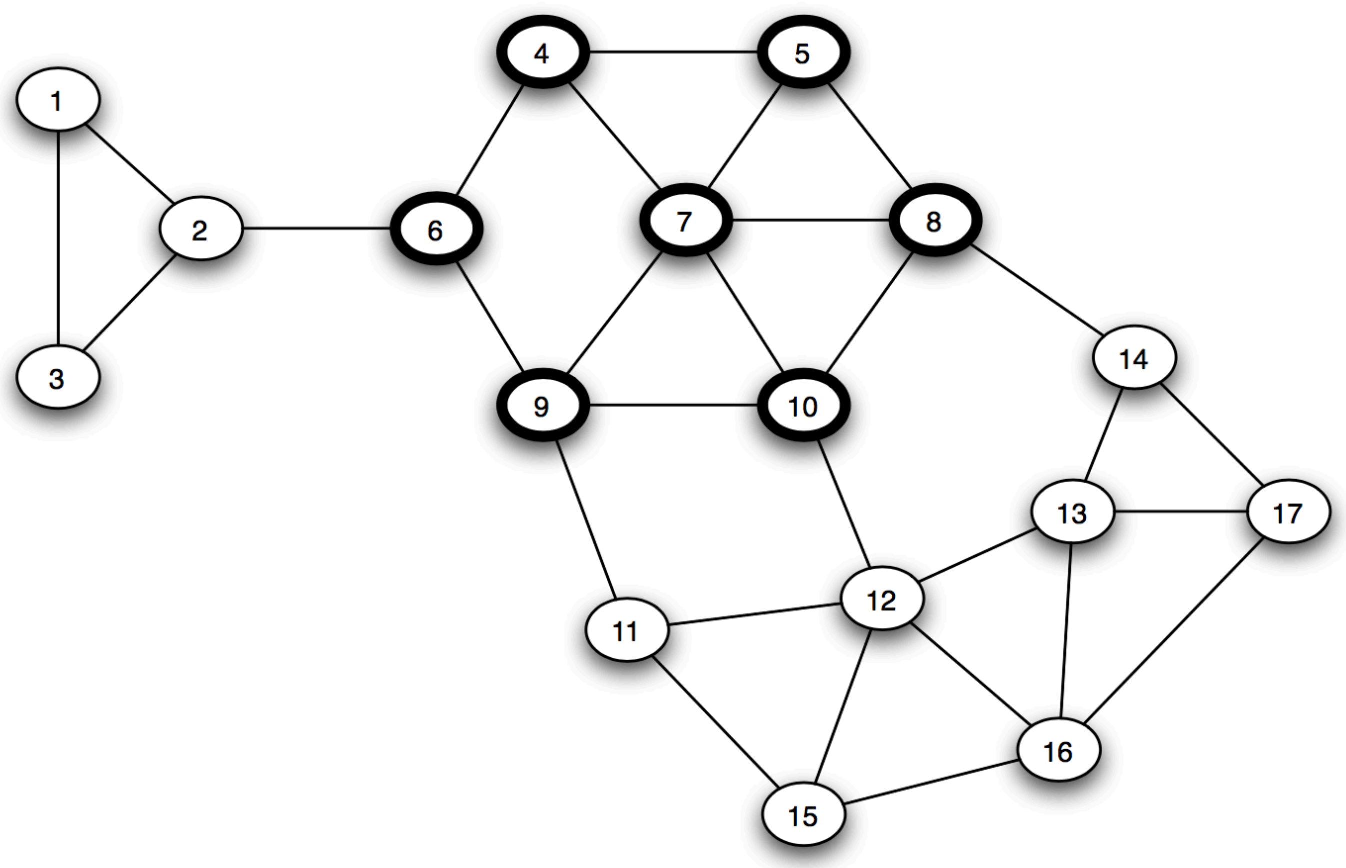
Another example with a=3 and b=2

A spread to nodes with sufficiently dense internal connectivity

But it could never bridge the "gaps" that separate nodes 8–10 and 11–14, and node 6 and node 2

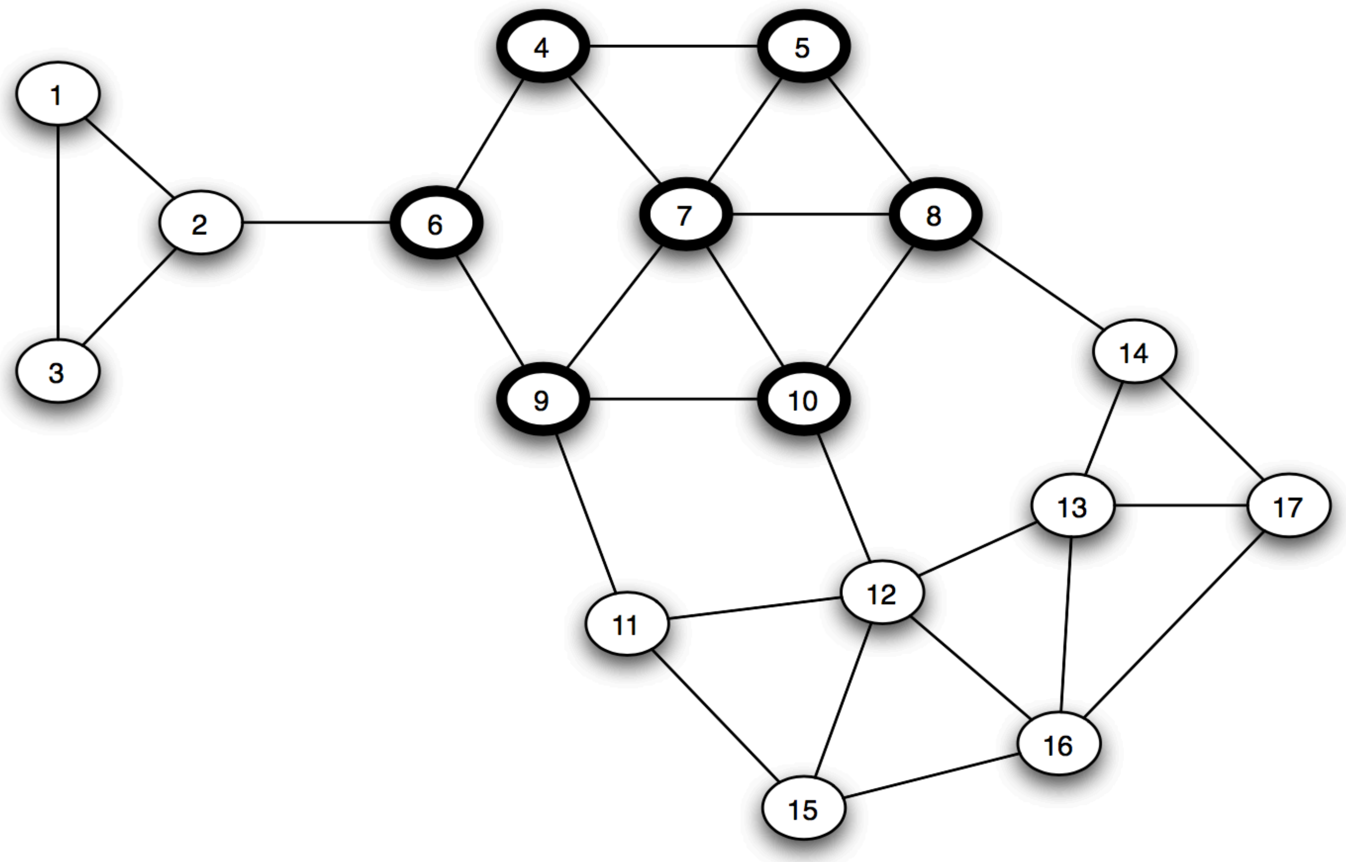
Result: coexistence of **A** and **B**, boundaries in the network where the two meet

- Different dominant political/religious views between adjacent communities
- Different social networking sites dominated by different age groups and lifestyles
- Windows vs. Mac (there are industries that heavily use Mac, even though Windows generally dominates)



Another example with a=3 and b=2

What could **A** do to improve its reach?



Raise quality of the product:

- If payoff in underlying coordination game improves from a=3 to a=4
- Threshold to switch drops from q=2/5 to q=1/3
- All nodes eventually switch to **A**

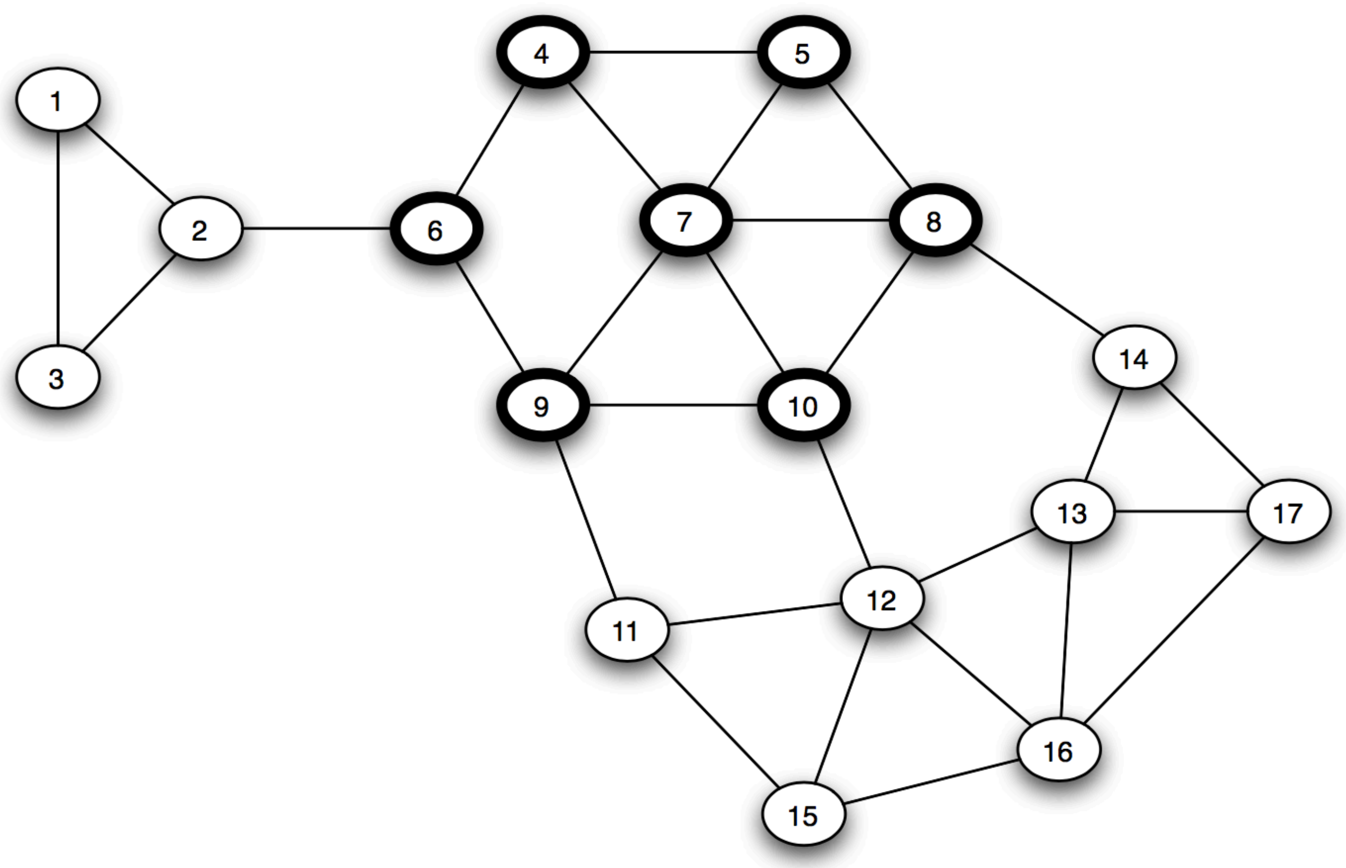
Slightly increasing the quality of innovations can dramatically alter their reach

Another example with a=3 and b=2

What could **A** do to improve its reach?

Convince key people to be early adopters

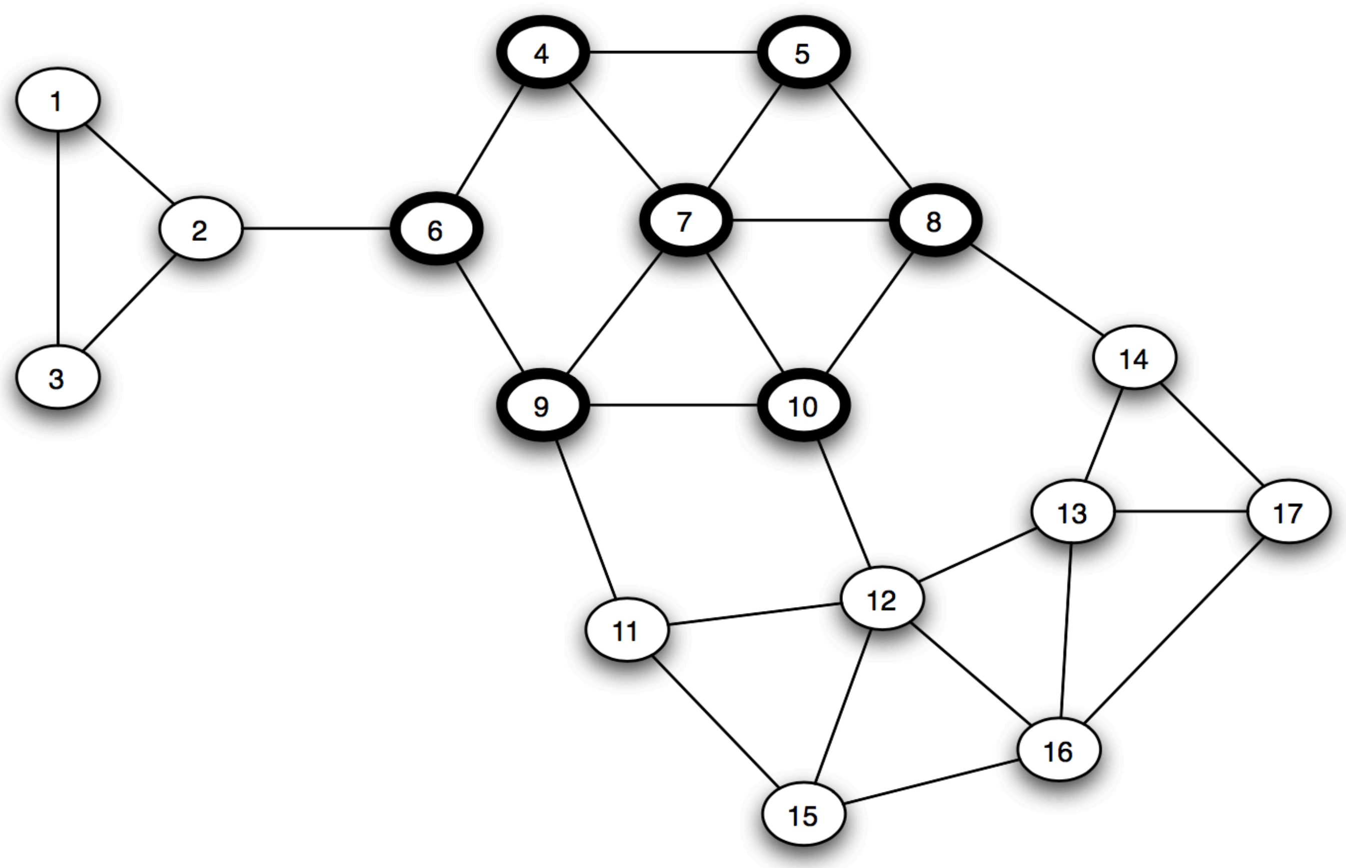
- Sometimes it's impossible to raise the quality any higher than it already is
- Threshold stays the same (here q=2/5)
- If 12 or 13 switch, then all nodes 11–17 switch
- If 11 or 14 switch, nothing else happens



Certain people occupy **structurally important positions**

Another example with a=3 and b=2

What are the impediments to spread?



Densely connected communities

- 1–3 are well-connected with each other but poorly connected to the rest of the network
- Similar story for 11–17
- **Homophily impedes diffusion**

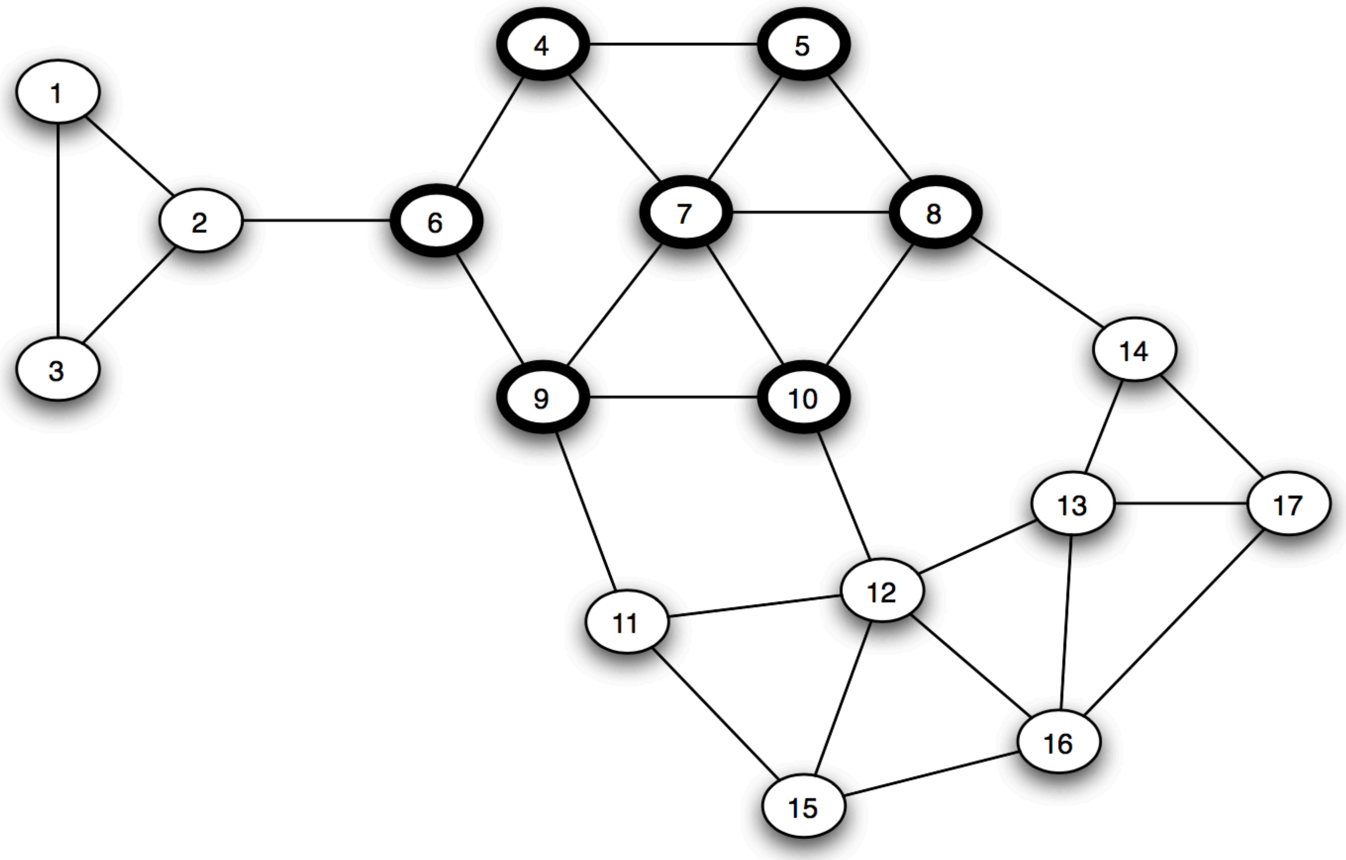
A **cluster of density *p*** is a set of nodes such that every node in the set has at least a p fraction of its neighbours in the set

Nodes {1,2,3} are a cluster of density p = ?

Nodes {11,12,13,14,15,16,17} are a cluster of density p = ?

Another example with a=3 and b=2

What are the impediments to spread?



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- **Homophily impedes diffusion**

A **cluster of density *p*** is a set of nodes such that every node in the set has at least a p fraction of its neighbours in the set

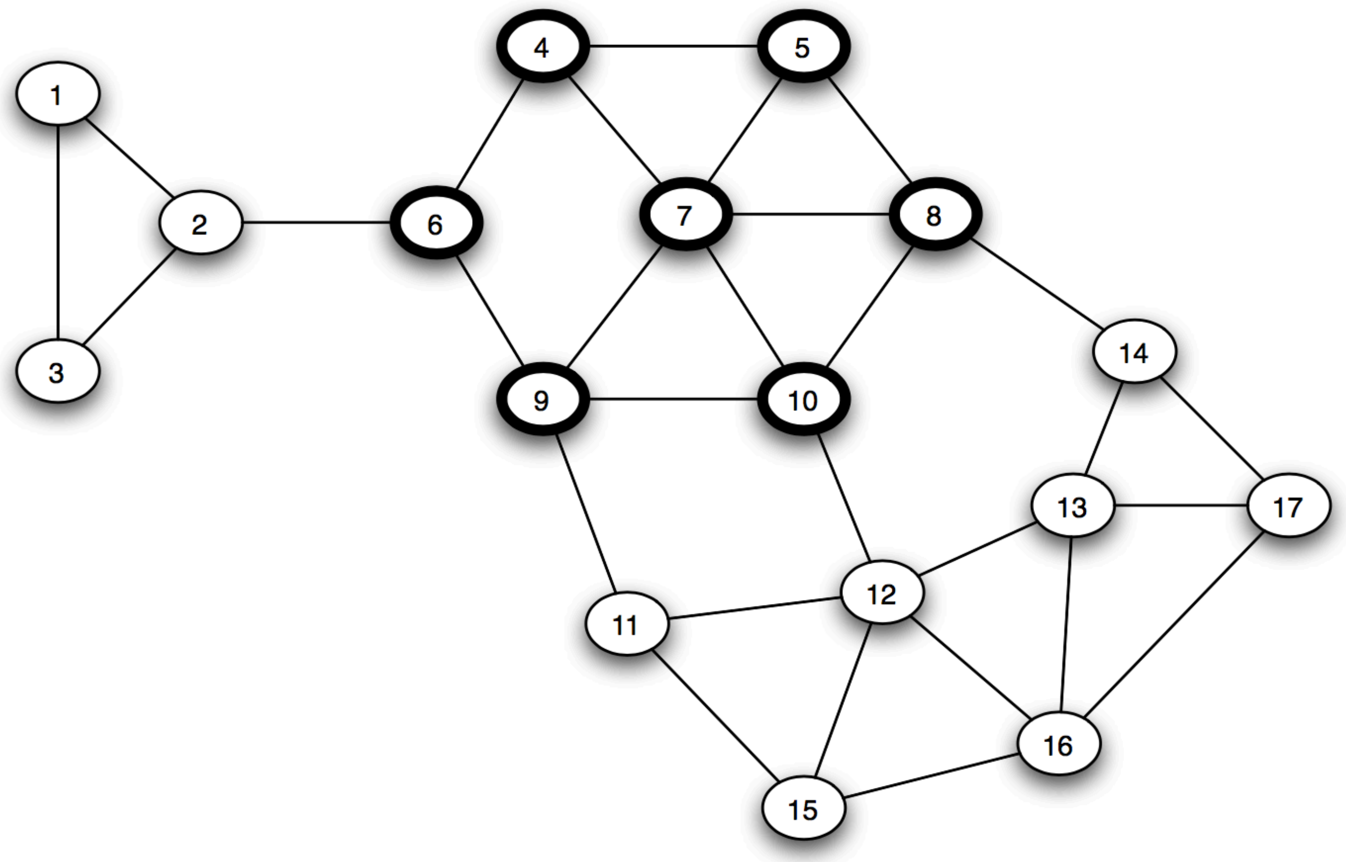
Nodes {1,2,3} are a cluster of density $p = 2/3$

Nodes {11,12,13,14,15,16,17} are a cluster of density $p = 2/3$

Another example with a=3 and b=2

<u>Fact</u>: Consider a set of initial adopters of behavior A, with a threshold of q for nodes in the remaining network to adopt behavior A.

- If the remaining network contains a cluster of density greater than 1−q, then the set of initial adopters will not cause a complete cascade.
- Moreover, whenever a set of initial adopters does not cause a complete cascade with threshold q, the remaining network must contain a cluster of den- sity greater than 1−q



In this model, densely connected communities are impediments to diffusion — and they are the only impediments to diffusion

Monotonic Spreading

Observation: Use of A spreads monotonically
(Nodes only switch B→A, but never back to B)

Why? Proof sketch:

Nodes keep switching from B to A: *B*→*A*

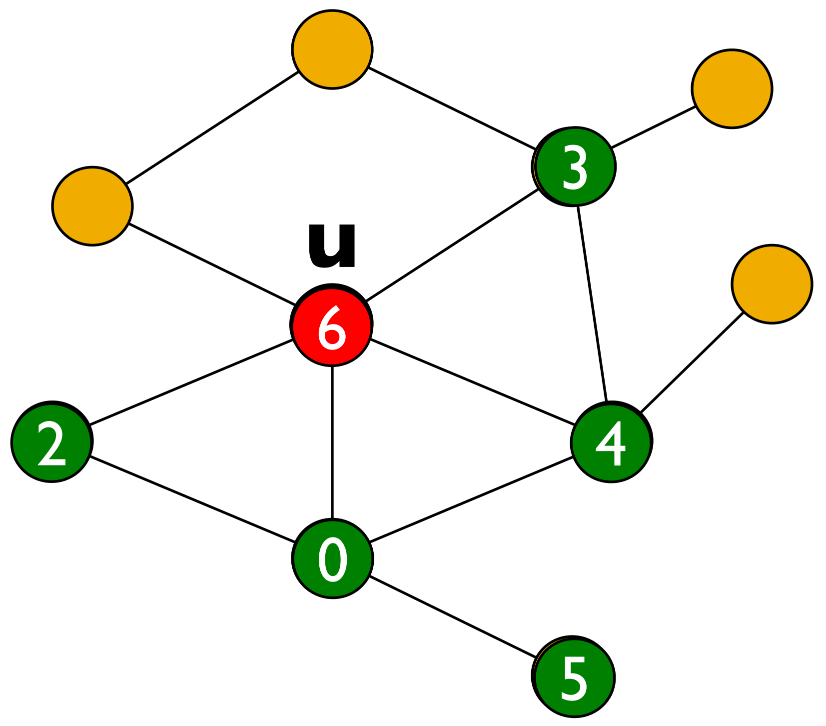
Now, suppose some node switched back from *A*→*B*, consider the first node *u* to do so (say at time *t*)

Earlier at some time *t'* (*t'*<*t*) the same node *u* switched *B*→*A*

So at time *t' u* was above threshold for *A*

But up to time *t* no node switched back to B, so node *u* could only have more neighbors who used *A* at time *t* compared to *t'*.
There was no reason for *u* to switch at the first place!

!! Contradiction !!



Infinite Graphs

Consider <u>infinite</u> graph *G*
(but each node has finite number of neighbors!)

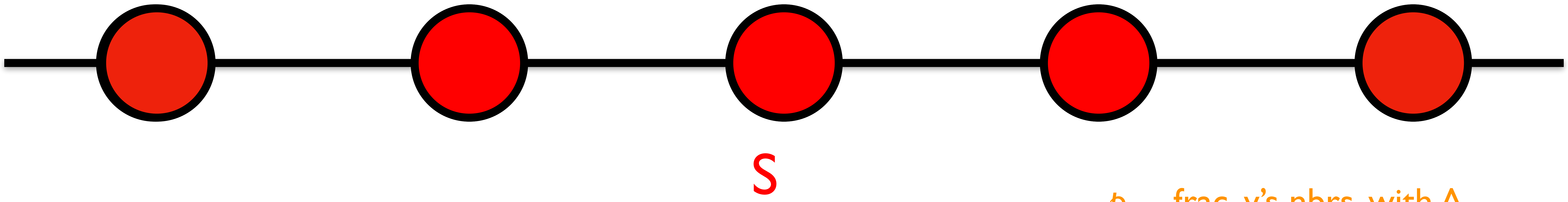
v chooses *A* if $p>q$

$$q = \frac{b}{a+b}$$

We say that a finite set ***S*** causes a **complete cascade** in *G* with **threshold *q*** if, when ***S*** adopts ***A***, eventually **every node in G adopts *A***

Example: **Path**

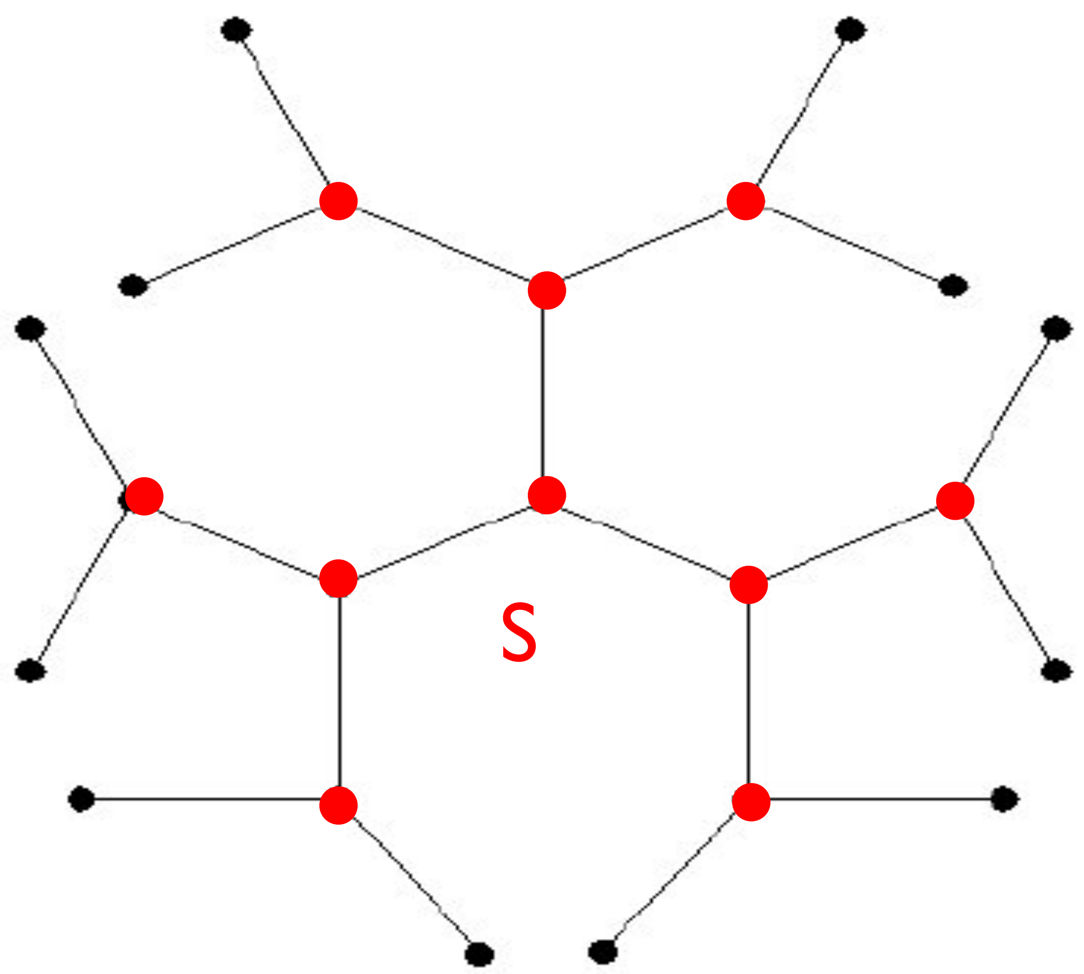
If $q<1/2$ then cascade occurs



p… frac. v's nbrs. with A
q… payoff threshold

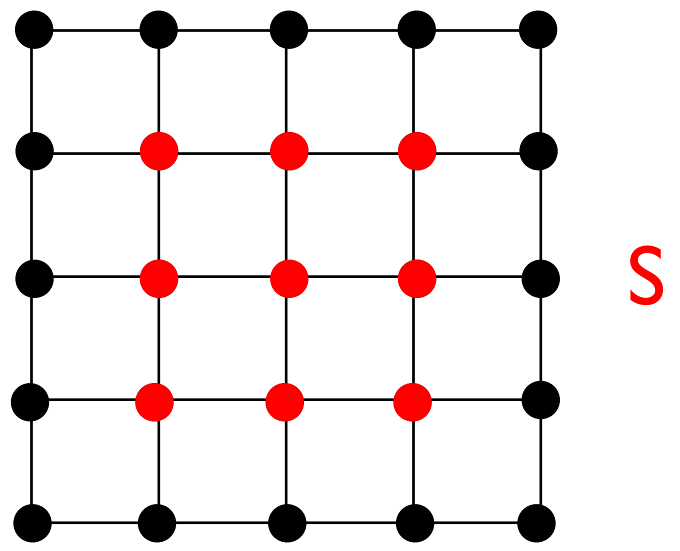
Infinite Graphs

Infinite Tree:



If $q<1/3$ then cascade occurs

Infinite Grid:



If $q<1/4$ then cascade occurs

Cascade Capacity

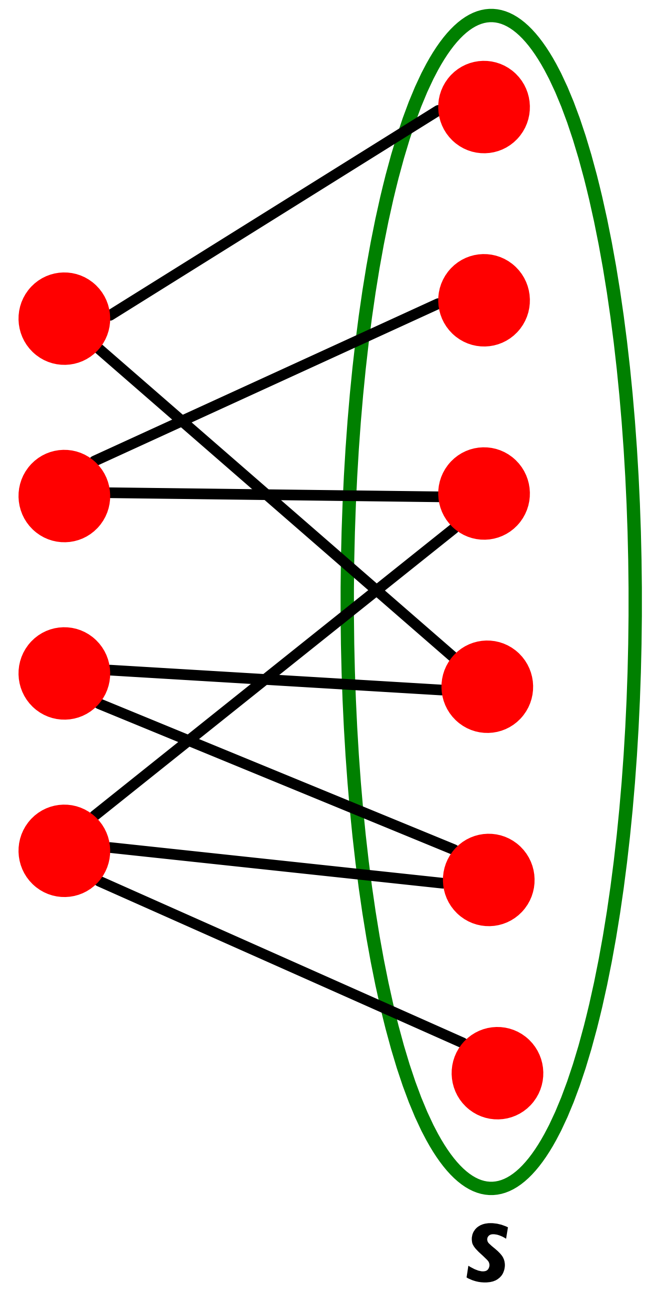
Def: The **cascade capacity** of a graph **G** is the **largest *q*** for which some **finite set S** can cause a **complete cascade**

Fact: There is no (infinite) **G** where cascade capacity **> ½**

Proof idea:

Suppose such **G** exists: **q>½**, finite **S** causes cascade

Show contradiction: Argue that nodes stop switching after a finite # of steps



Cascade Capacity

Fact: There is no *G* where cascade capacity **>** ½

Proof sketch:

Suppose such **G** exists: **q>½**, finite **S** causes cascade

Contradiction: Switching stops after a finite # of steps

Define "potential energy"

Argue that it starts finite (non-negative) and strictly decreases at every step

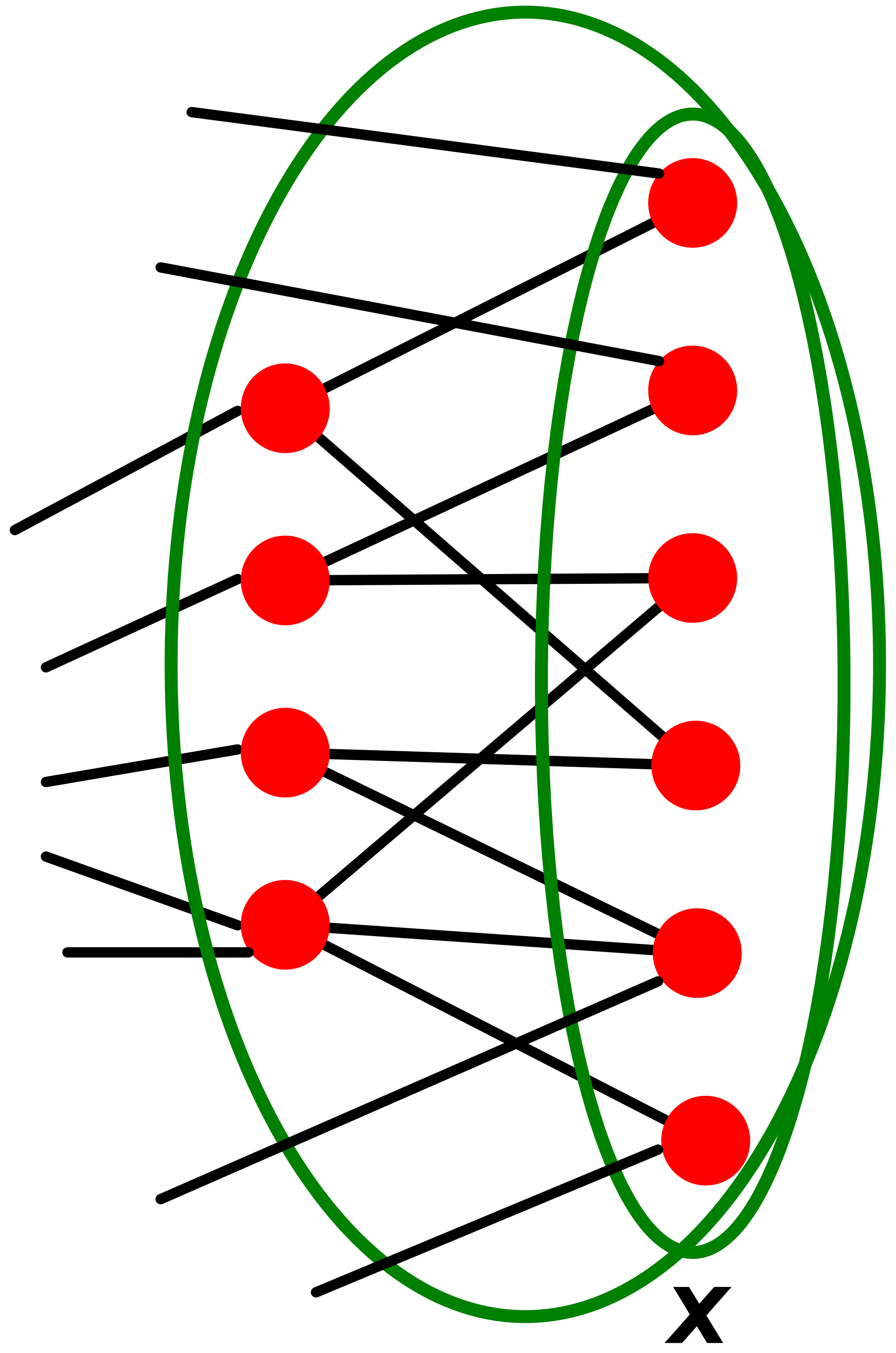
"Energy": = $|d^{out}(X)|$

$|d^{out}(X)|$:= # of outgoing edges of active set *X*

The only nodes that switch have a strict majority of its neighbors in ***S***

$|d^{out}(X)|$ strictly decreases

It can do so only a finite number of steps



Today: Game Theory in the Wild and Influence Through Networks

- If people are connected through a network, it's possible for them to influence each other's behaviour and actions
- Today: why?
  - Informational
  - Direct benefit
  - Social conformity