

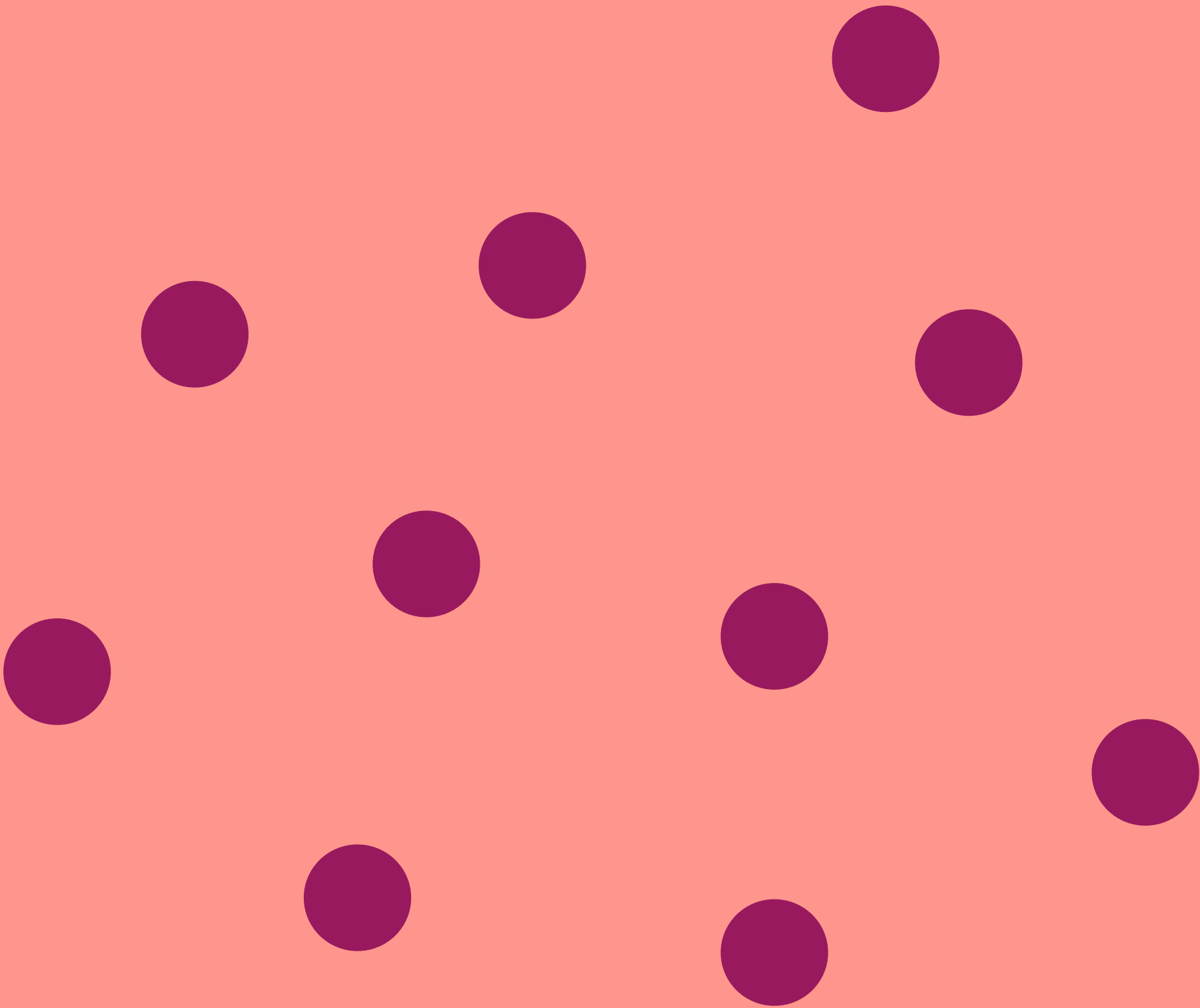
Social and Information Networks

**CSCC46H, Fall 2025
Lecture 1**

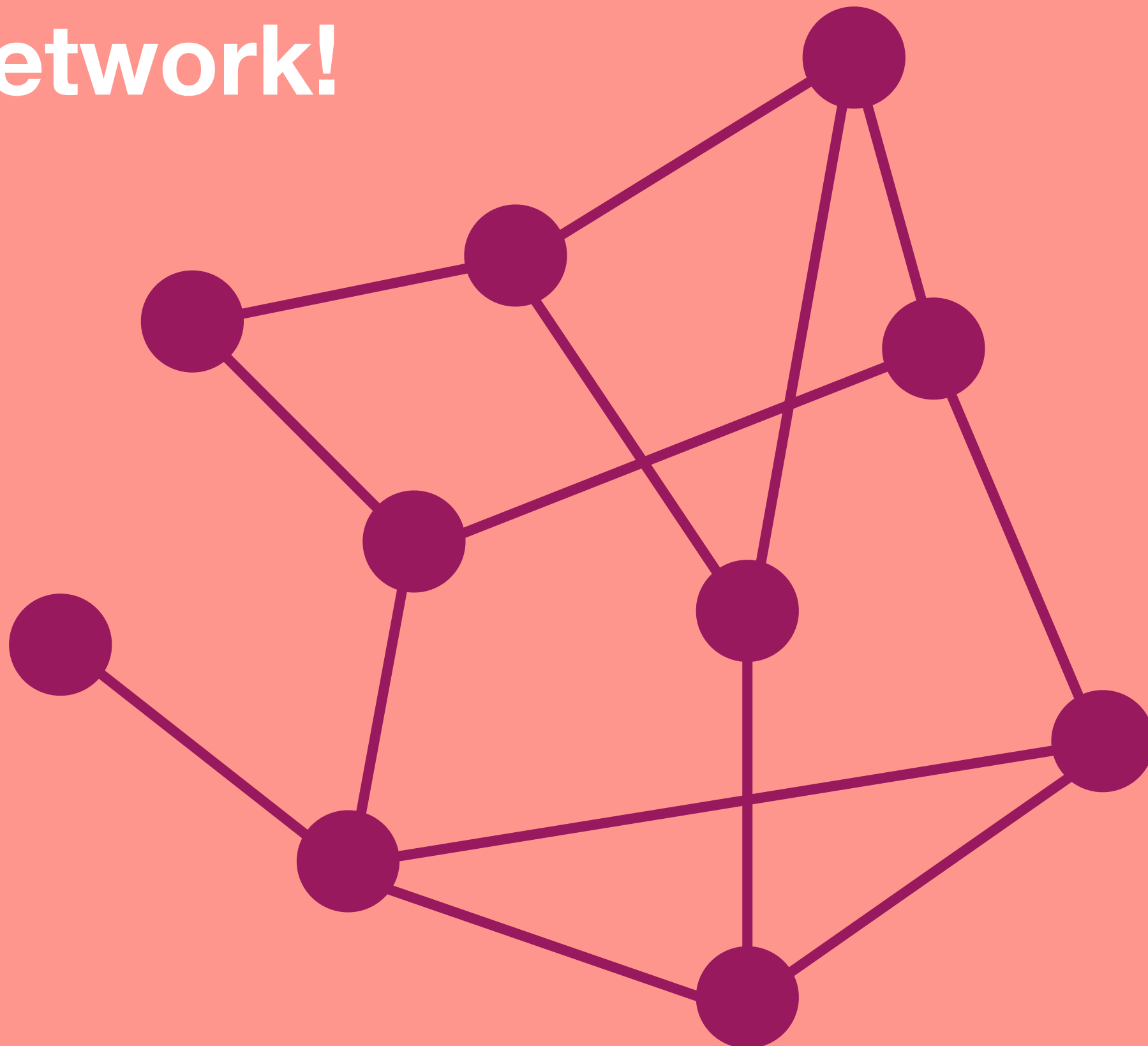
Prof. Ashton Anderson
ashton@cs.toronto.edu

Why Networks?

Networks are a language for representing, describing, and understanding complex, interconnected systems



A Network!





How to understand the behaviours, decisions, beliefs, etc. of millions of people?



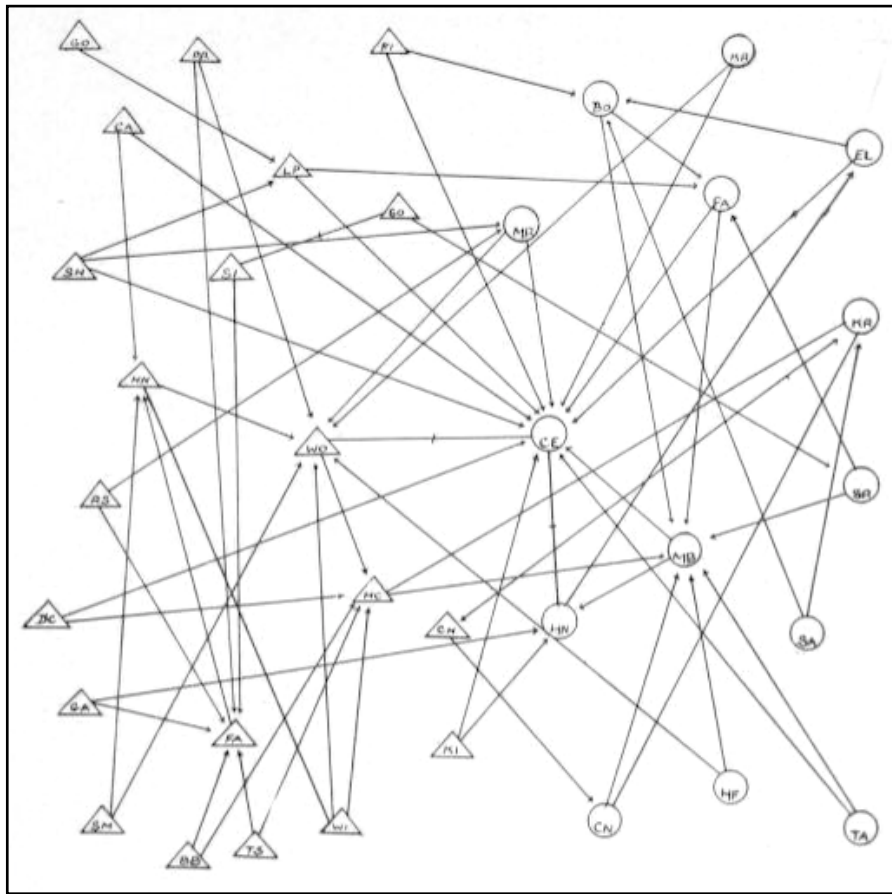
Social network



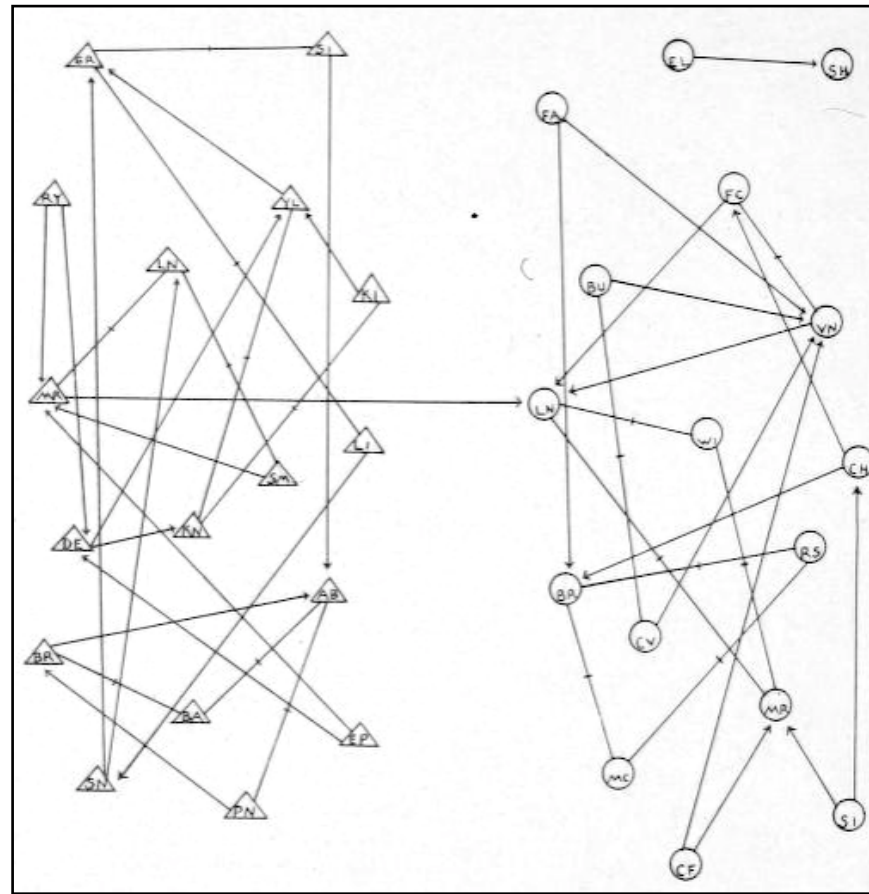
Educational settings: people learn and interact with each other in complex ways

EMOTIONS MAPPED BY NEW GEOGRAPHY

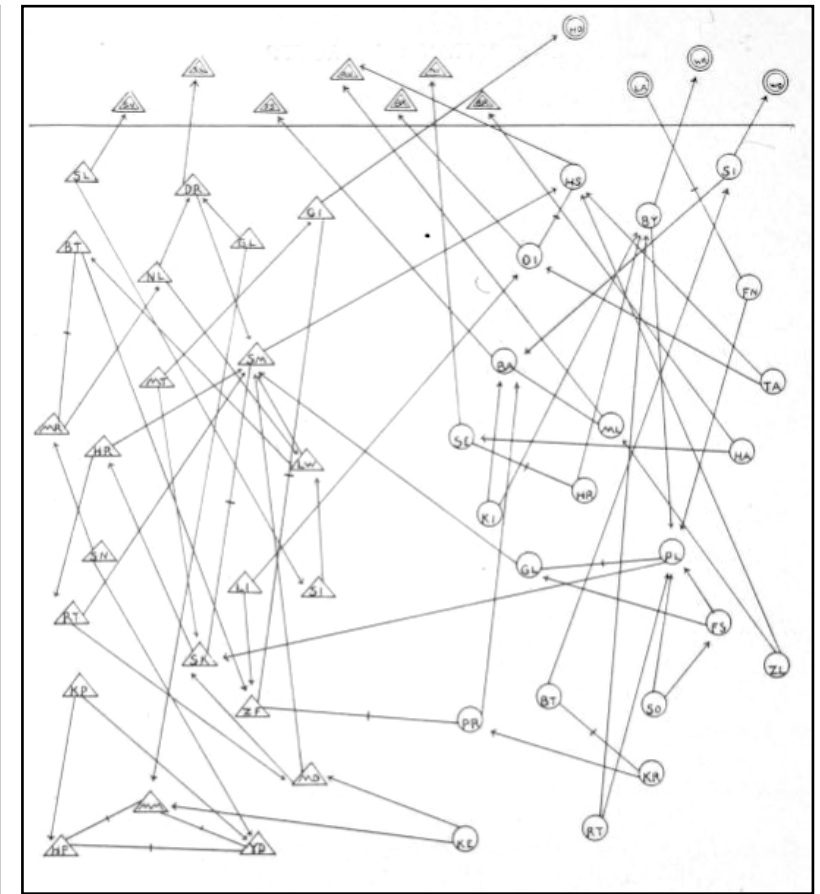
Charts Seek to Portray the
Psychological Currents of
Human Relationships.



1st grade



4th grade

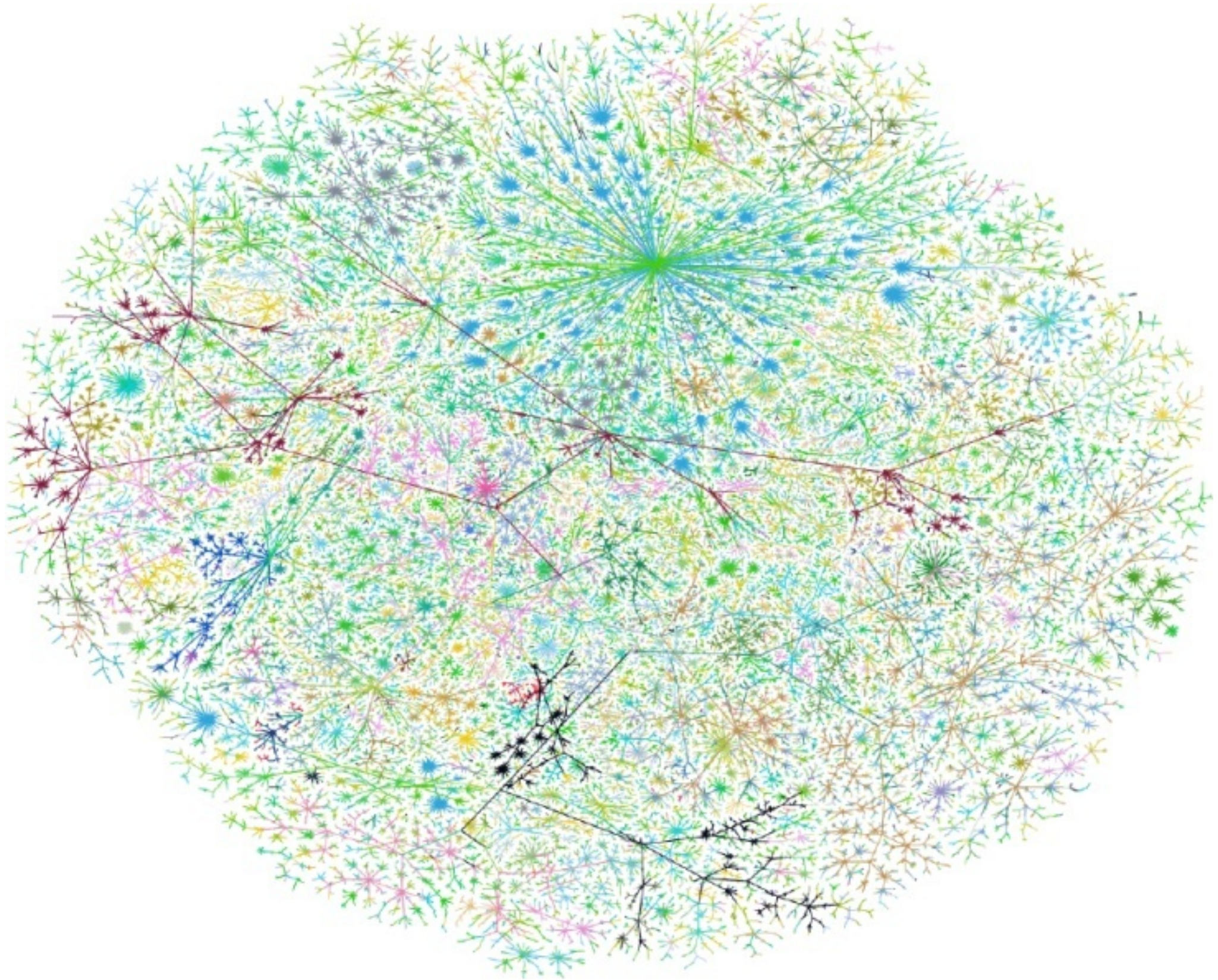


8th grade

Moreno's sociograms, 1934



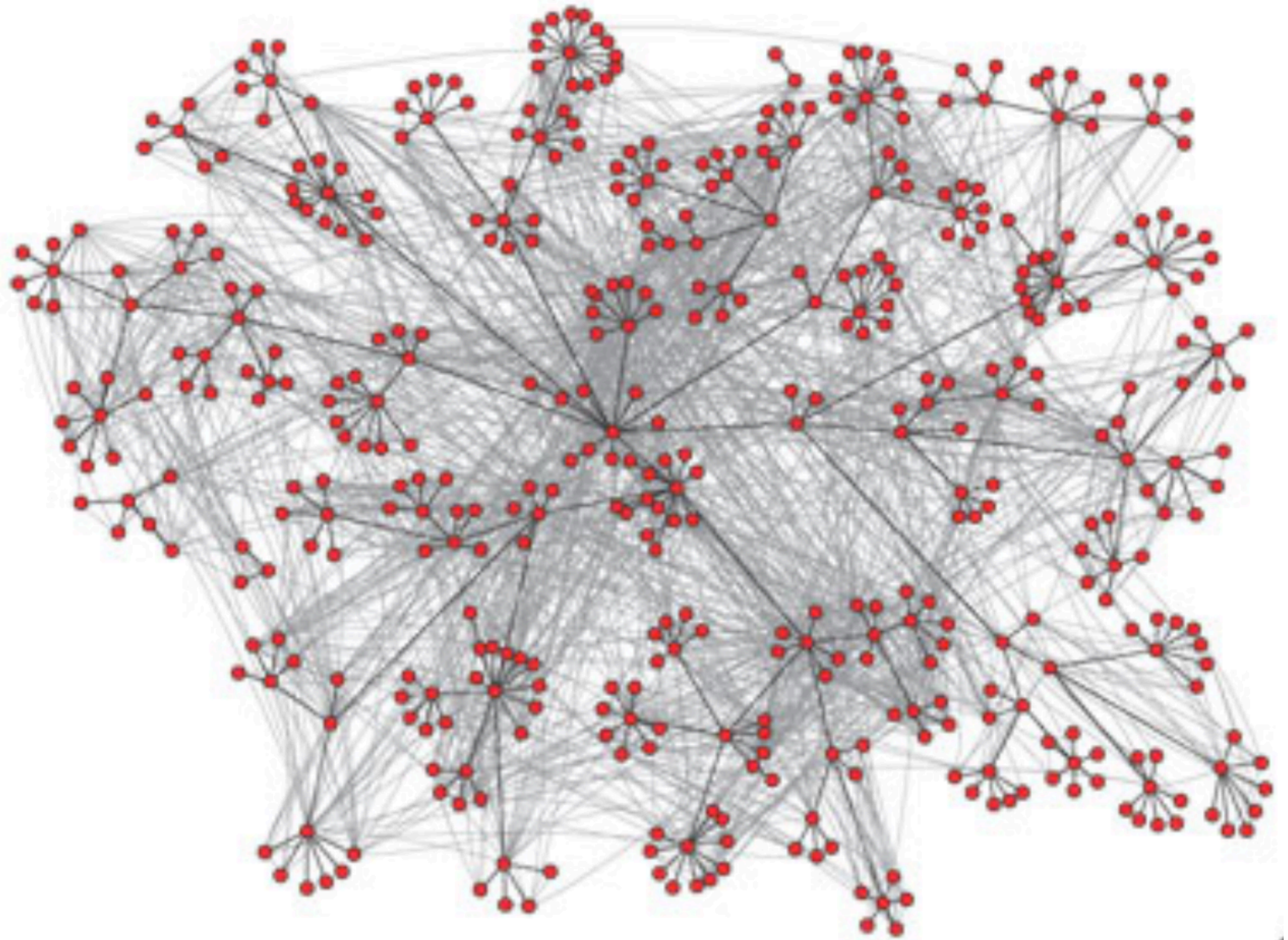
Everything on the Internet is passed through autonomous systems (routers, etc.)



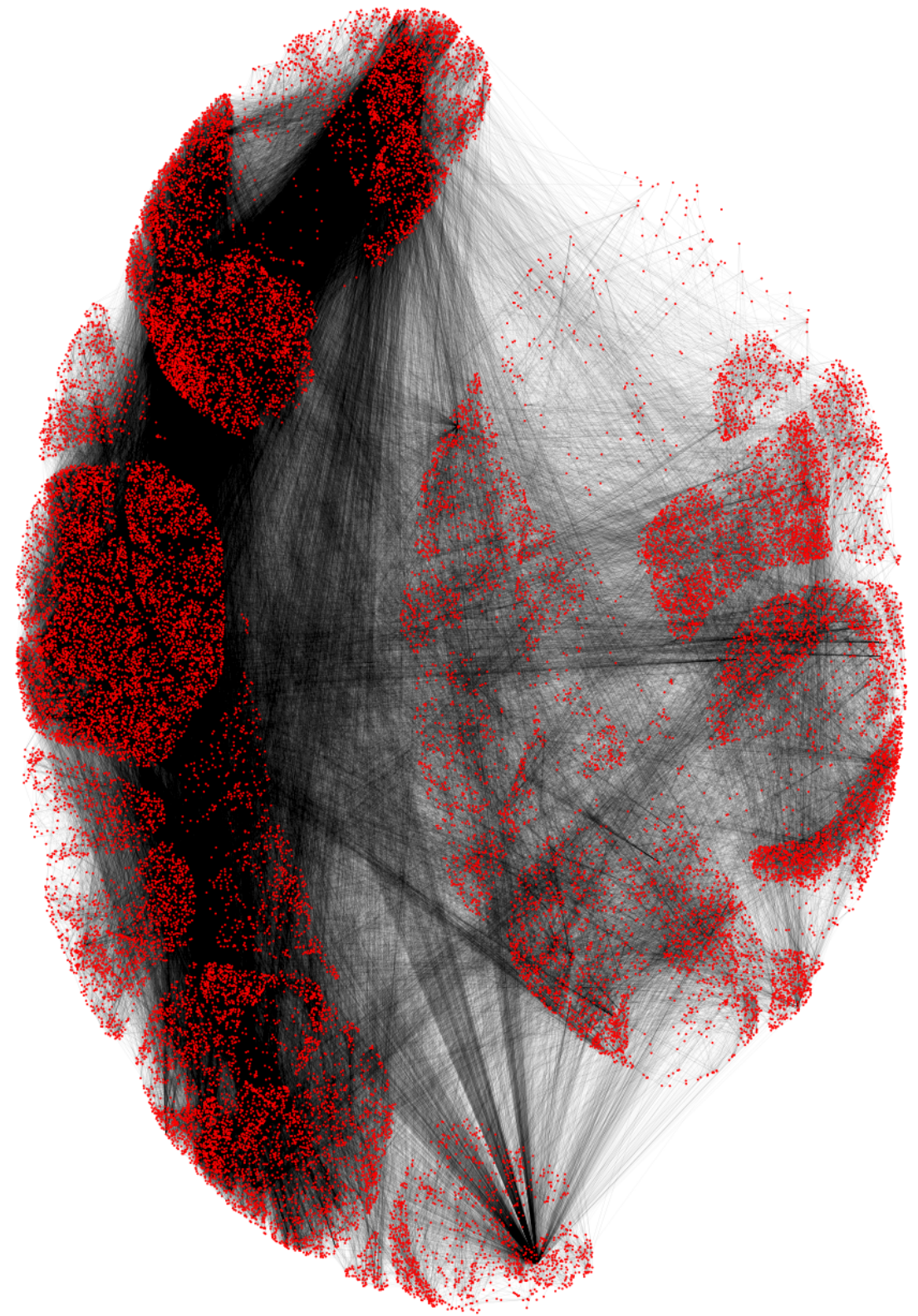
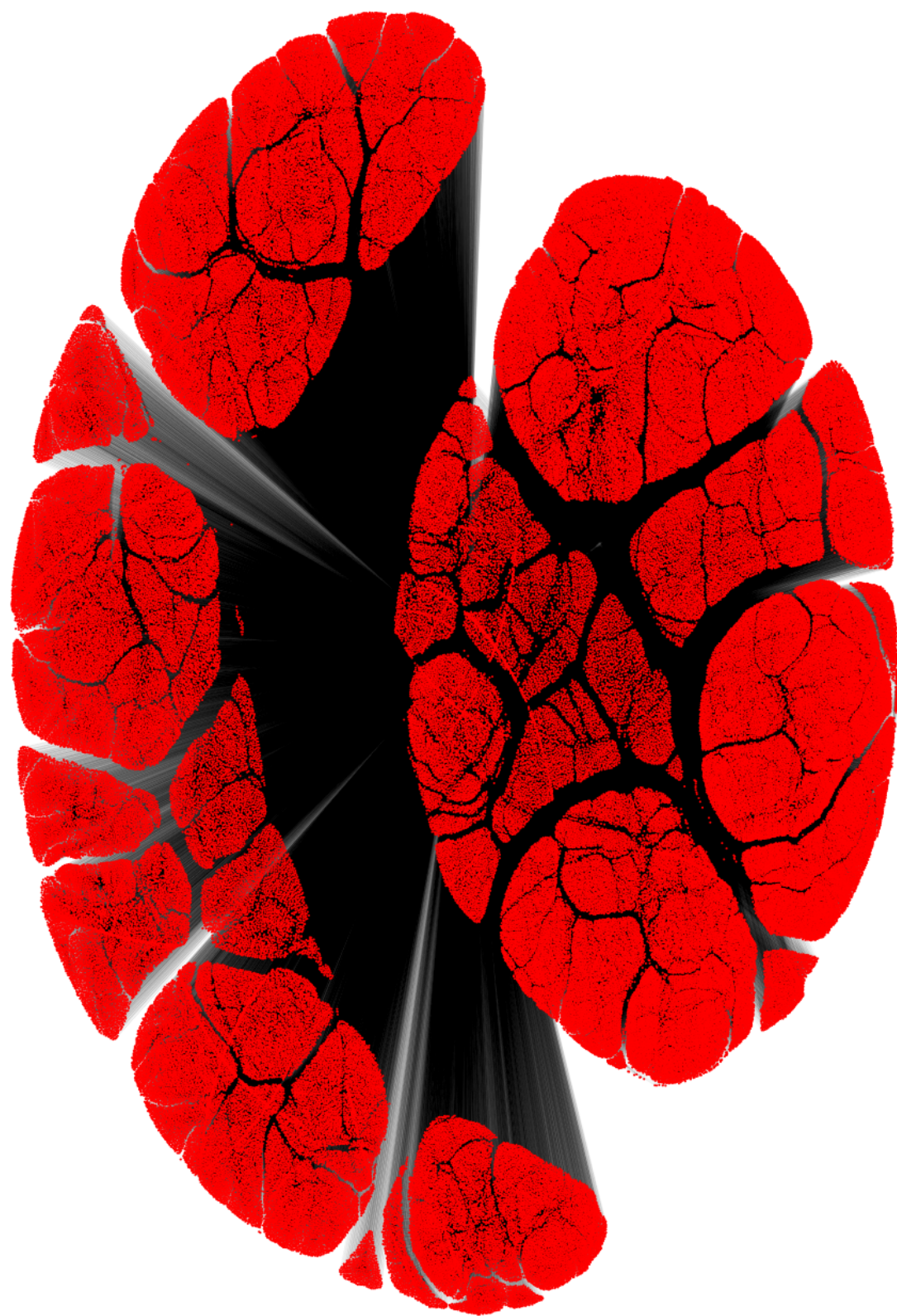
Graph of the Internet (Autonomous Systems)



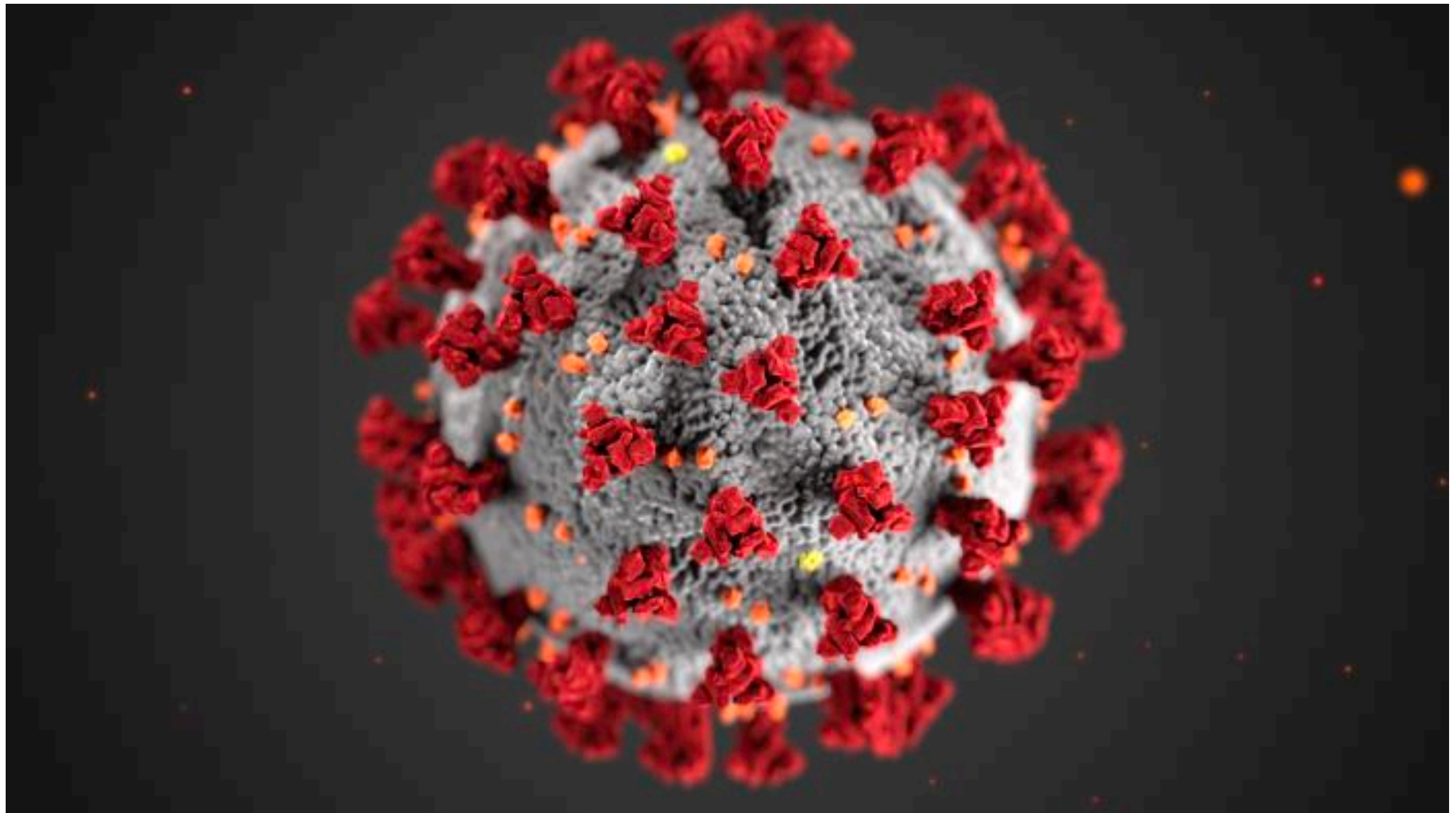
In companies, people communicate and work together in large hierarchies and structures



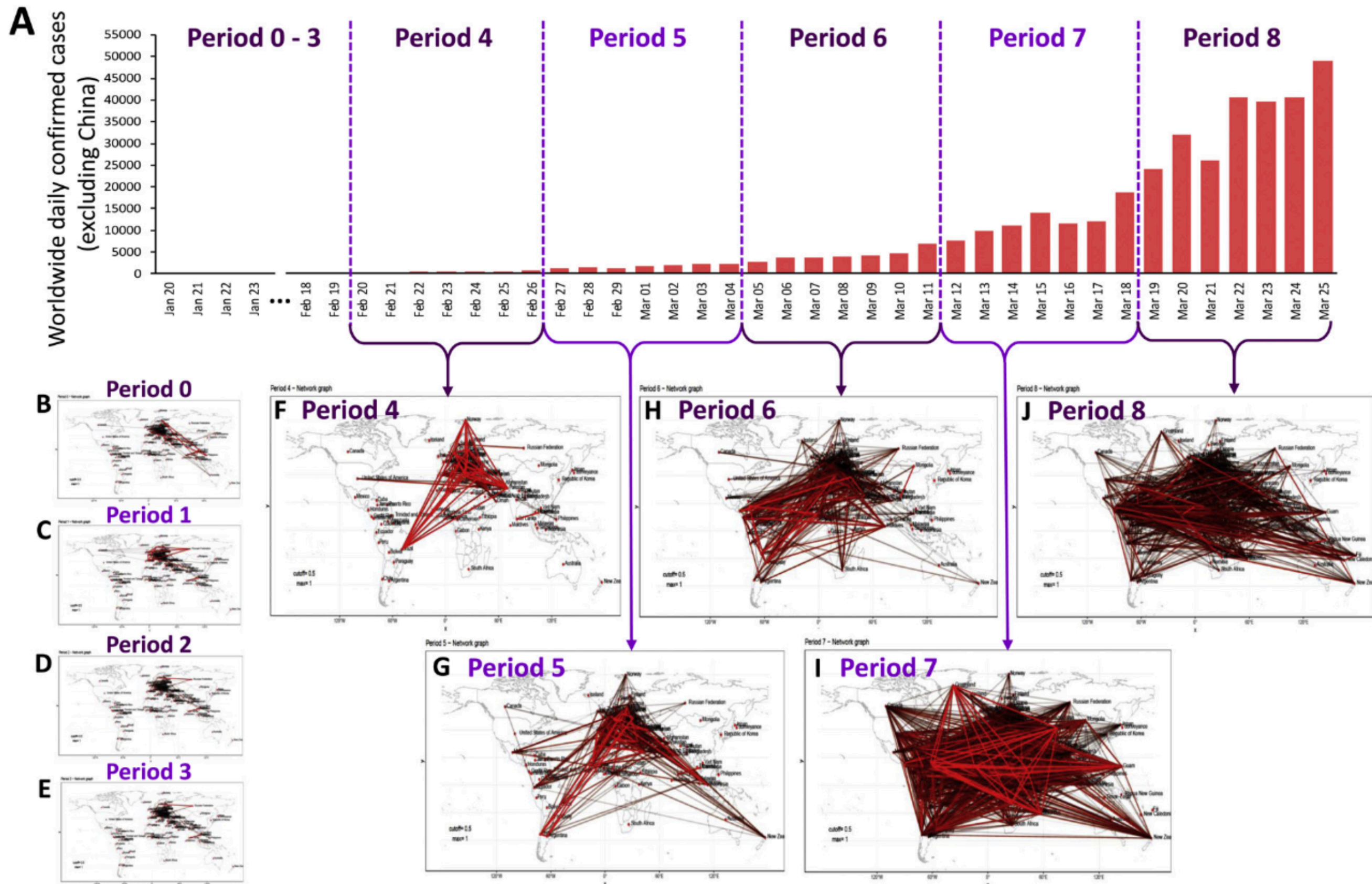
Email communication network
(HP Research, 436 employees)



Org chart (left) and Email communication network (right)
Microsoft, 200,000 employees [Jake Hofman, 2018]



Many diseases are transmitted socially (e.g. COVID-19)



Global spread of COVID-19



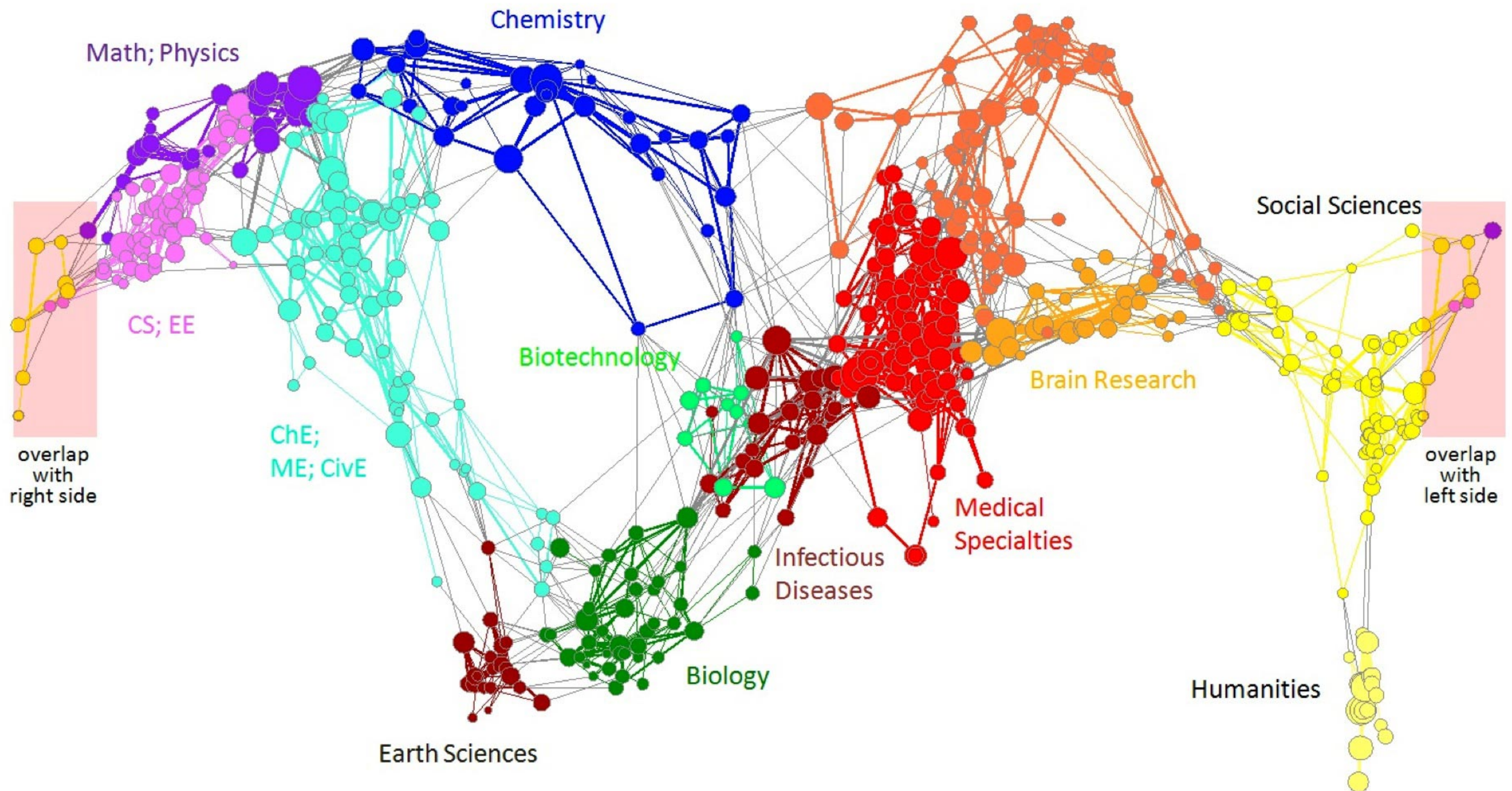
Power is transported everywhere with interconnected stations and lines



The power grid: a network



Science is a complex system of academics working together and being influenced by each other

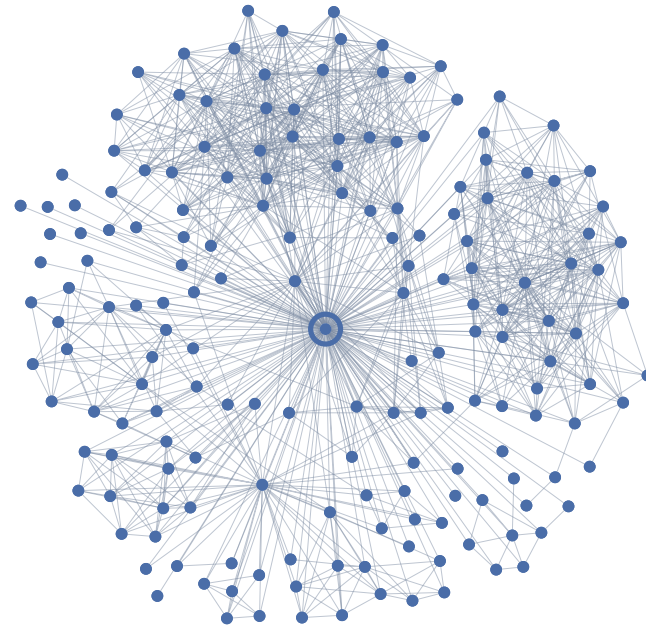


Citation networks and maps of science [Börner et al., 2012]

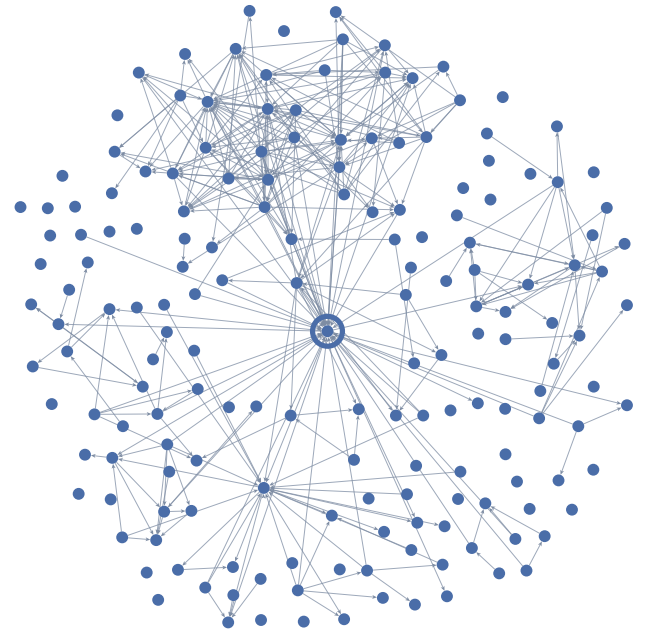


A single person's interactions with friends and family are a huge part of their life

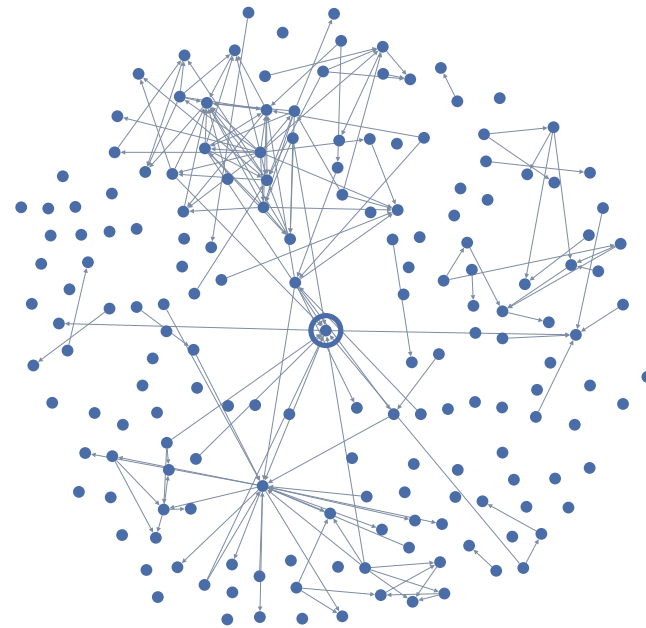
All Friends



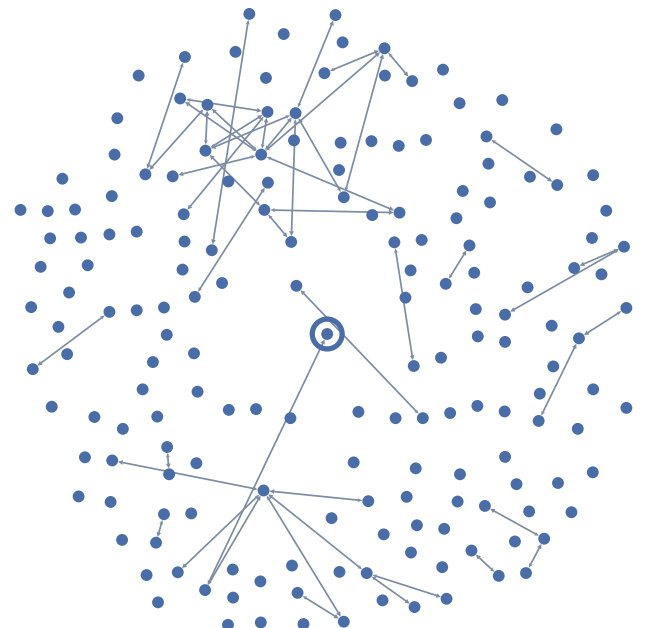
Maintained Relationships



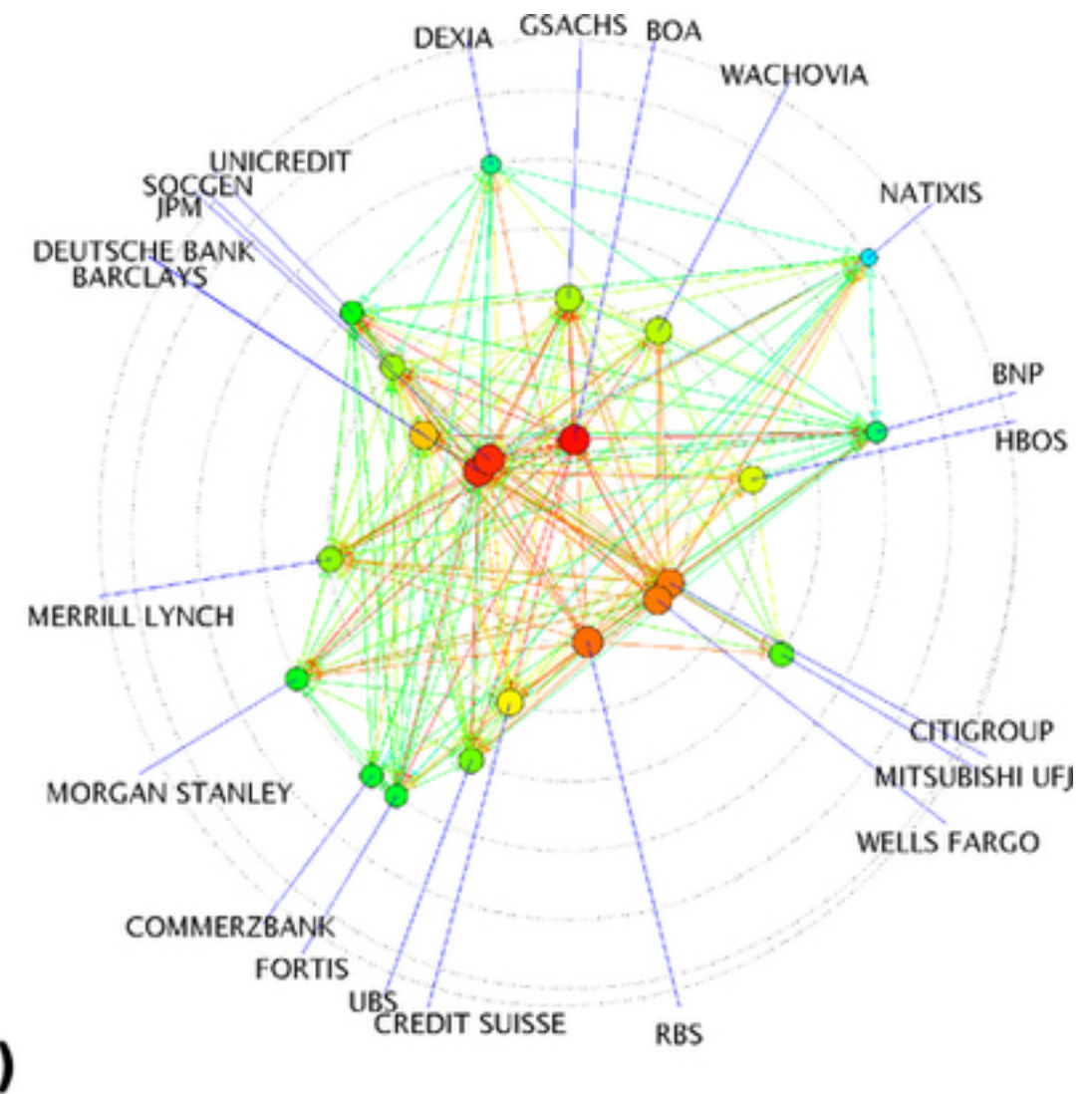
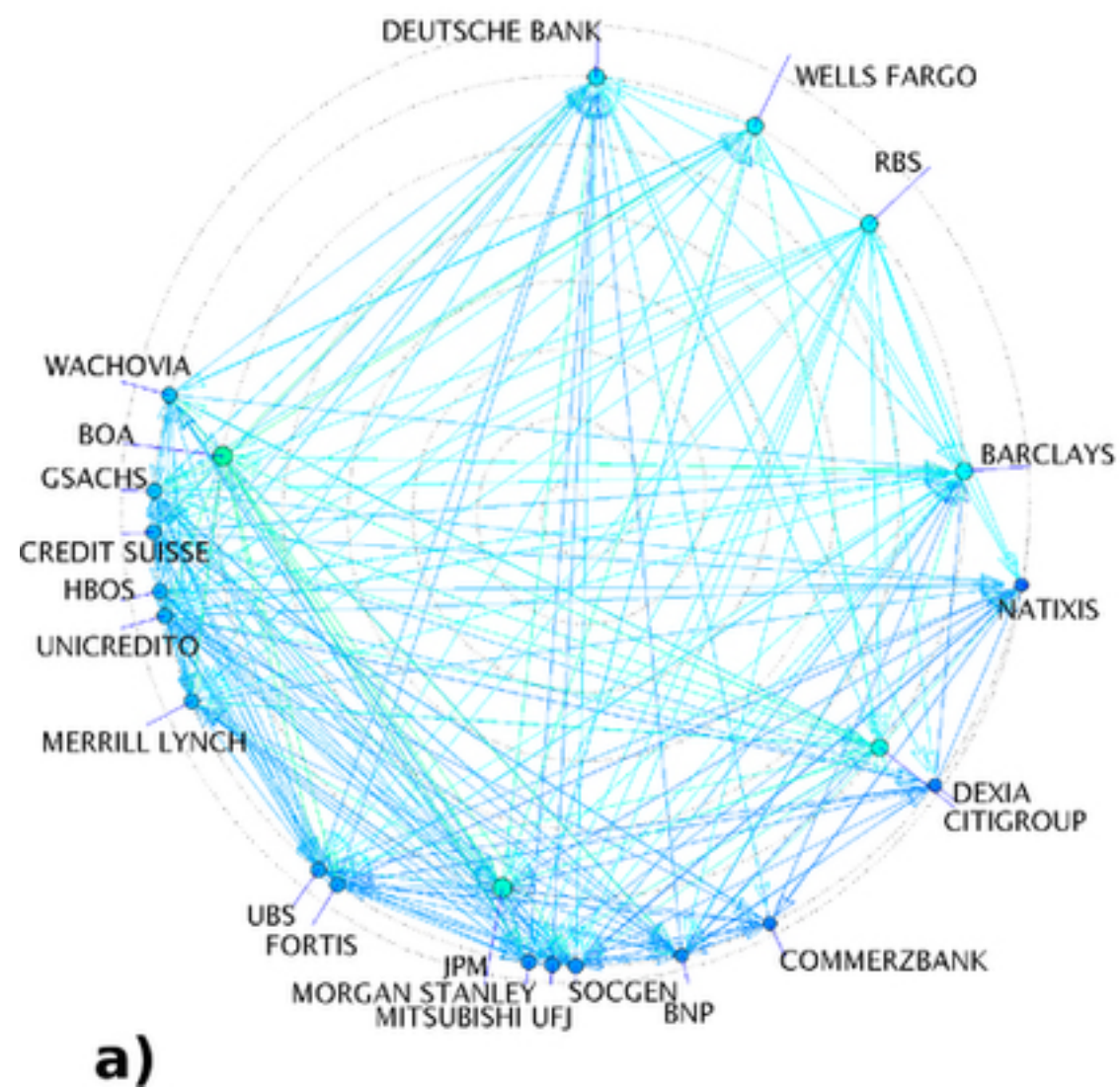
One-way Communication



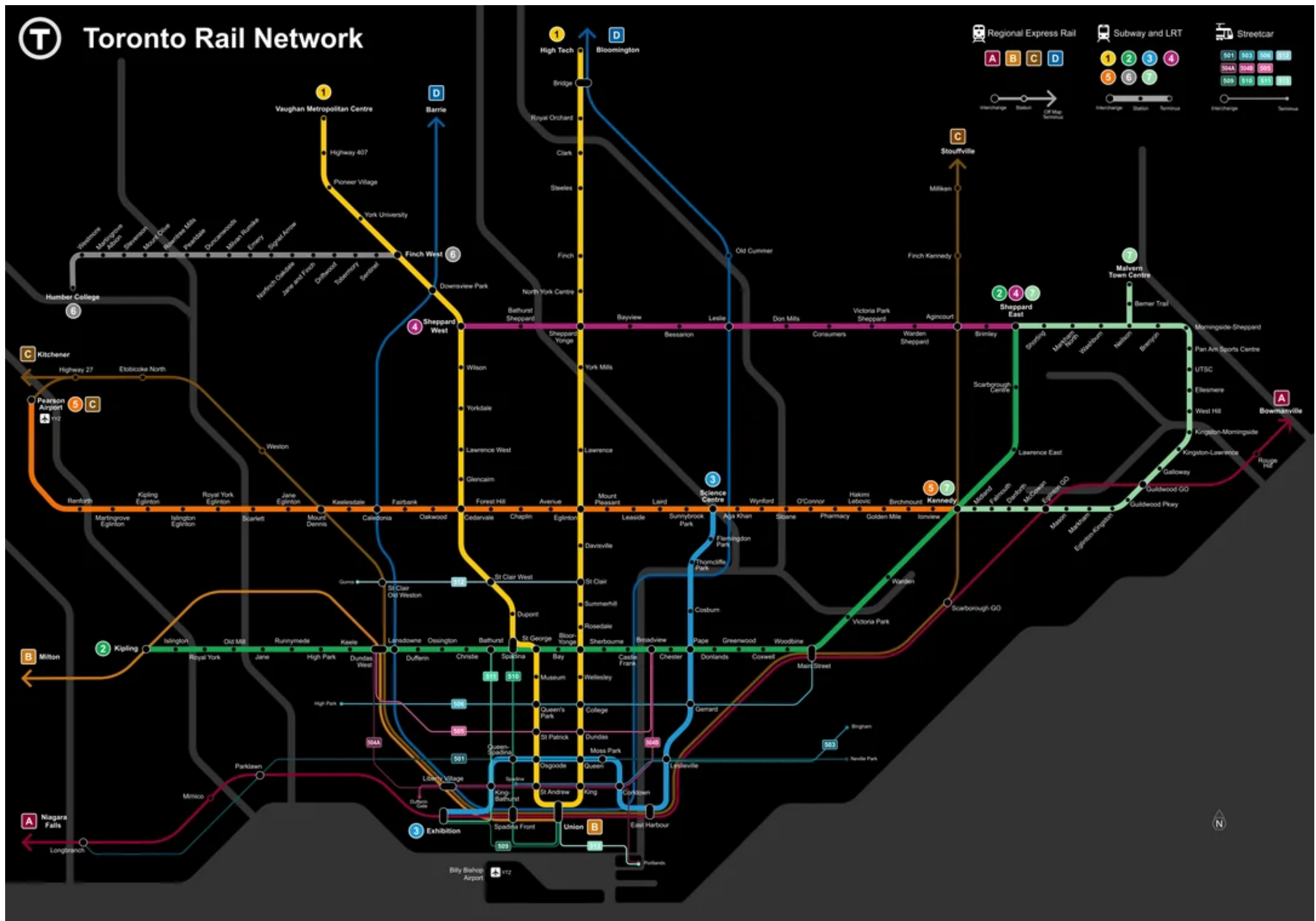
Mutual Communication



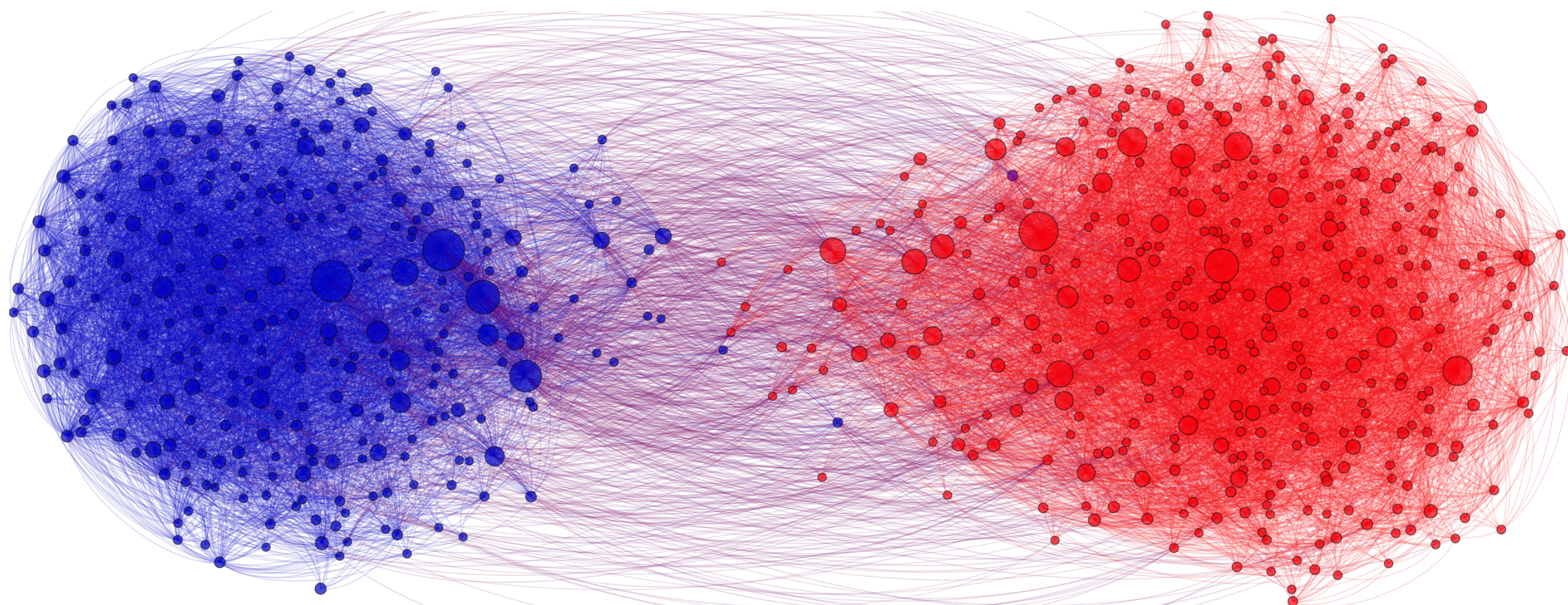
**An “ego network”: the neighbourhood
around a single individual**



**The economy is a network:
e.g. Bank lending**



Transportation network



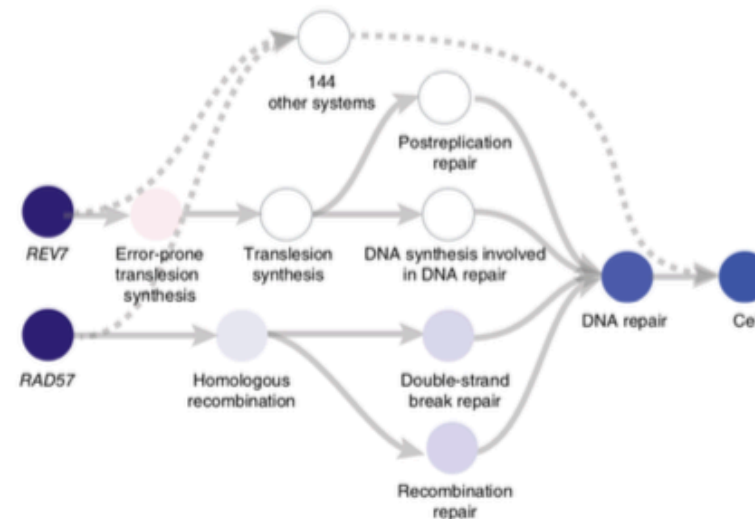
Political blogs prior to 2004 US Presidential election



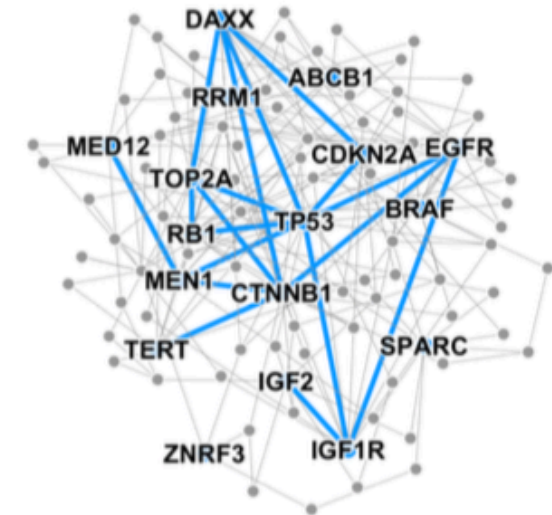
The human brain has between 10-100 billion neurons connected to each other in complex ways



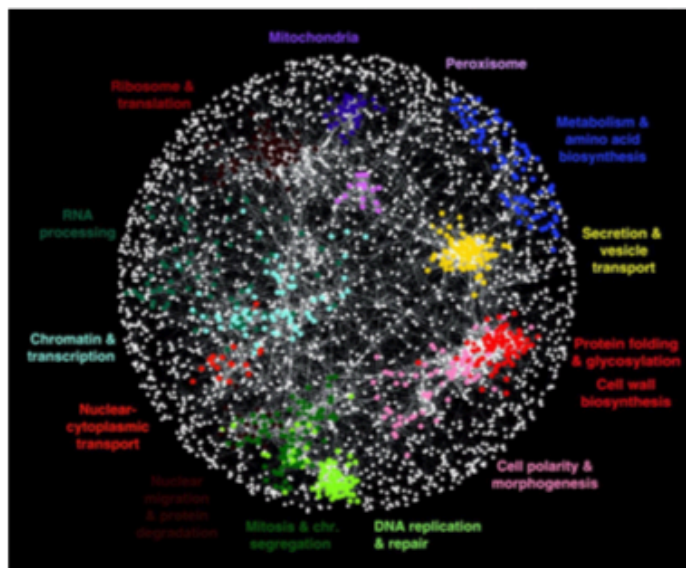
Patient networks



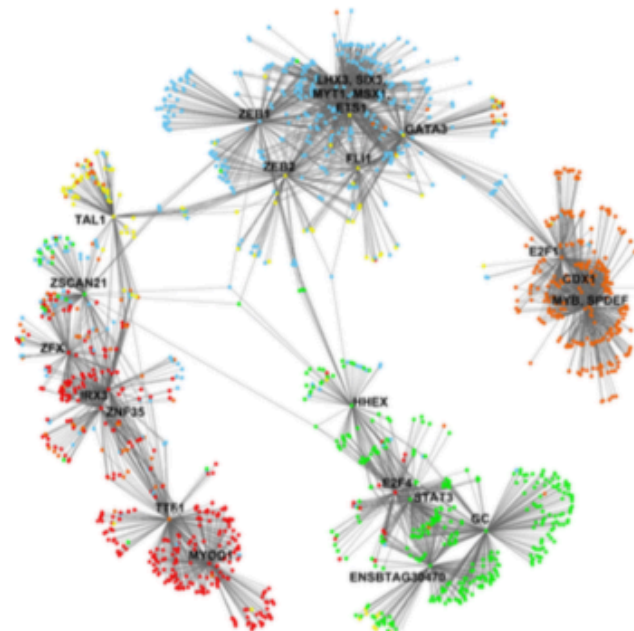
Hierarchies of cell systems



Disease pathways



Genetic interaction networks



Gene co-expression networks



Cell-cell similarity networks

Many, many more examples

But why should **I care
about networks?**

Why study networks?

Complex systems are all around us

- **Society** is a collection of eight billion people
- **Communications systems** link electronic devices
- **Information and knowledge** is organized and linked
- Interactions between thousands of **genes** regulate life
- Our thoughts and selves are manifested in the connections between billions of neurons in the **brain**
- **Information and diseases** spread from person to person

Why study networks?

Networks are a **universal language for describing complex data**

Networks from science, nature, and technology are more similar than you might expect

Shared vocabulary between fields

CS, finance, tech, social sciences, physics, economics, statistics, biology

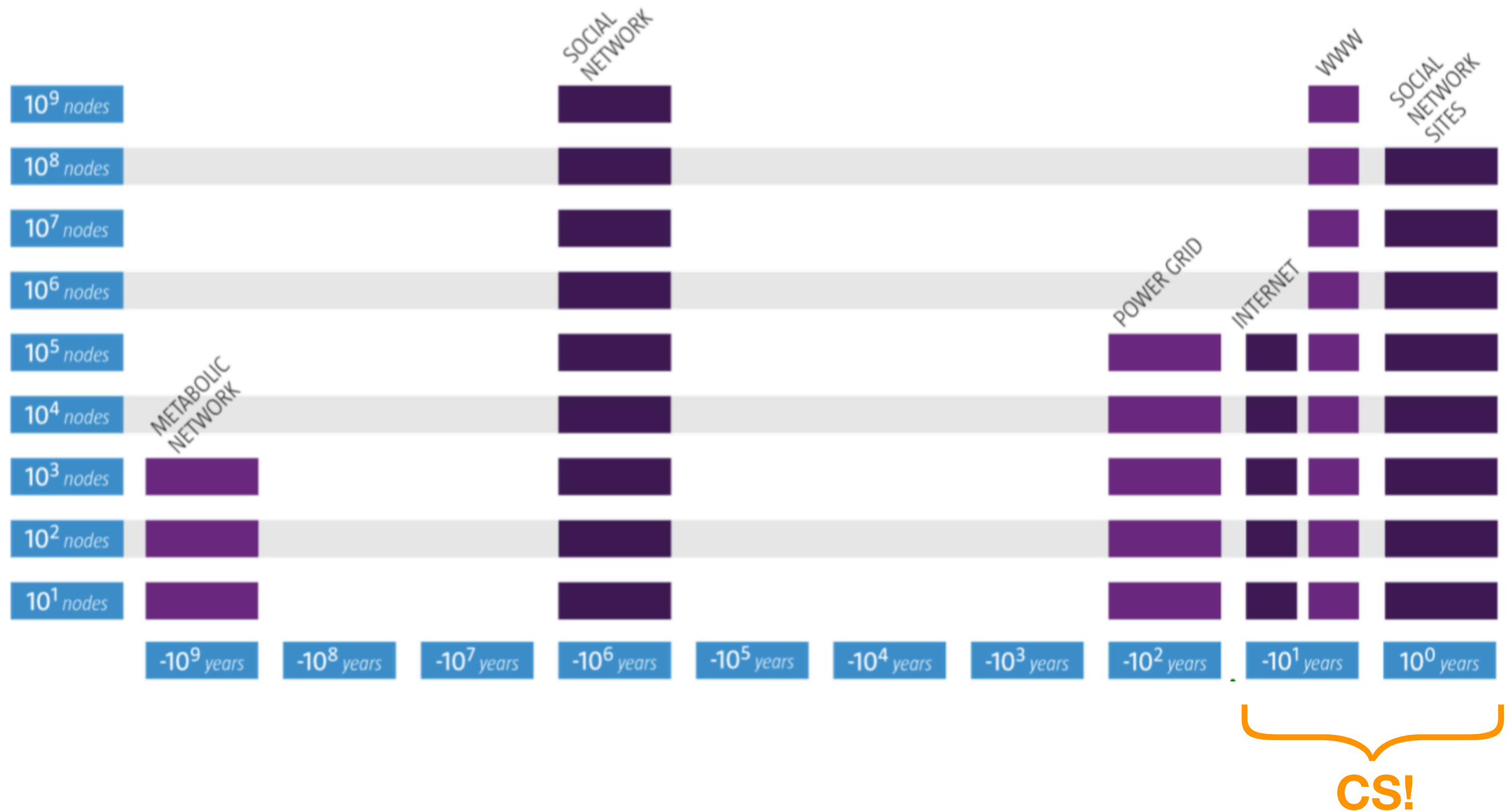
Data availability (and computational challenges)

Web/mobile, bio, health, medical

Impact!

Social networking, social media, drug design

Why now?



Age and size of networks

Networks: Impact



Google/Alphabet
Dominant tech giant

Facebook/Meta
Dominant social
platforms

Networks and Applications

Ways to Analyze Networks

Predict the **type** of a given node (*node classification*)

Predict whether two nodes are **linked** (*link prediction*)

Identify densely **linked clusters** of nodes (*community detection*)

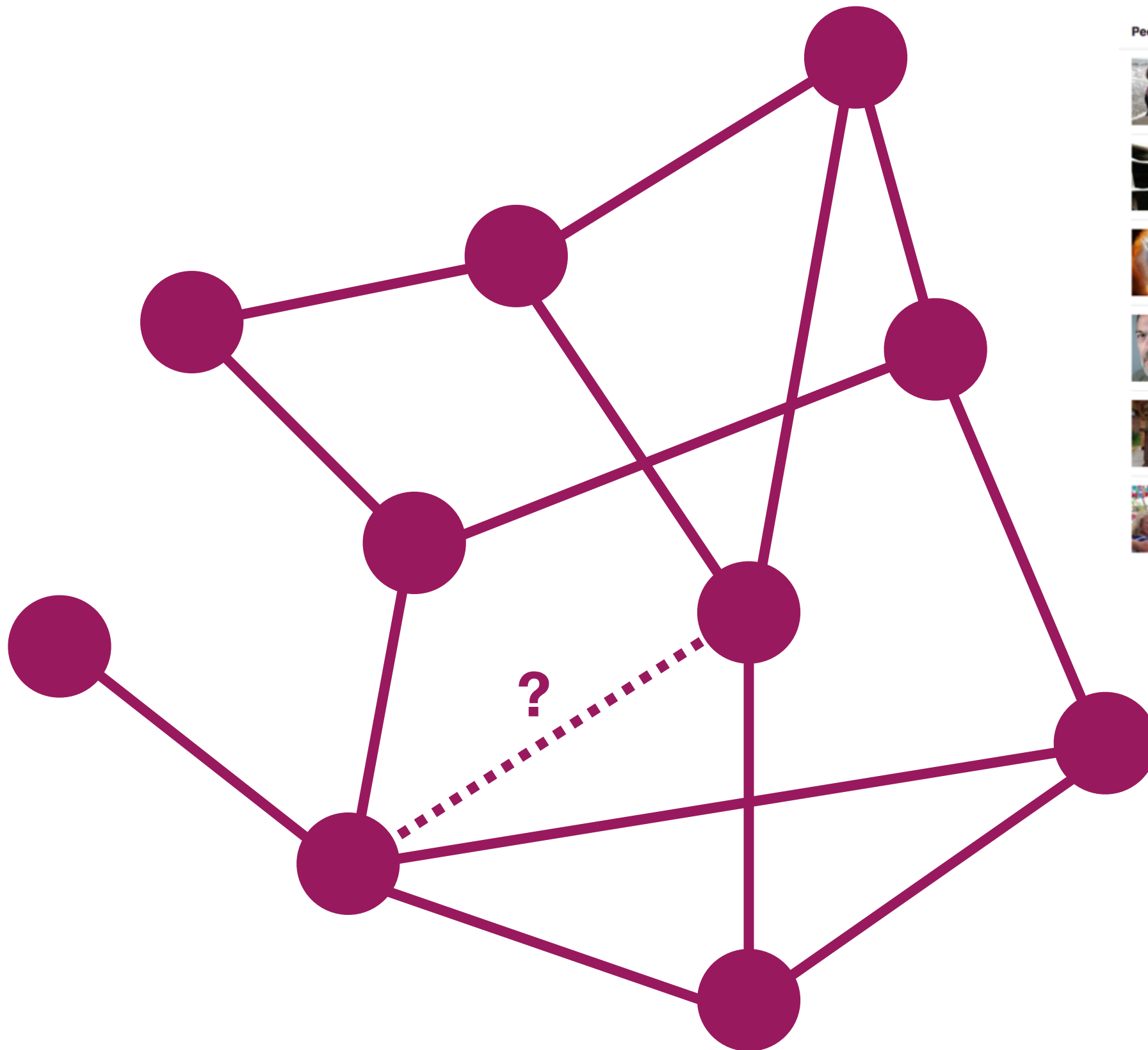
Predict **common pathways** (*social influence/propagation*)

Measure **similarity** between nodes/networks (*network similarity*)

(1) Networks: Social



Application: Friend Prediction



People you may know



Sara Anderson Severance

📍 Denver, Colorado

Rachelle Albright and 10 other mutual friends

👤 Add Friend

Remove

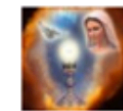


Anne Walker (Anne Anderson)

Sarah Frederick and 6 other mutual friends

👤 Add Friend

Remove



Paul Dube

Ryan Dube is a mutual friend.

👤 Add Friend

Remove



Mark Rieder

📍 Lord Beaverbrook High School

Justin Pot is a mutual friend.

👤 Add Friend

Remove



Nancy Mescher

Maggie Flynn is a mutual friend.

👤 Add Friend

Remove



Becky Williams Swenson

📍 Denver, Colorado

Rachelle Albright and 3 other mutual friends

👤 Add Friend

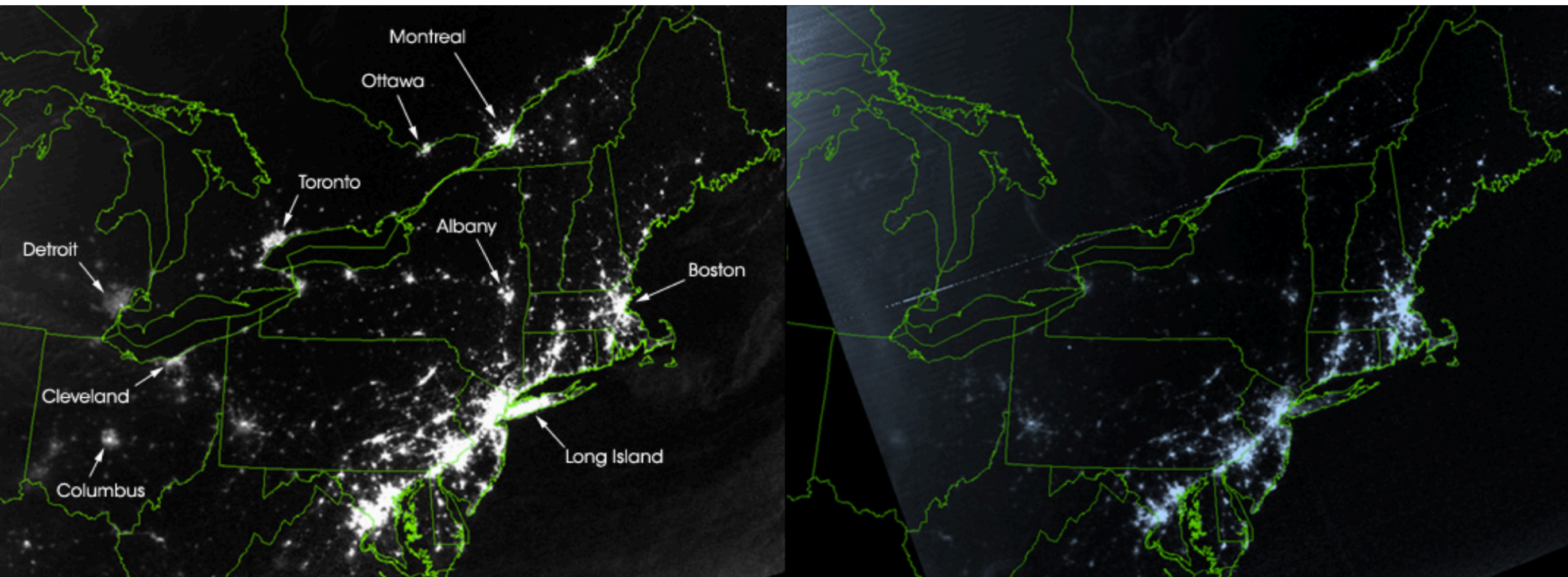
Remove

(2) Networks: Infrastructure



Power grids connect you to electricity

(2) Networks: Infrastructure



Aug 14, 2003, 9:29pm
20 hours before

Aug 15, 2003, 9:14pm
4 hours after

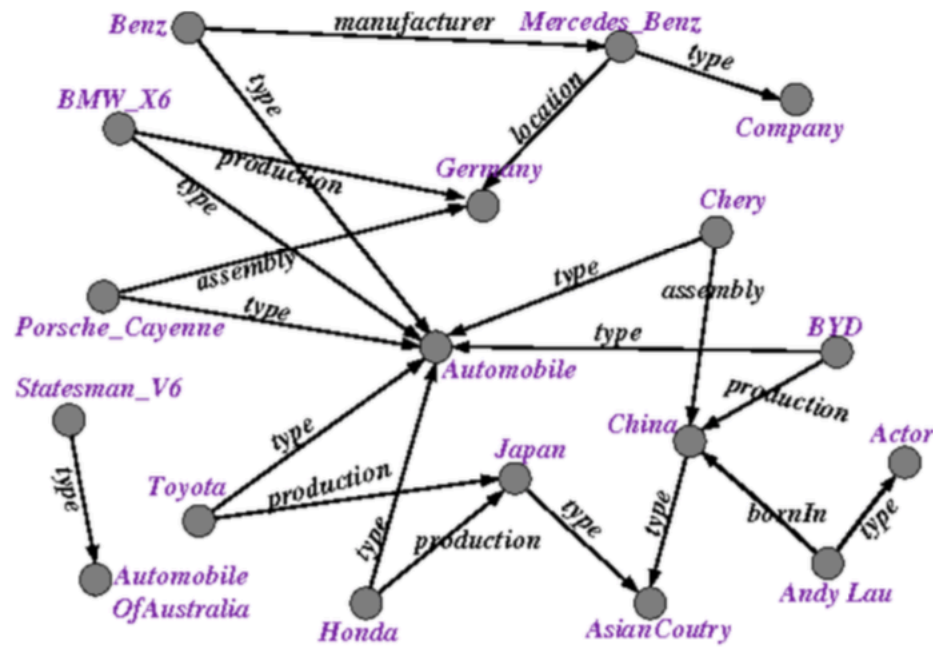
August 2003 blackout

(2) Networks: Infrastructure

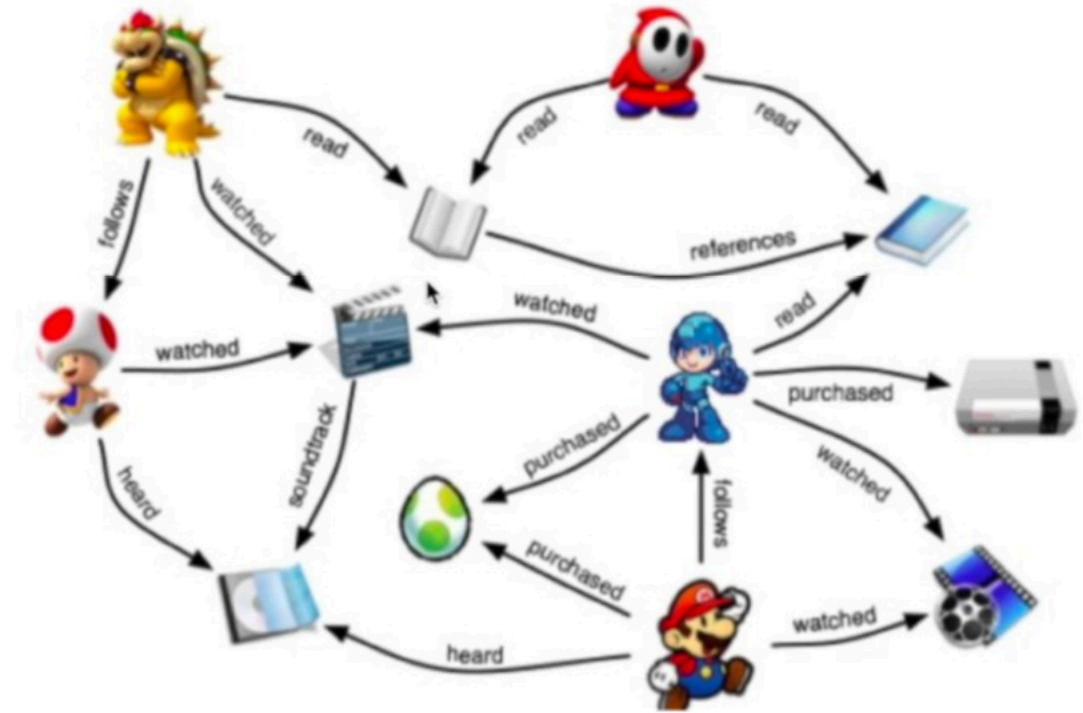
This illustrates important themes of this class:

- We must **understand** how network structure affects the system
- We will **develop quantitative tools** to assess the interplay between network structure and the dynamic processes that happen on networks
- We will learn that in reality failures follow reproducible laws, and can be **quantified**, and to some extent **predicted**, using the language of network analysis

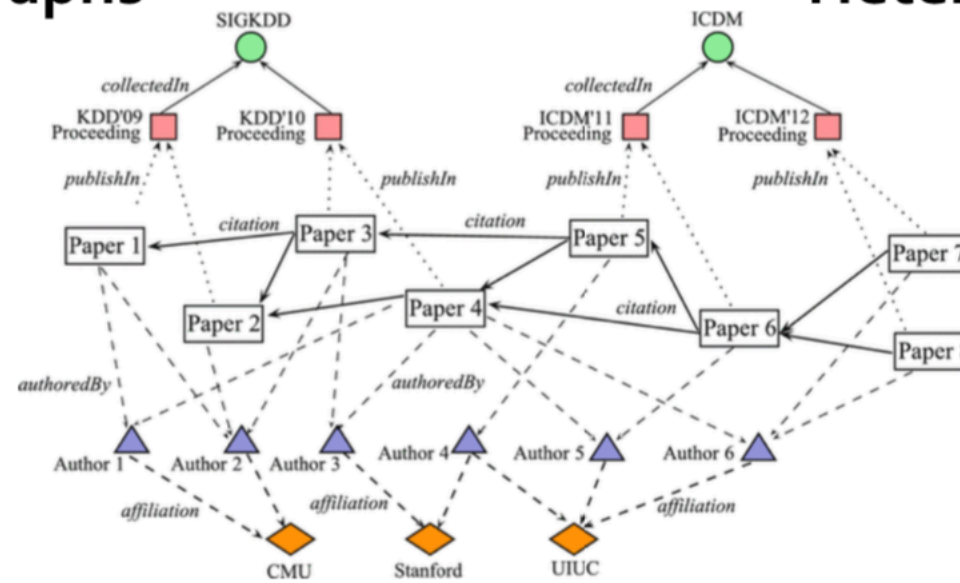
(3) Networks: Information and Knowledge



Knowledge Graphs

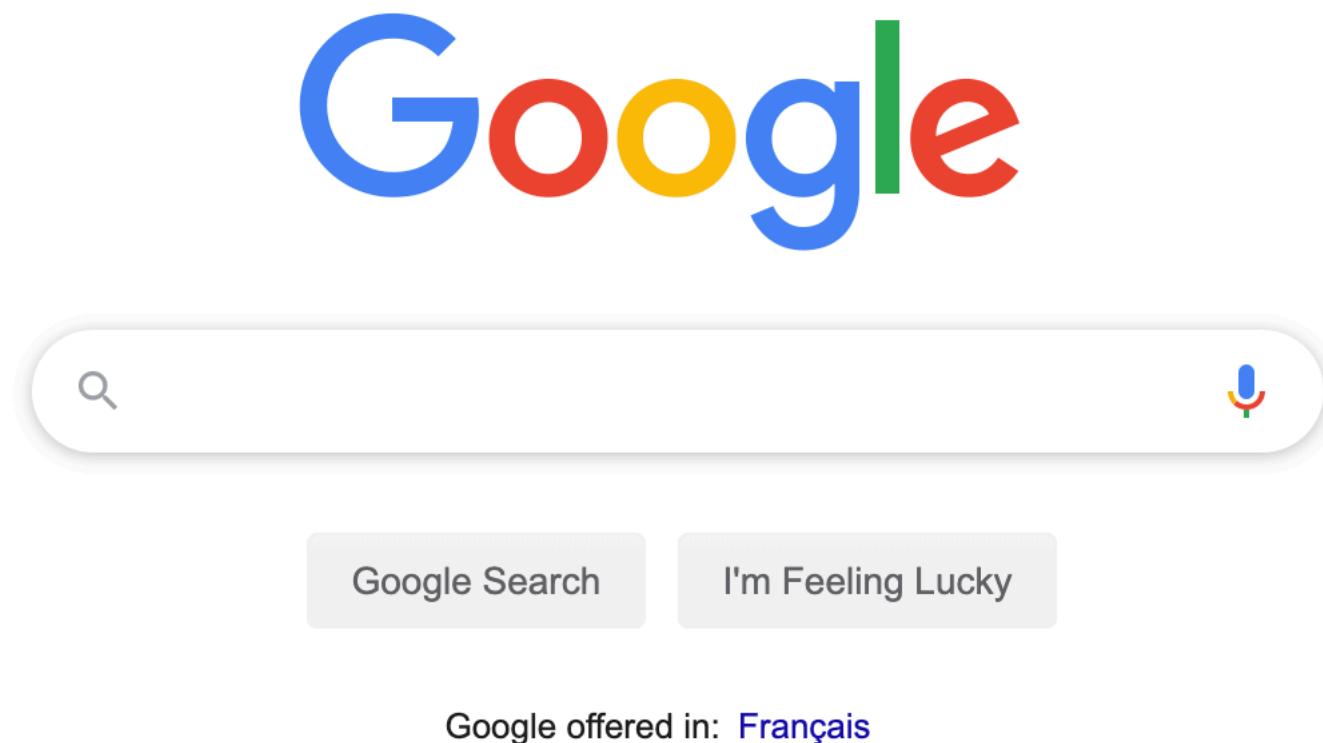


Heterogeneous Graphs



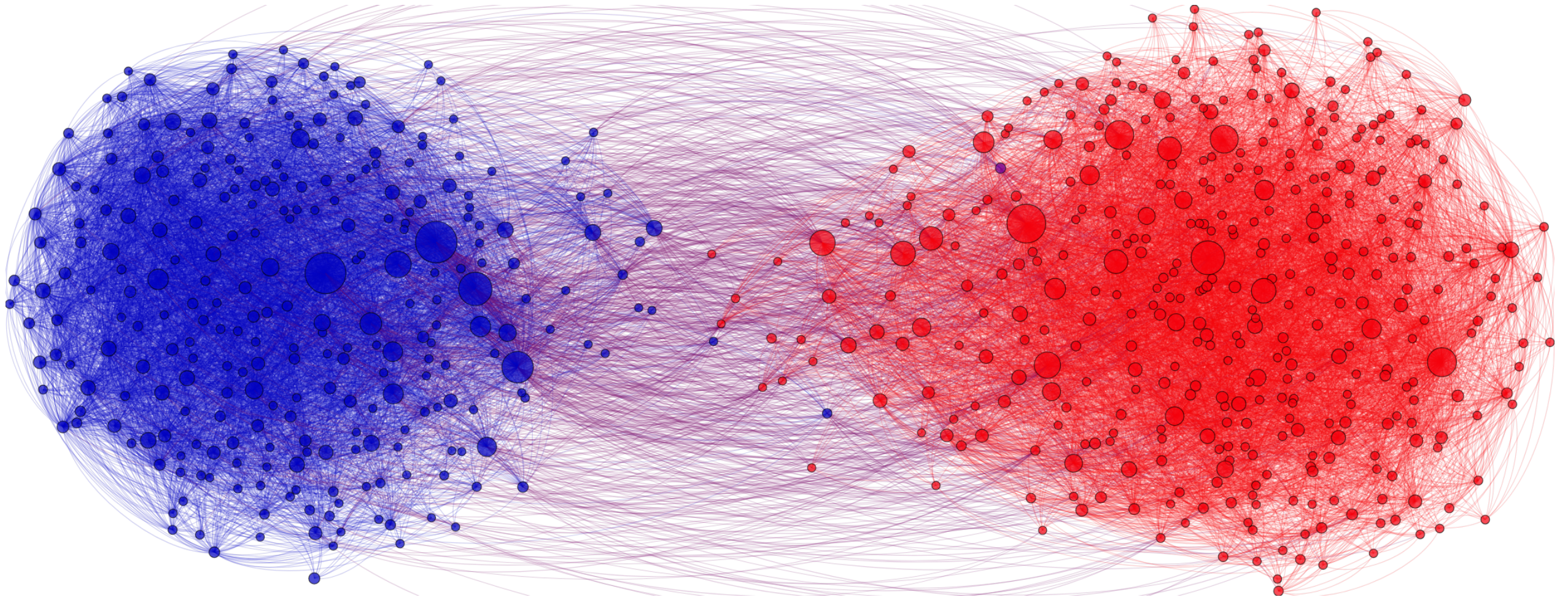
Multimodal Graphs

Application: Web Search



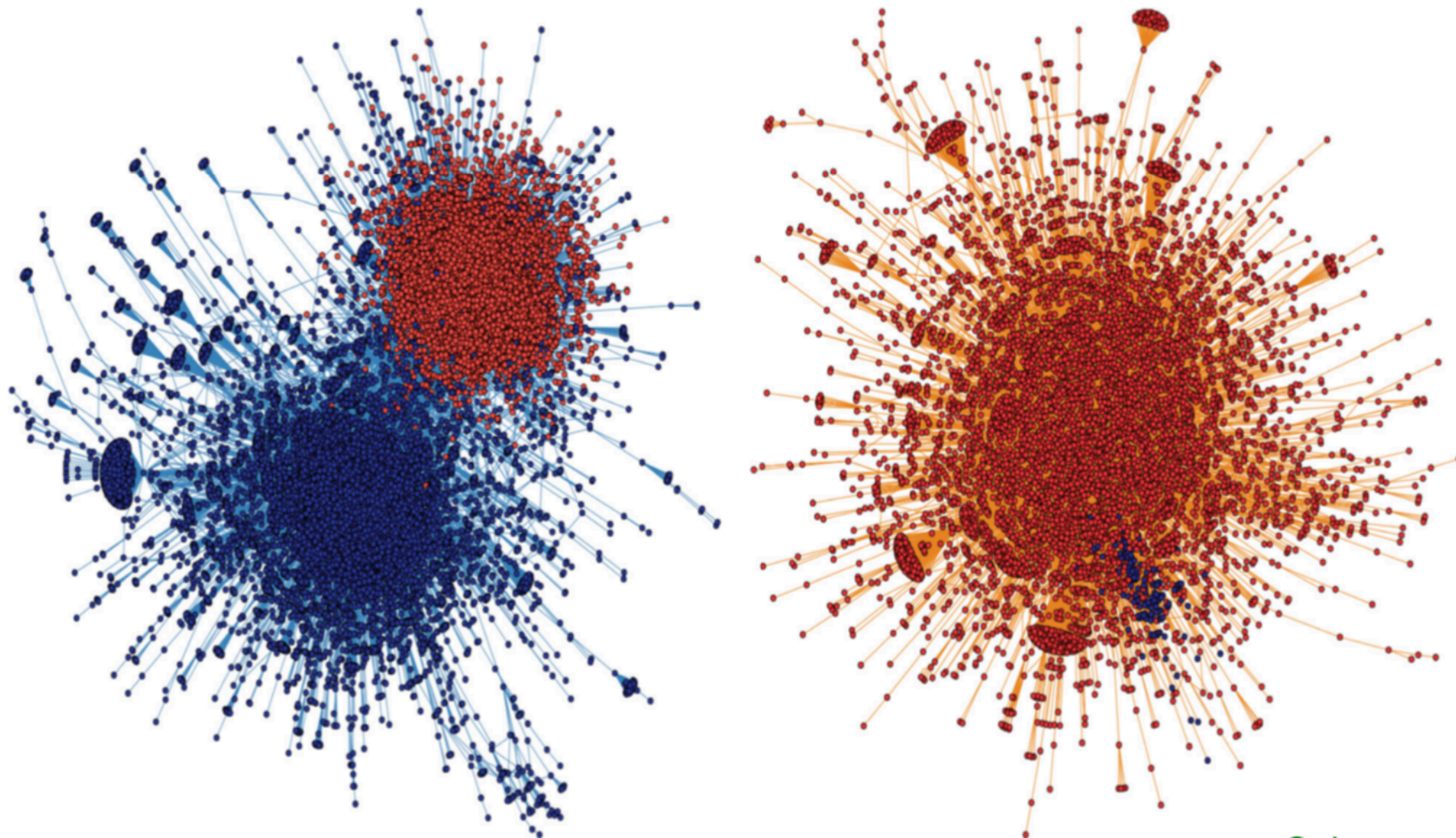
How do you go from a tiny text string to the 10 most relevant sites out of billions of pages?

(4) Networks: Online Media



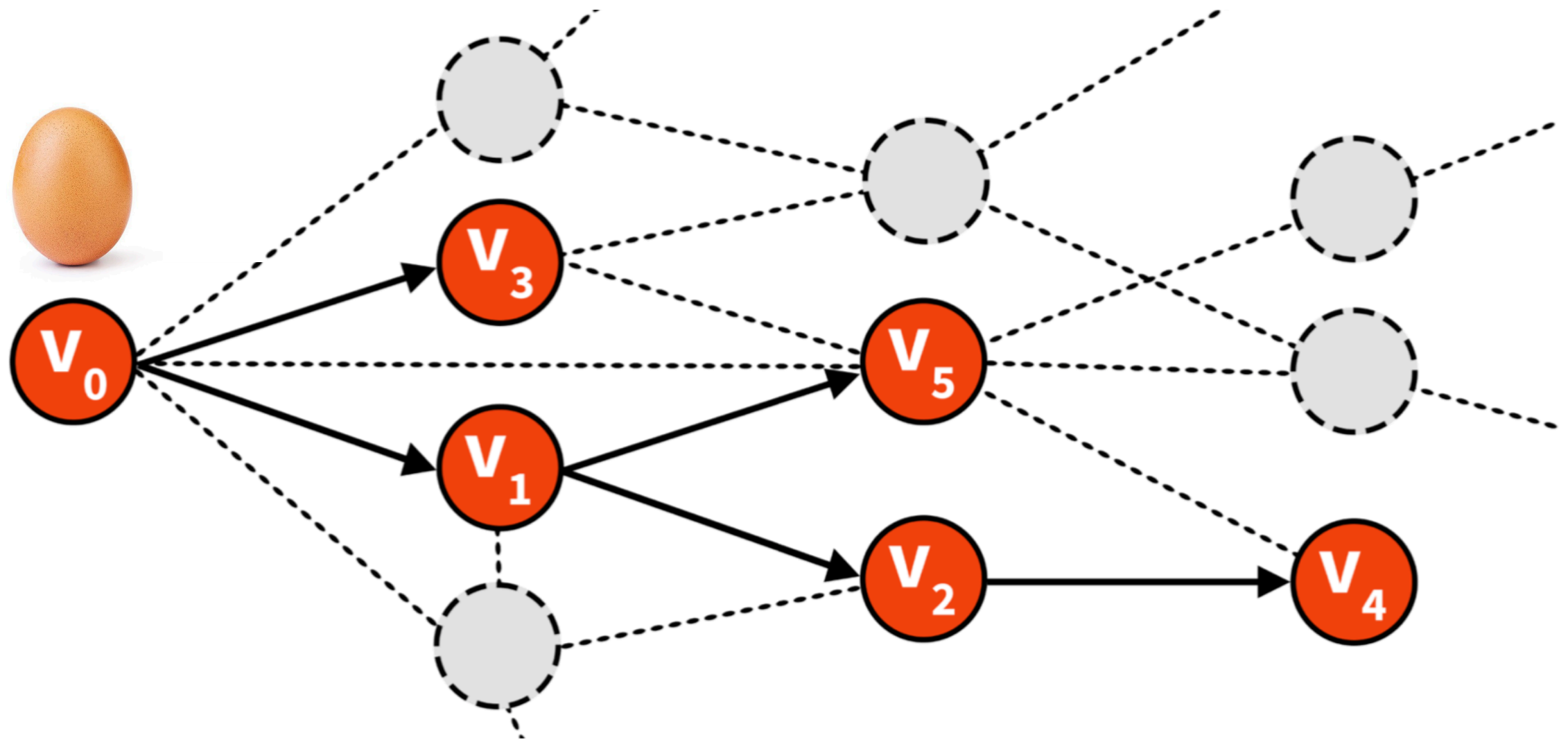
Connections between political blogs

Application: Polarization on Twitter



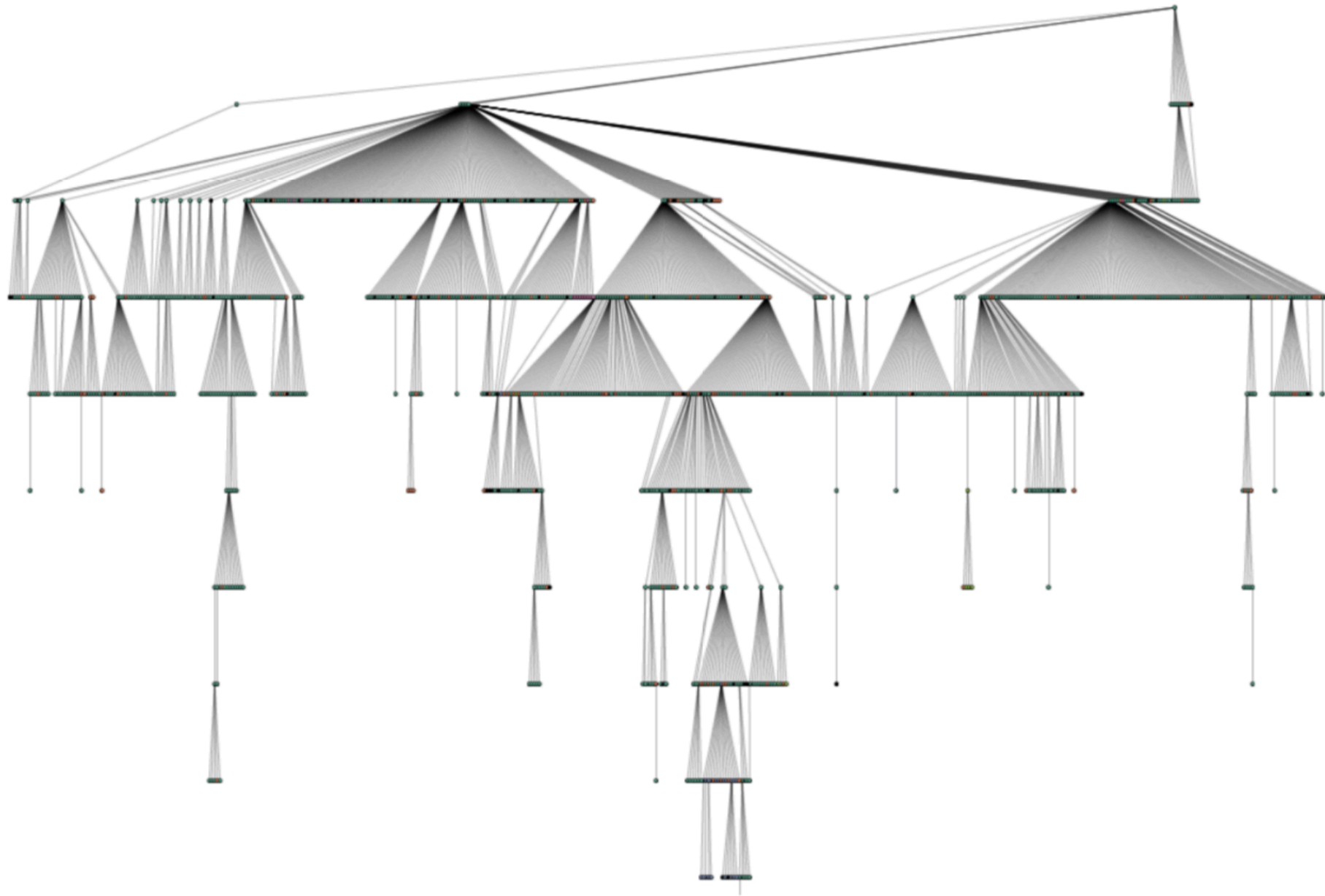
Retweet networks: polarized (left) and unpolarized (right)

Application: Understanding Virality



Information cascades in networks

Application: Product Adoption



Invitation cascades: 60–90% of LinkedIn users signed up due to an invitation from another user

[Anderson et al., WWW '15]

Networks matter

If you want to **predict the spread of a disease**, you need to know who is in contact with whom

If you want to **understand the structure of the Web** (or serve relevant search results), you have to analyze the links

If you want to understand the **dissemination of news or the evolution of science**, you have to follow the flow

About CSCC46

Ways to Analyze Networks

What do we hope to achieve from studying networks?

Develop the language of interconnectedness

Learn the patterns and statistical properties of network data

Understand design principles and models of networks

Develop algorithmic understanding of processes in networked systems

Networks: Structure, Dynamics, Incentives

What do we study in networks?

Structure and evolution

What is the structure of networks?

Why and how do they come to have such structure?

How do we harness the structure to extract useful information?

Processes and dynamics

Networks are the “skeleton” on which information, behaviours, and diseases spread

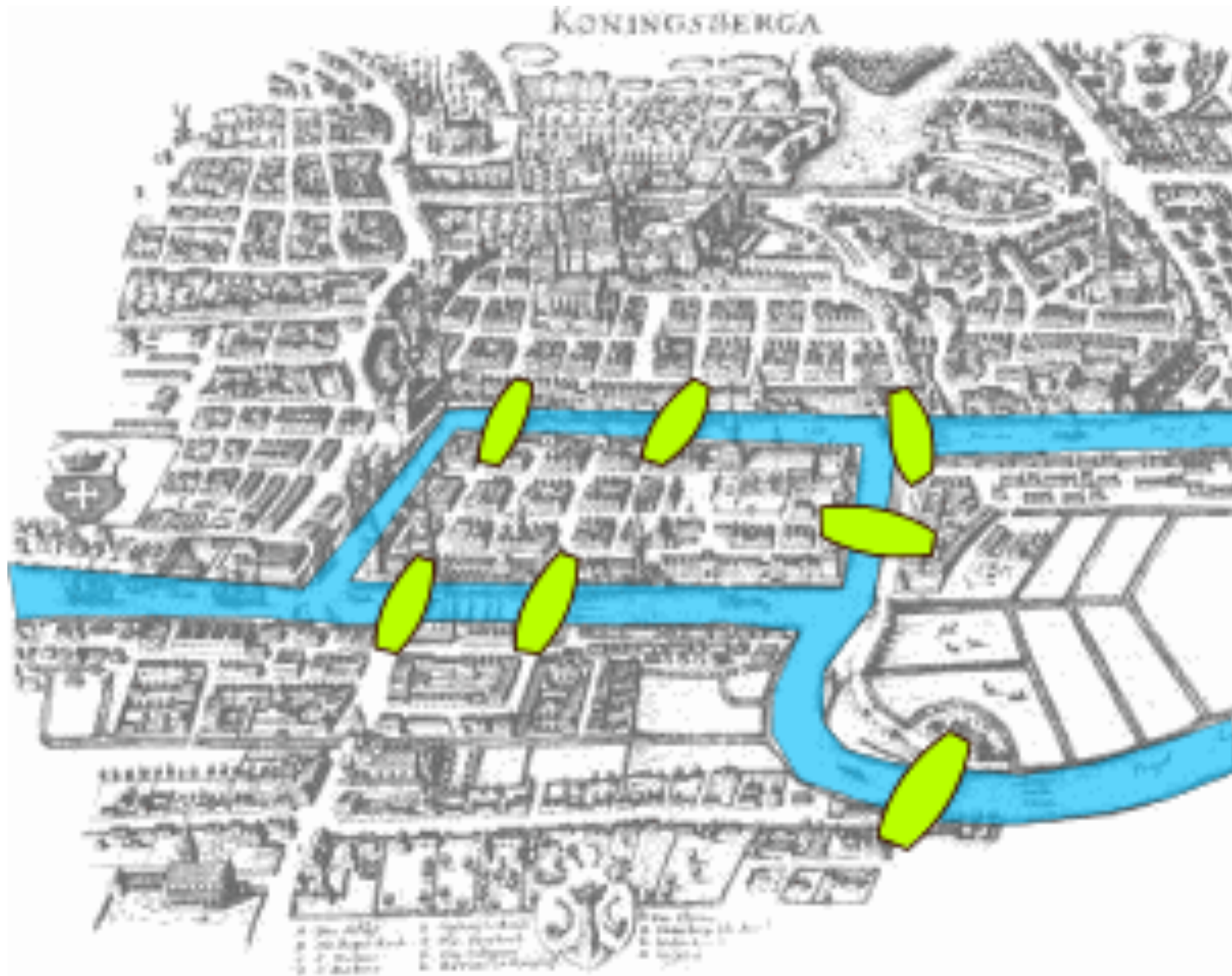
How do information and diseases spread?

Incentives in networks

Behaviour is interconnected by depending on what others do

How do decisions and behaviours depend on network structure and dynamics?

Main Tool: Graph Theory



Seven Bridges of Königsberg [Euler, 1735]

We'll make extensive use of graph theory in this course

Main Tool: Game Theory



The mathematical theory of strategic interaction

We'll also make extensive use of game theory in this course

(Tentative) Course Overview

- Week 1:** Course overview, Introduction to graph theory, The Web as a Network
- Week 2:** Network Representations, Affiliation, Homophily
- Week 3:** Strong and Weak Ties; Community Detection
- Week 4:** Signed Networks; Structural Balance; Homophily
- Week 5:** Six Degrees; Decentralized Search
- Week 6:** Power Laws and Rich-Get-Richer Phenomena
- Week 7:** Link Analysis; PageRank
- Week 8:** Game Theory
- Week 9:** Congestion; Decision Cascades; Information Cascades
- Week 10:** Contagion; Epidemics
- Week 11:** Voting
- Week 12:** Review

Course resources

Course webpage (<http://www.cs.toronto.edu/~ashton/csc46/>)

Quercus (course announcements, assignments)

MarkUs (assignment submission)

Course evaluation

25%	3 assignments
20%	Midterm
50%	Final exam
5%	Participation/Collegiality

Assignments are due on Thursdays at 5pm

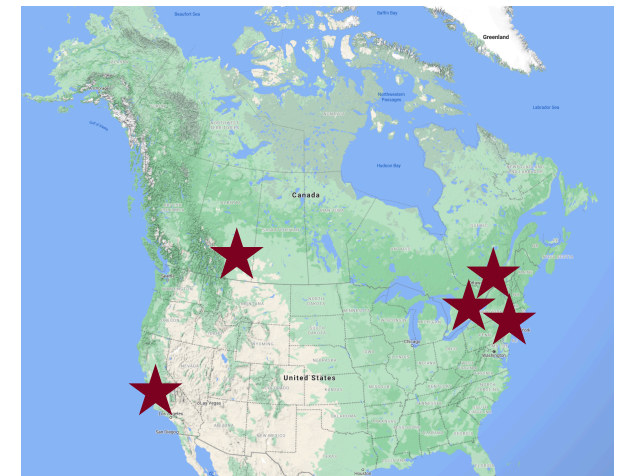
Because I understand sometimes stuff happens, you get 3 "flex days": 1 flex day is a 24-hour period that you can hand in assignments late with no penalty. After that, no late assignments will be accepted.

About Me

Calgary → Montreal → San Francisco → New York City → Toronto

1M → 4M → 7M → 20M → 6M

Now: Associate Professor of
Computer Science at U of T



Head of the Computational Social Science Lab
(researching questions in AI, data, and society) 🧐

(Want to get involved? Email me after the course!)

My path

Stage

Interests



McGill
B.Soft.Eng '08

Theoretical

Quantum algorithms and information

Anything practical was impure



Stanford

"Hmm...would be nice to feel more connected to the world"

Game theory: computational/economic lens on



Stanford Ph.D.

Discovered the joy and power of large-scale empirical analysis



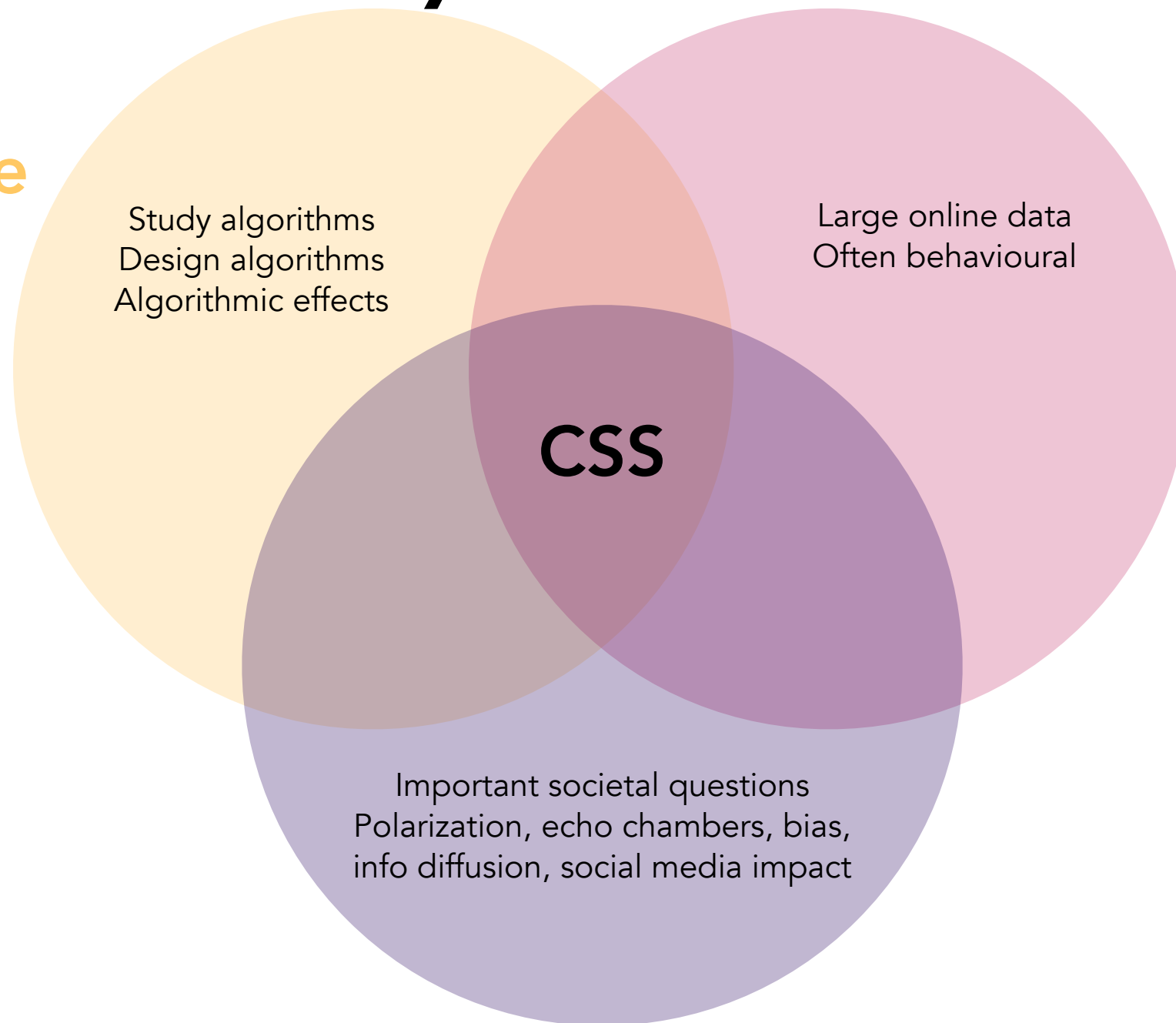
Computational social science: social research in the digital age

Mostly empirical analysis supplemented with theoretical modeling, experimentation, and surveys

My research

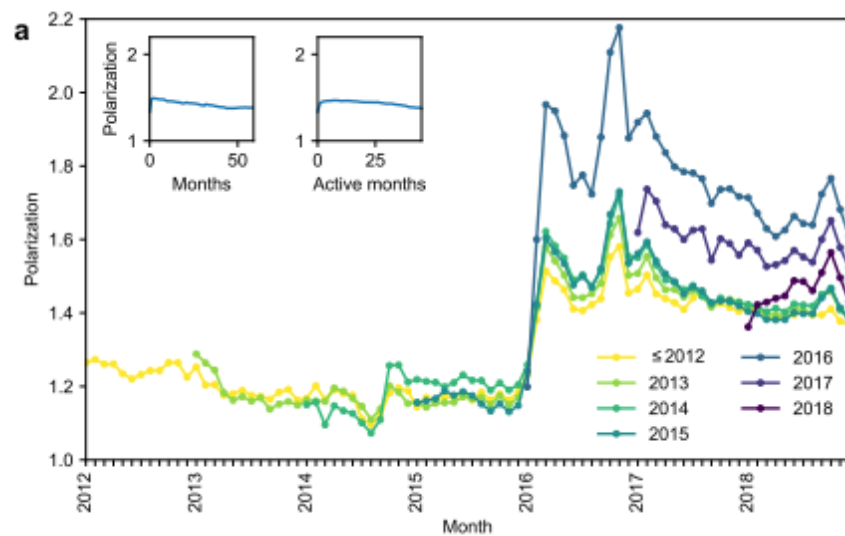
Artificial
Intelligence

Data

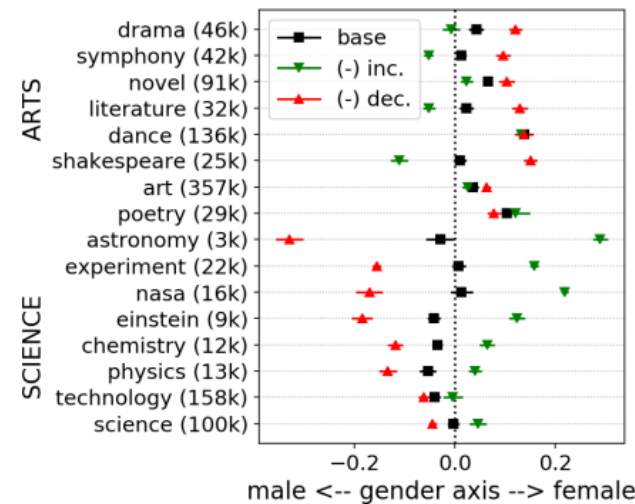


Society

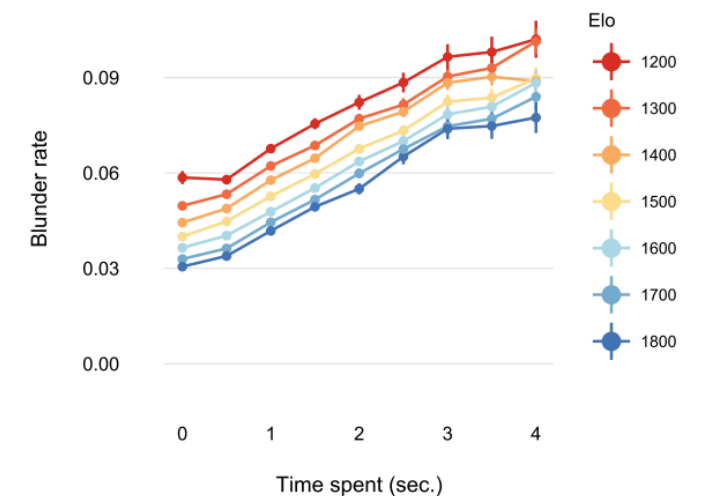
My research



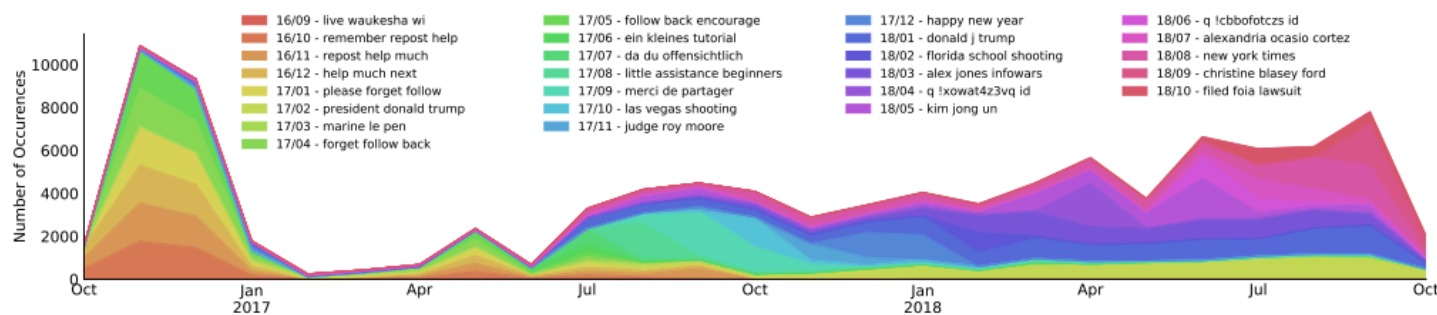
Political polarization on Reddit



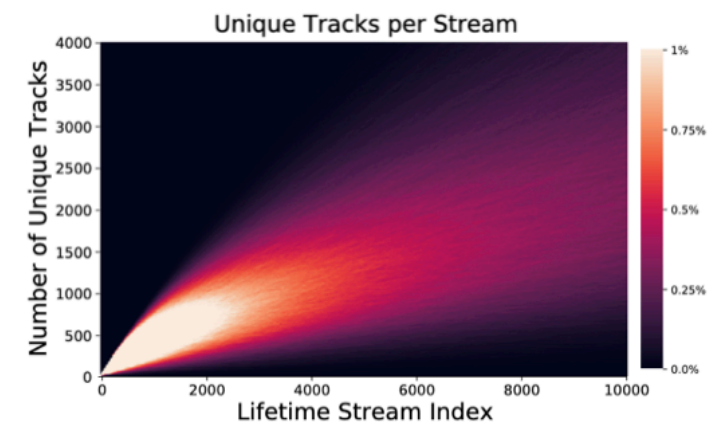
Gender bias in text algorithms



Nature of human error in chess



Discussion topics on Gab (alt-right platform)



Music exploration on Spotify

Tutorials

- Time: Mondays 12–1 and 1–2
- Required
- Mostly working through concrete examples as a group, Q&A (group office hours), and assignment help

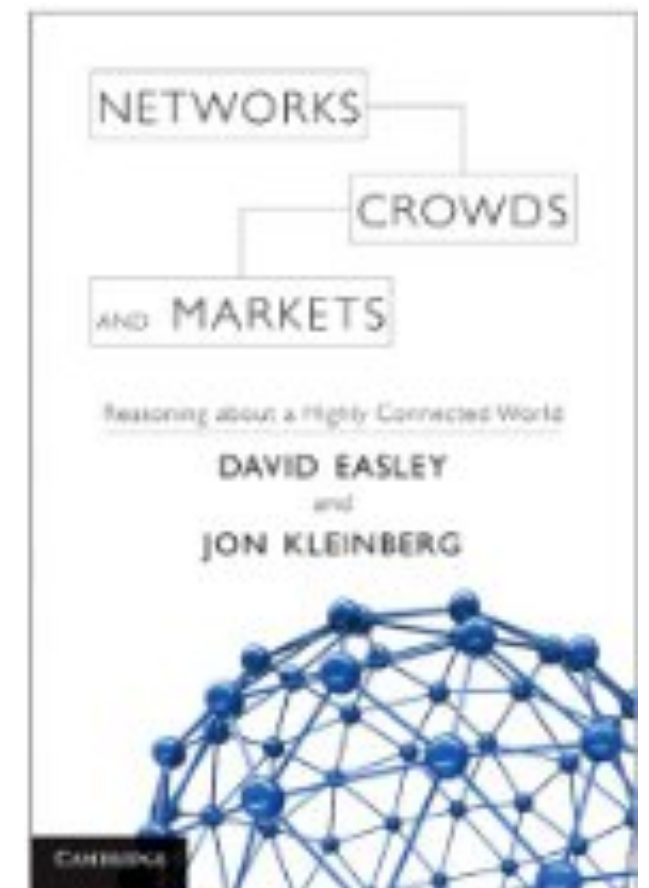
Textbook

“Networks, Crowds & Markets” Easley & Kleinberg

Available free online / reasonably-priced hardcover

Very readable, engaging text

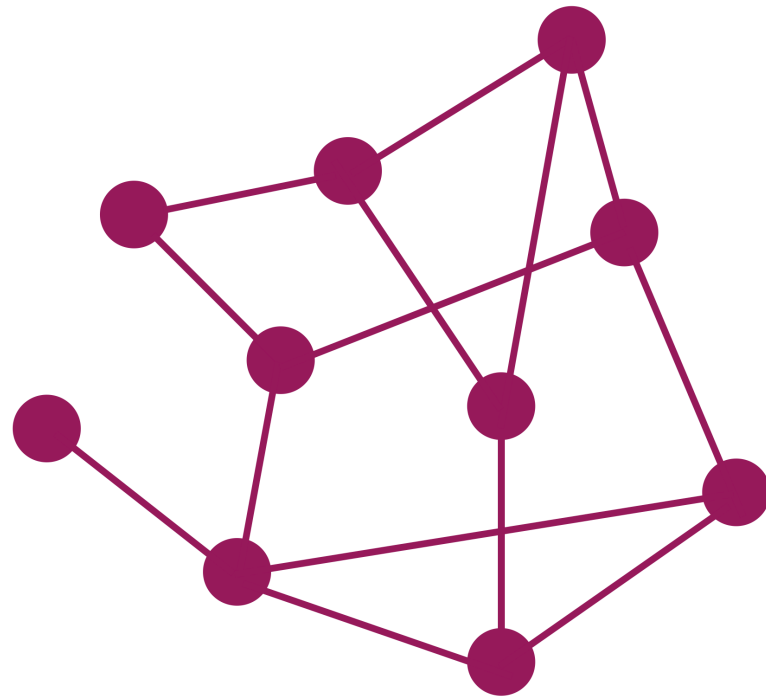
Some assignment exercises from the book



Questions?

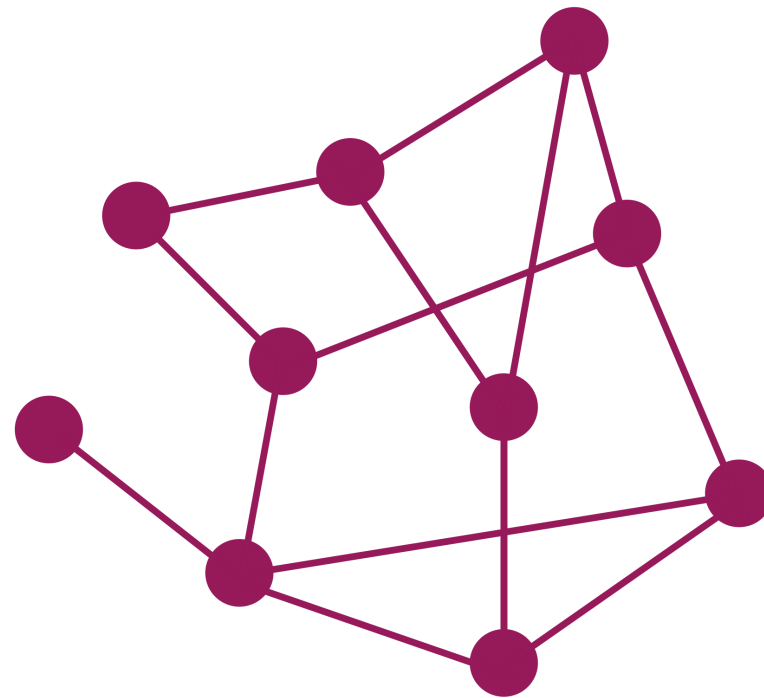
First topic: Network Analysis Fundamentals and The Structure of the Web

A Network



**A network is a collection of objects
where some pairs of objects are
connected by links**

Components of a Network



Objects: nodes, vertices

Interactions: links, edges

System: network, graph

N

E

G(N,E)

Networks or Graphs?

Network often refers to real systems

Web, Social network, Metabolic network

Language: Network, node, link

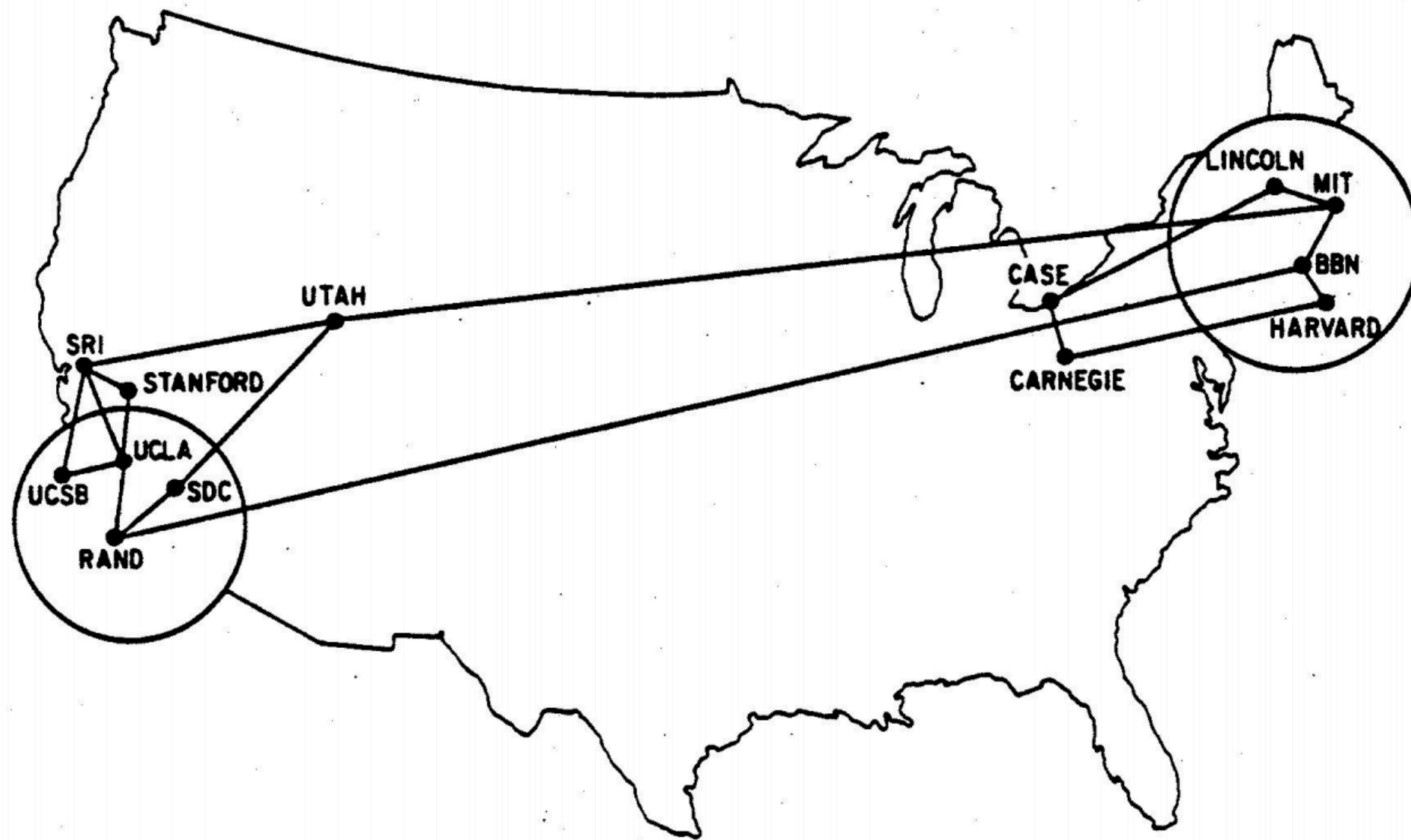
Graph is mathematical representation of a network

Web graph, Social graph (a Facebook term)

Language: Graph, vertex, edge

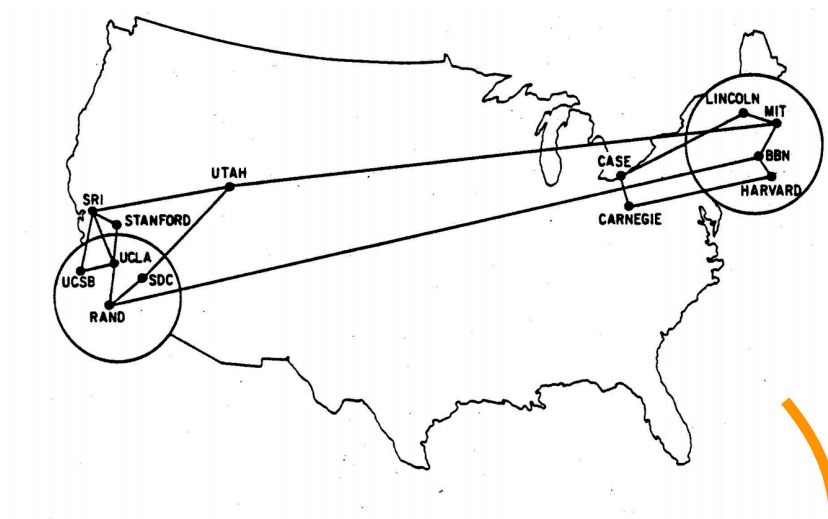
We will try to make this distinction whenever it is appropriate

A first example

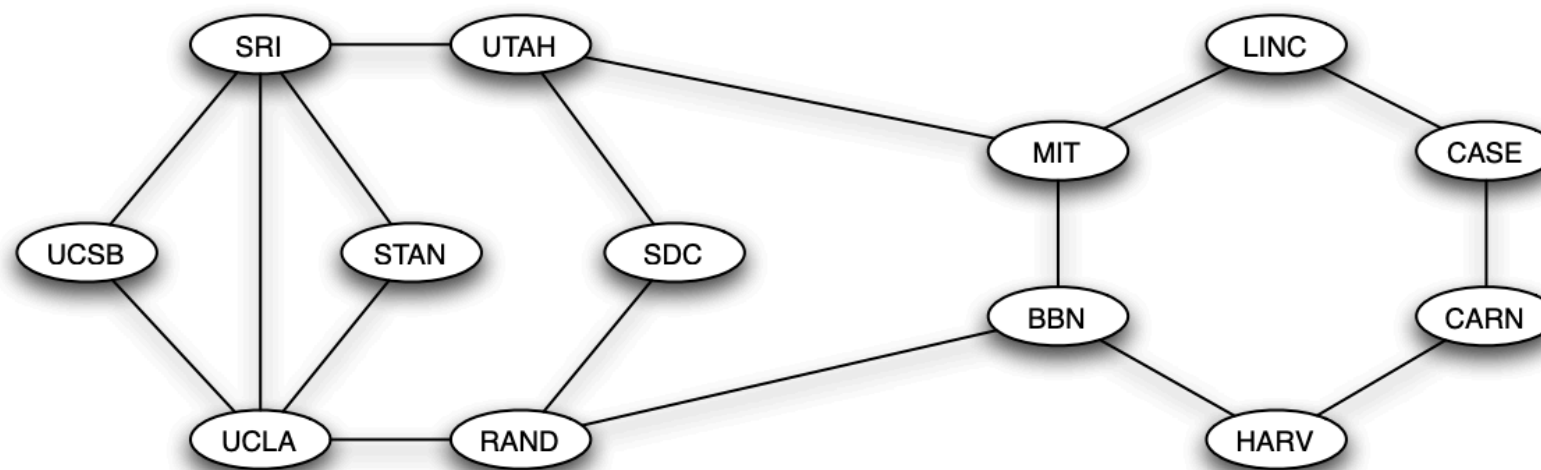


The Internet in 1970

A first example

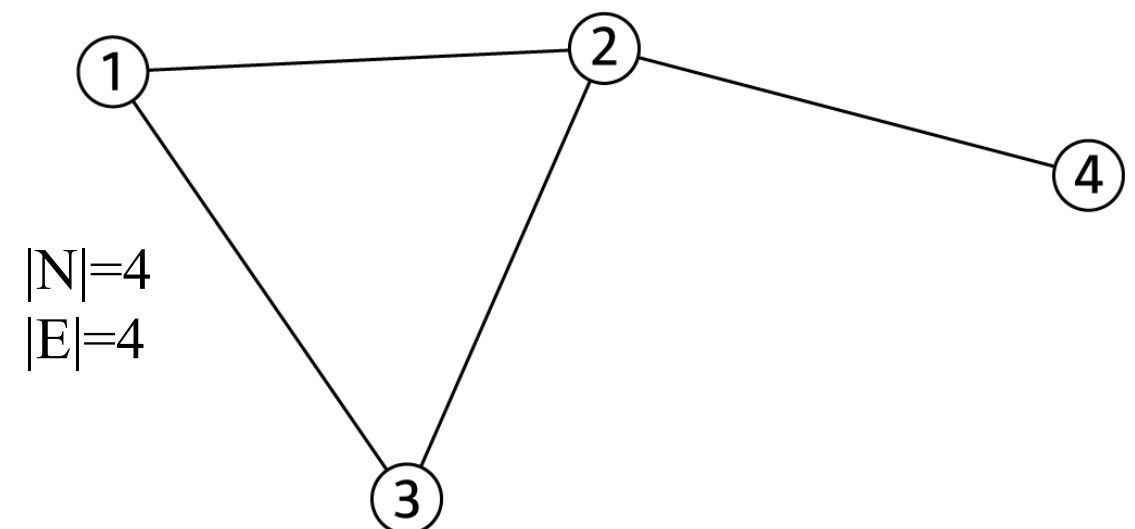
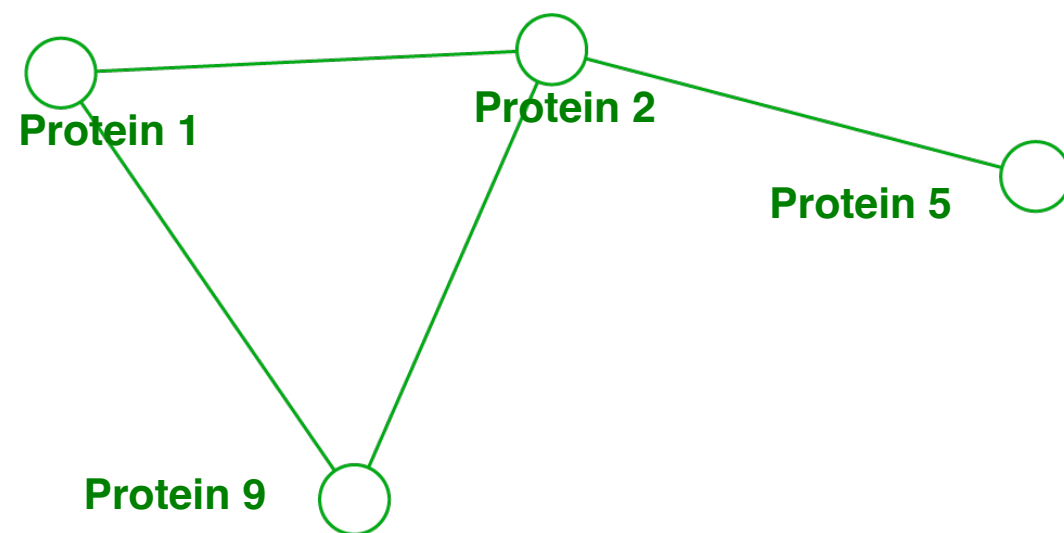
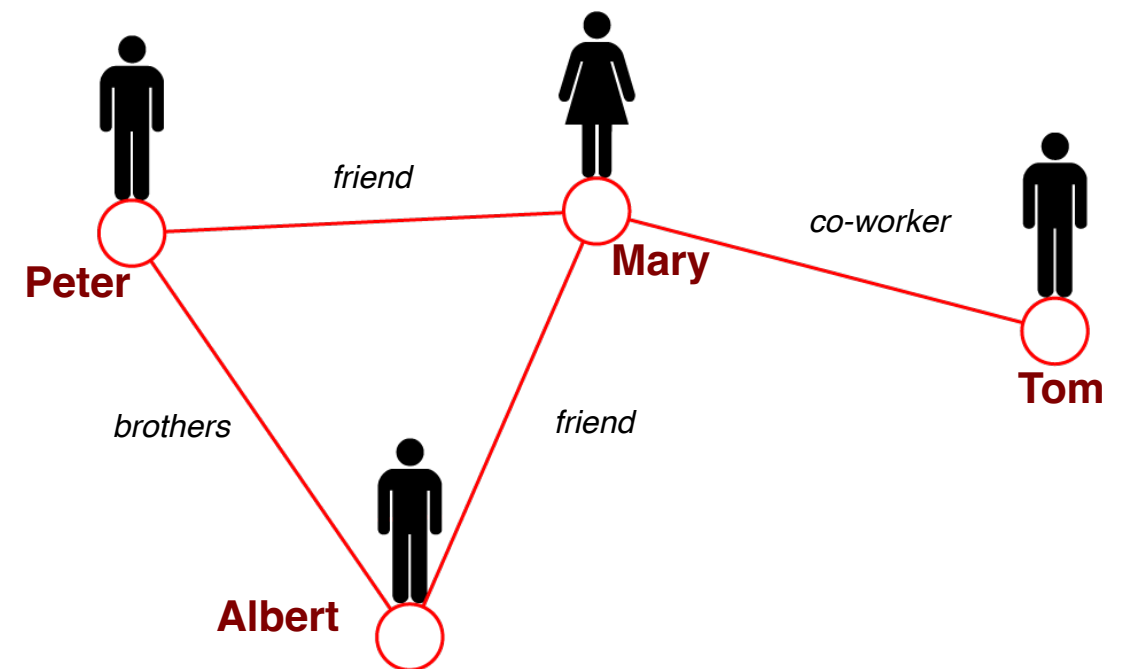
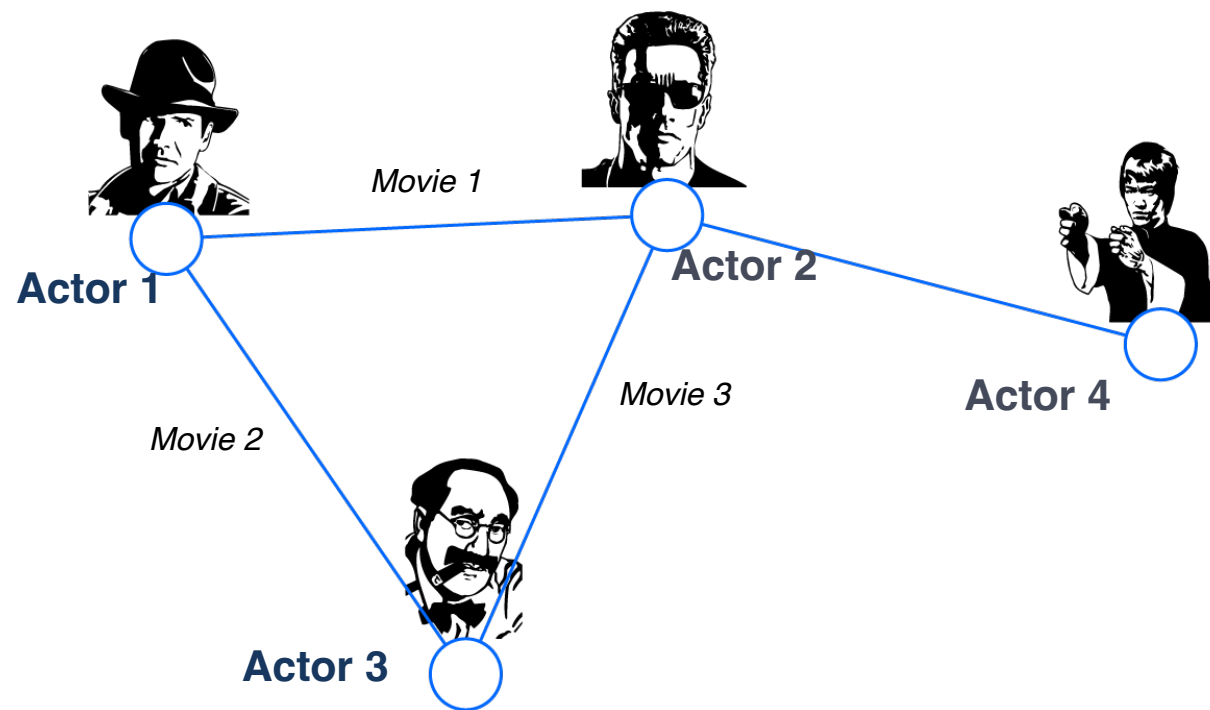


Translation



The Internet in 1970

Networks: a shared language



Choosing a Proper Representation

- **How to build a graph:**

- What are nodes?
- What are edges?

- The choice of the proper network representation of a given domain/problem **determines our ability to use networks successfully:**

- In some cases there is a unique, unambiguous representation
- In other cases, the representation is by no means unique
- The way you assign links will determine the nature of the question you can study

Choosing a Proper Representation

- If you connect individuals that work with each other, you will be exploring a **professional network**
- If you connect those that have a friendship relationship, you will be exploring a **friendship network**
- If you connect scientific papers that cite each other, you will be studying a **citation network**

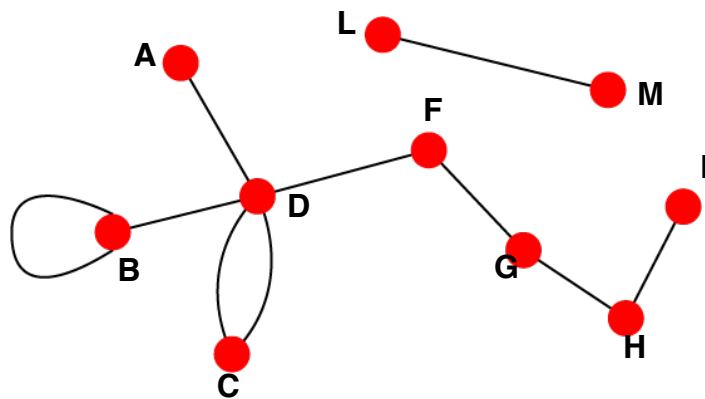
If you connect all people with first names that share the same first letter, what are you studying?

It is a network, but is it meaningful?

Undirected and Directed Networks

Undirected

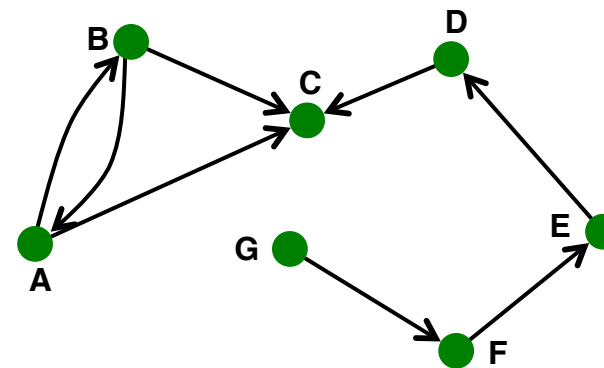
- Links: undirected (symmetrical, reciprocal)



- Examples:
 - Collaborations
 - Friendship on Facebook

Directed

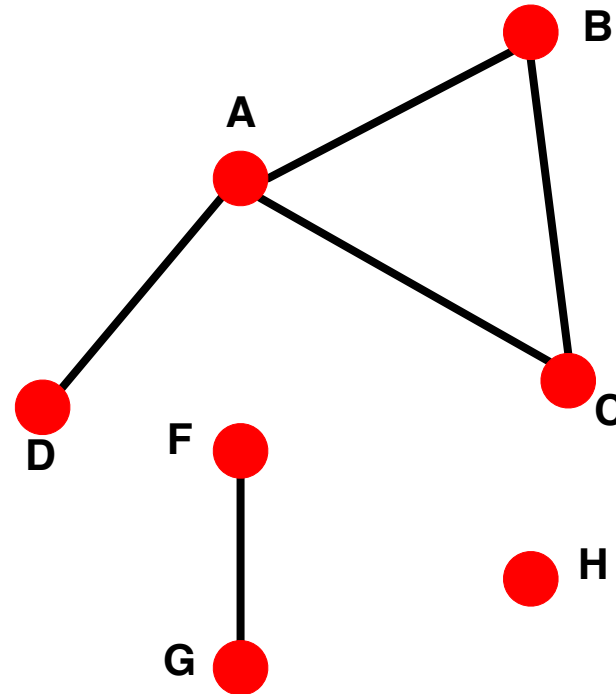
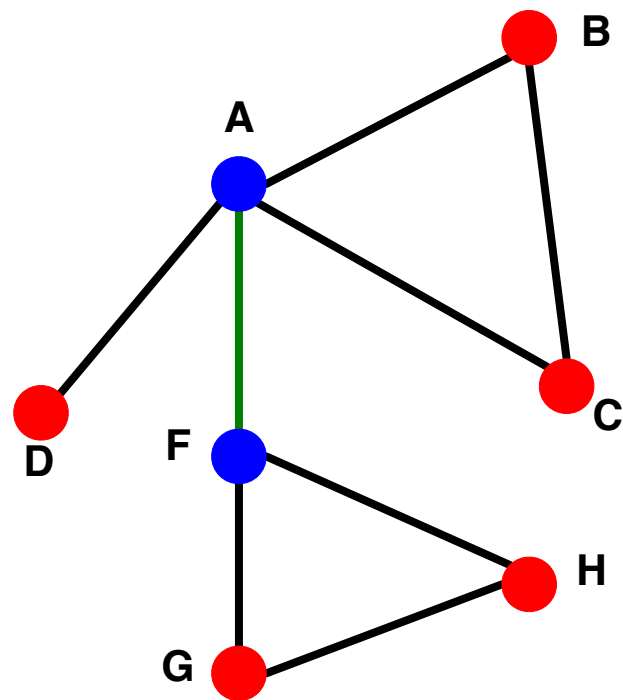
- Links: directed (arcs)



- Examples:
 - Phone calls
 - Following on Twitter

Connectivity of Graphs

- **Connected component (undirected):**
 - Any two vertices can be joined by a path
 - No superset with the same property
- A disconnected graph is made up of two or more connected components

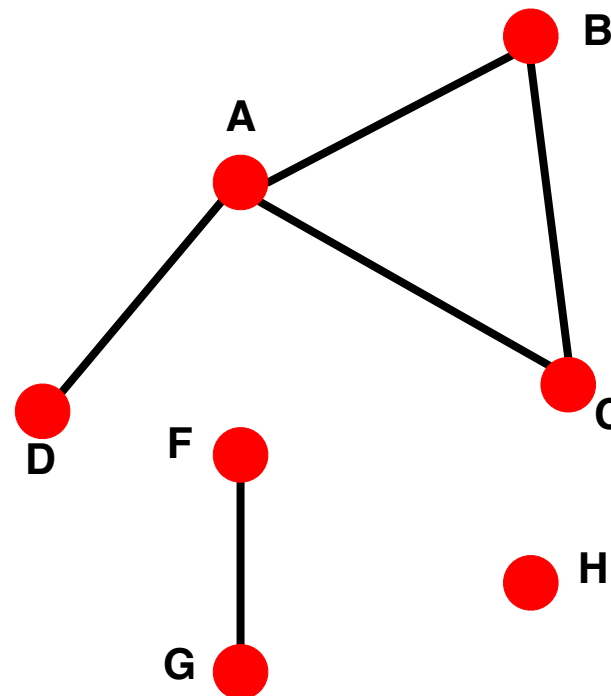
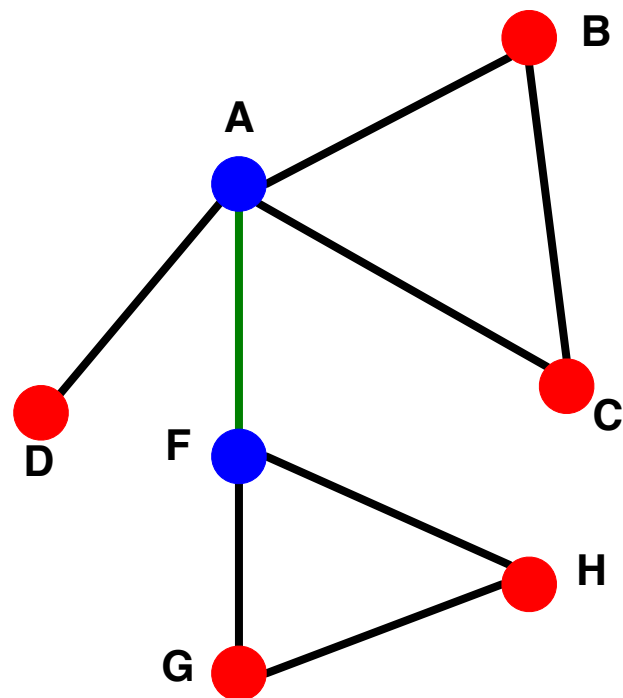


Largest Component:
Giant Component

Isolated node (node H)

Connectivity of Graphs

- **Connected component (undirected):**
 - Any two vertices can be joined by a path
 - No superset with the same property
- A disconnected graph is made up of two or more connected components



Largest Component:
Giant Component

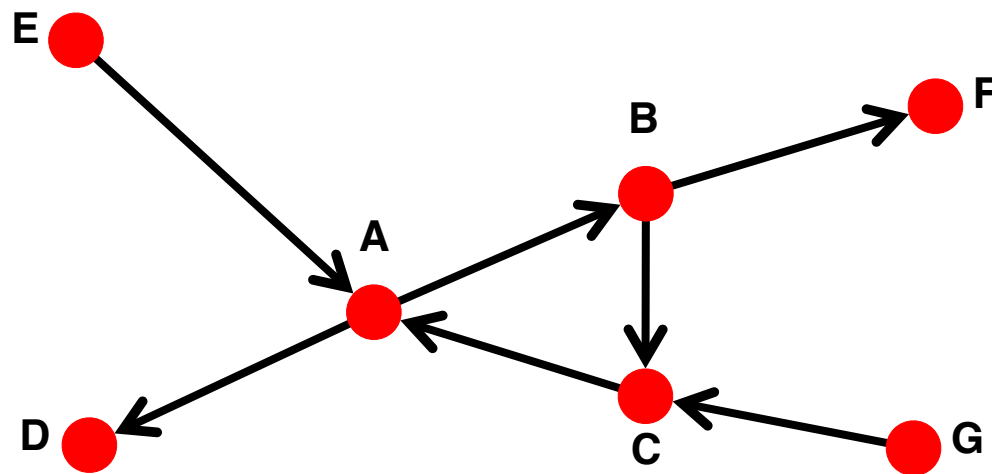
Isolated node (node H)

Bridge edge: If we erase it, the graph becomes disconnected.

Articulation point: If we erase it, the graph becomes disconnected.

Connectivity of Directed Graphs

- **Strongly connected directed graph**
 - has a path from each node to every other node and vice versa (e.g., A-B path and B-A path)
- **Weakly connected directed graph**
 - is connected if we disregard the edge directions



Is this graph weakly connected?
Strongly connected?

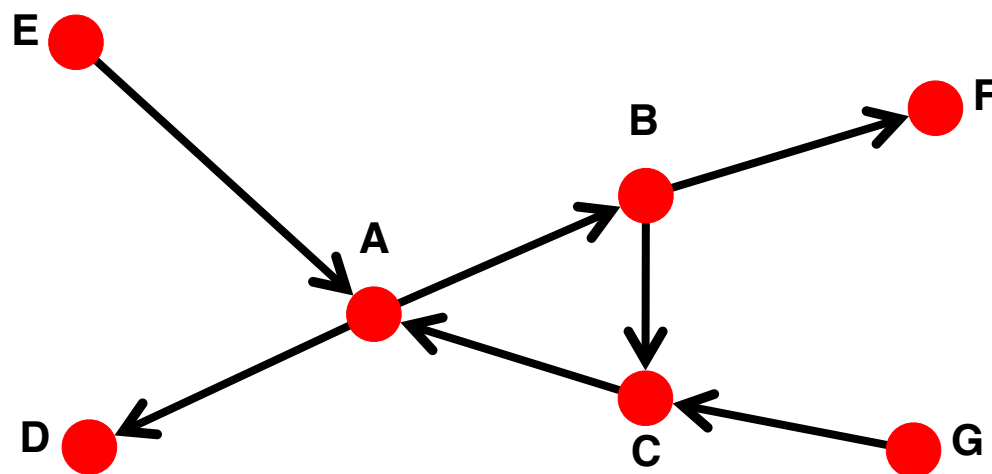
Connectivity of Directed Graphs

■ Strongly connected directed graph

- has a path from each node to every other node and vice versa (e.g., A-B path and B-A path)

■ Weakly connected directed graph

- is connected if we disregard the edge directions



It is weakly connected but not strongly connected (e.g., there is no way to get from F to G by following the edge directions)

What is the large-scale structure of the Web?

The Structure of the Web

- Q: What does the Web “look like”?



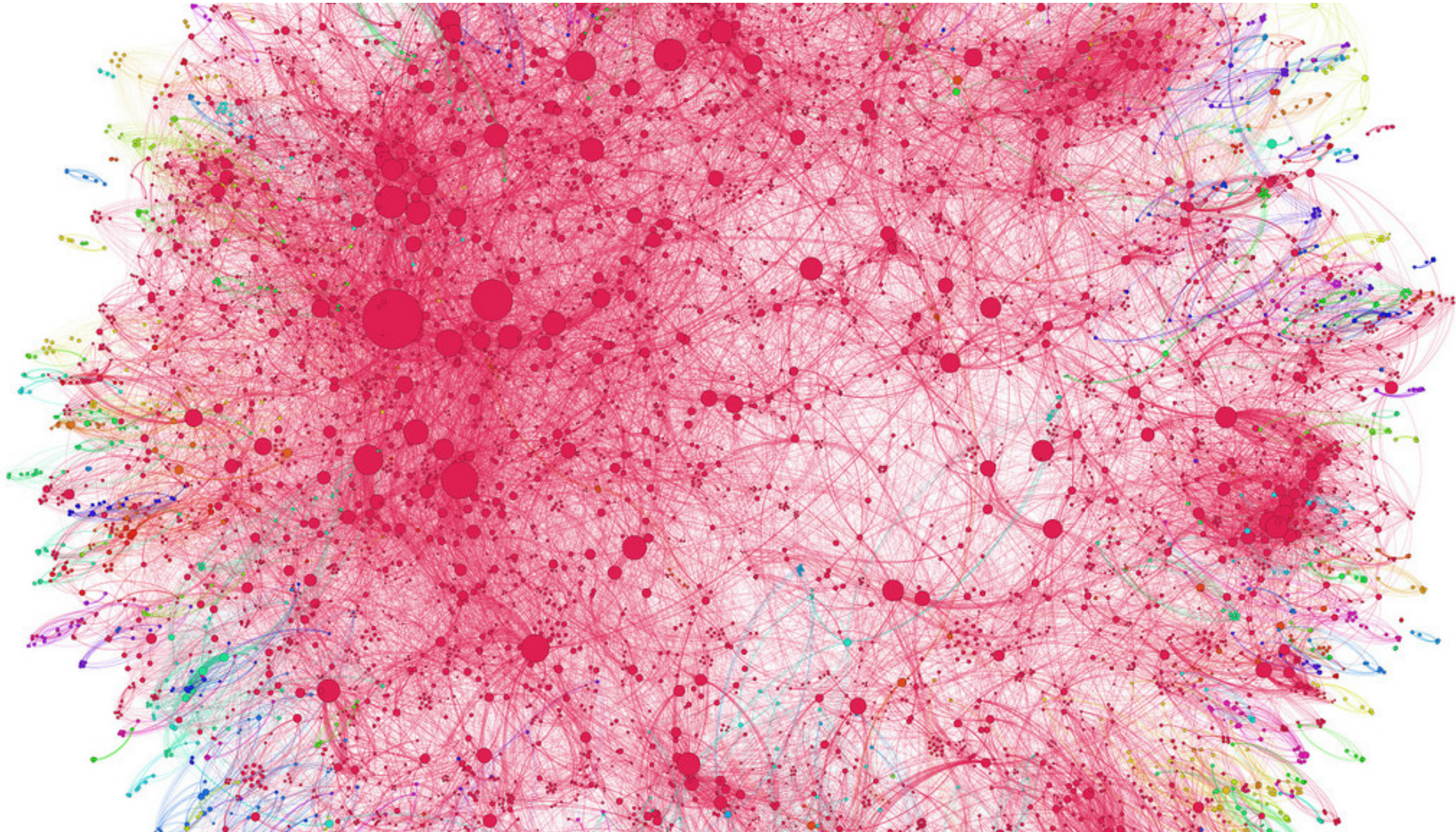
The Structure of the Web

■ Q: What does the Web “look like”?



The Structure of the Web

A network!



Web as a Graph

Here is what we will do next:

- We will take a real system (i.e., the Web)
- We will represent the Web as a graph
- We will use language of graph theory to reason about the structure of the graph
- Do a computational experiment on the Web graph
- **Learn something about the structure of the Web!**



Web as a Graph

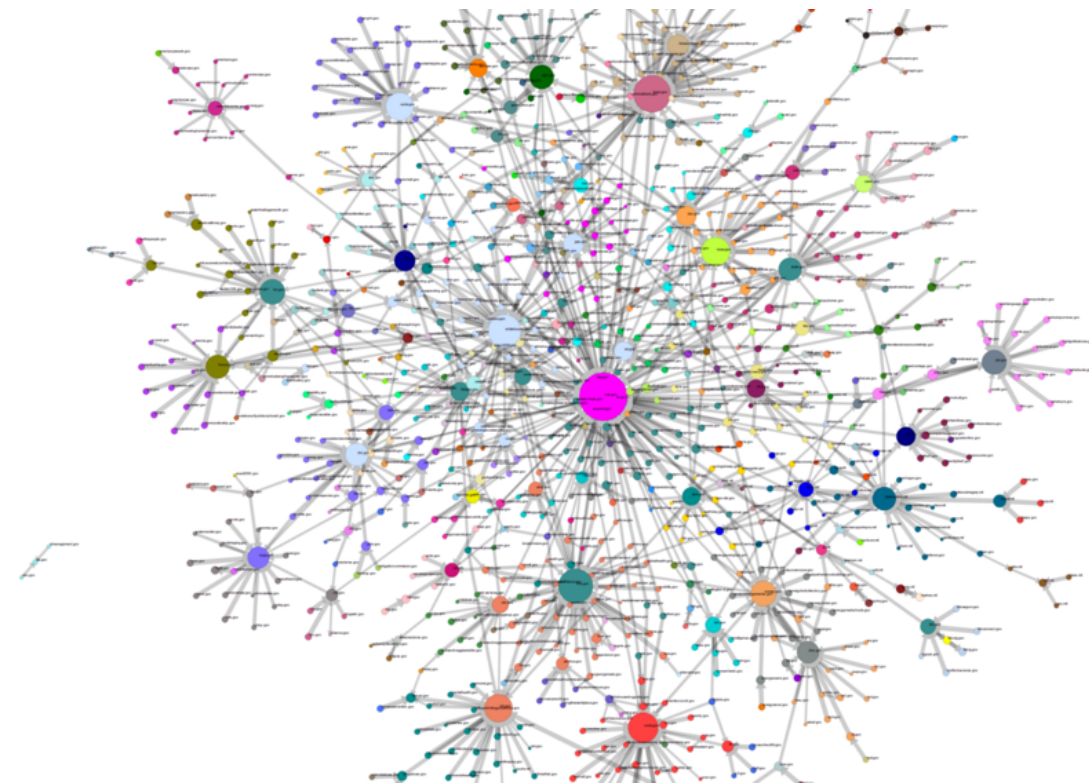
Q: What does the Web “look like” at a global level?

- **Web as a graph:**

- Nodes = web pages
- Edges = hyperlinks

- **Side issue:** What is a node?

- Dynamic pages created on the fly
- “dark matter” – inaccessible database generated pages



The Web as a Graph

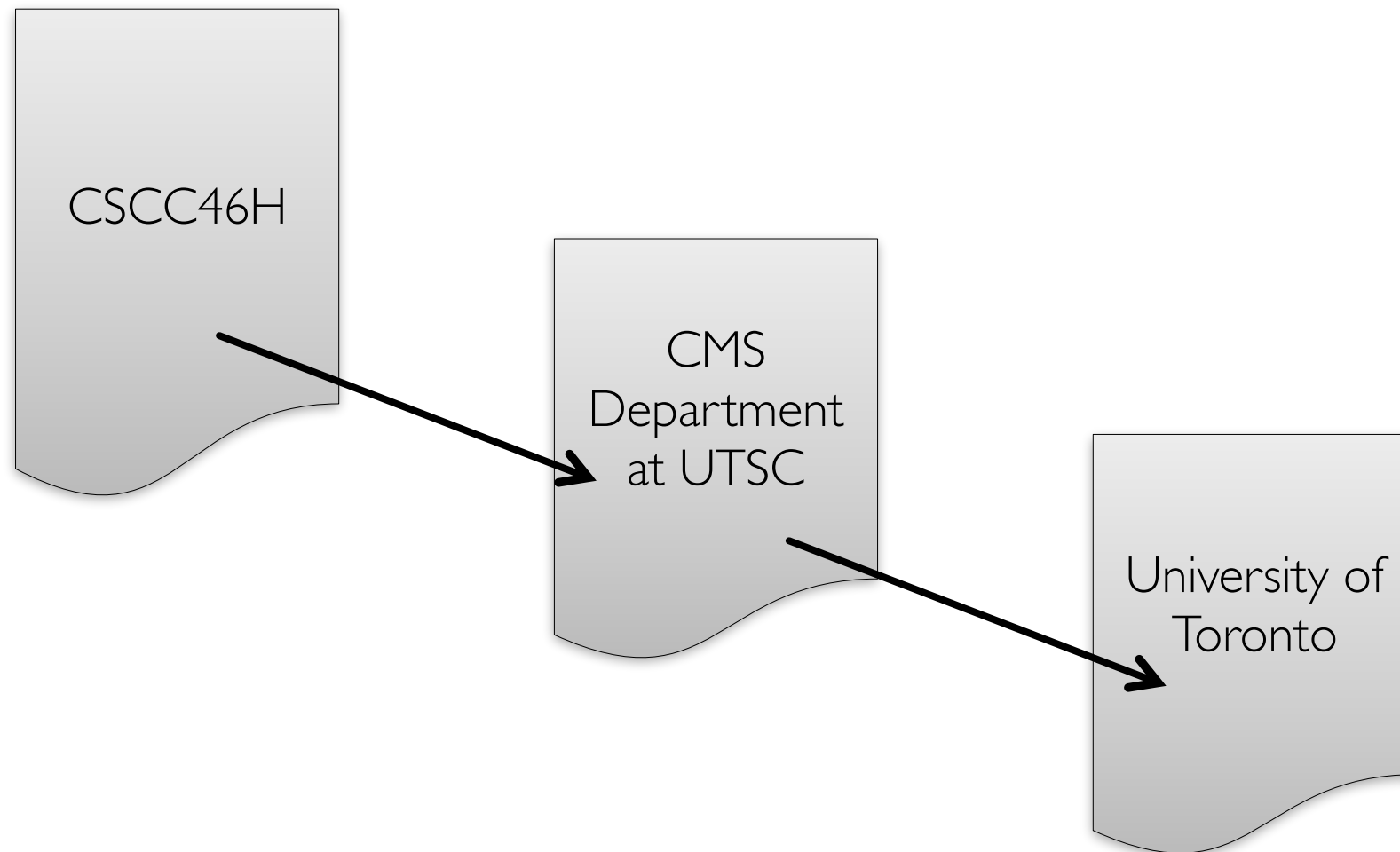


CSCC46H

CMS
Department
at UTSC

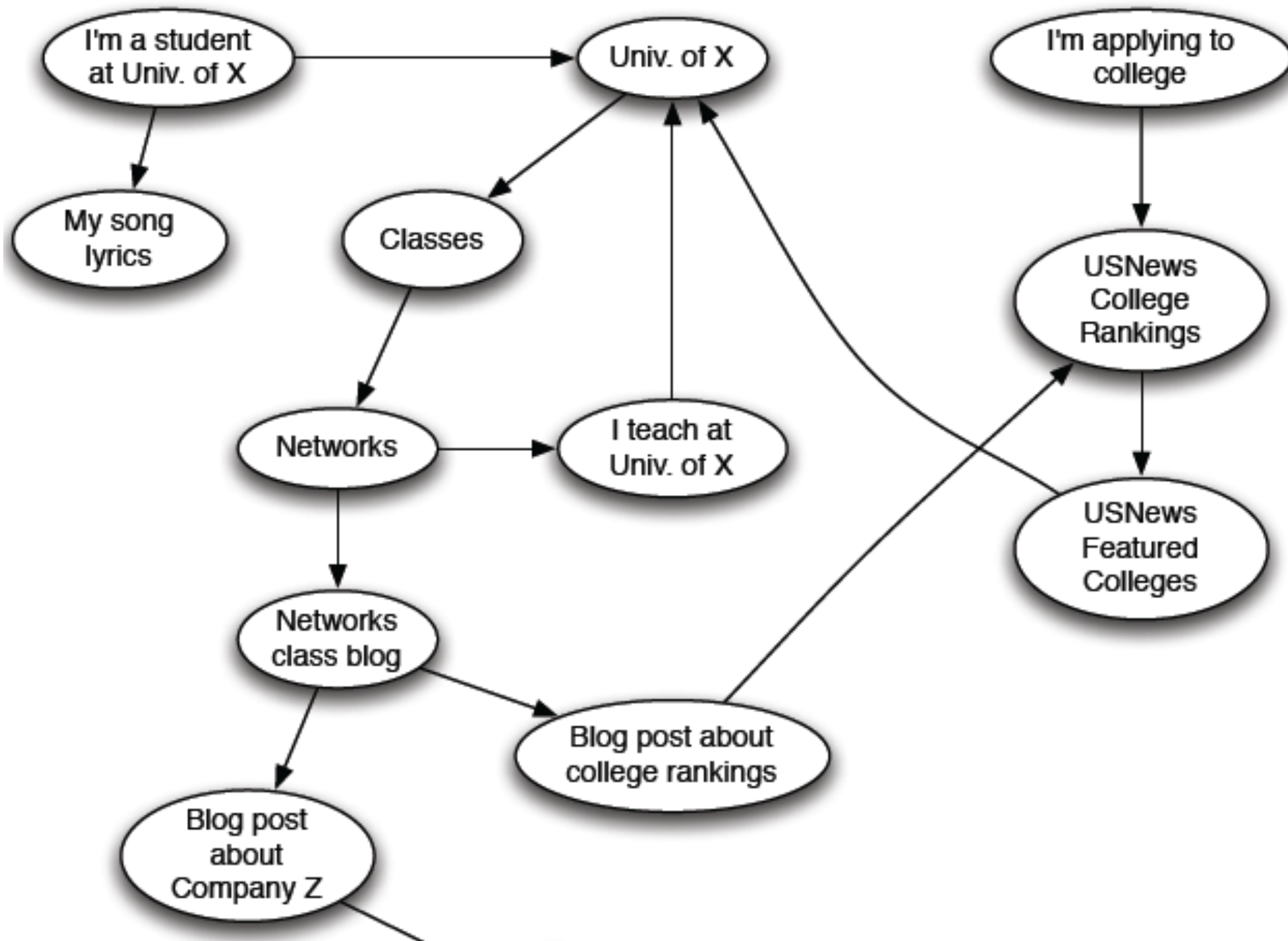
University of
Toronto

The Web as a Graph

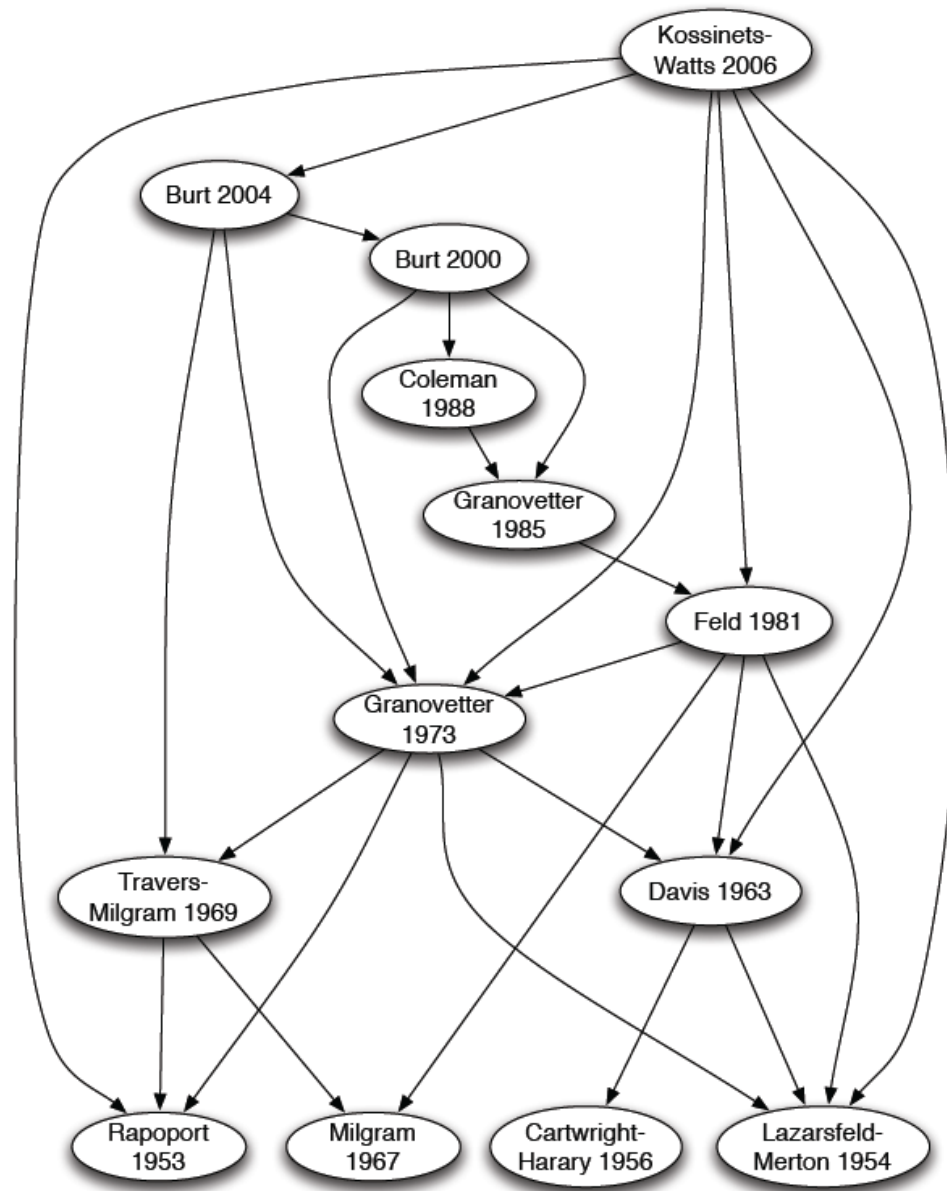


- In early days of the Web links were **navigational**
- Today many links are **transactional**

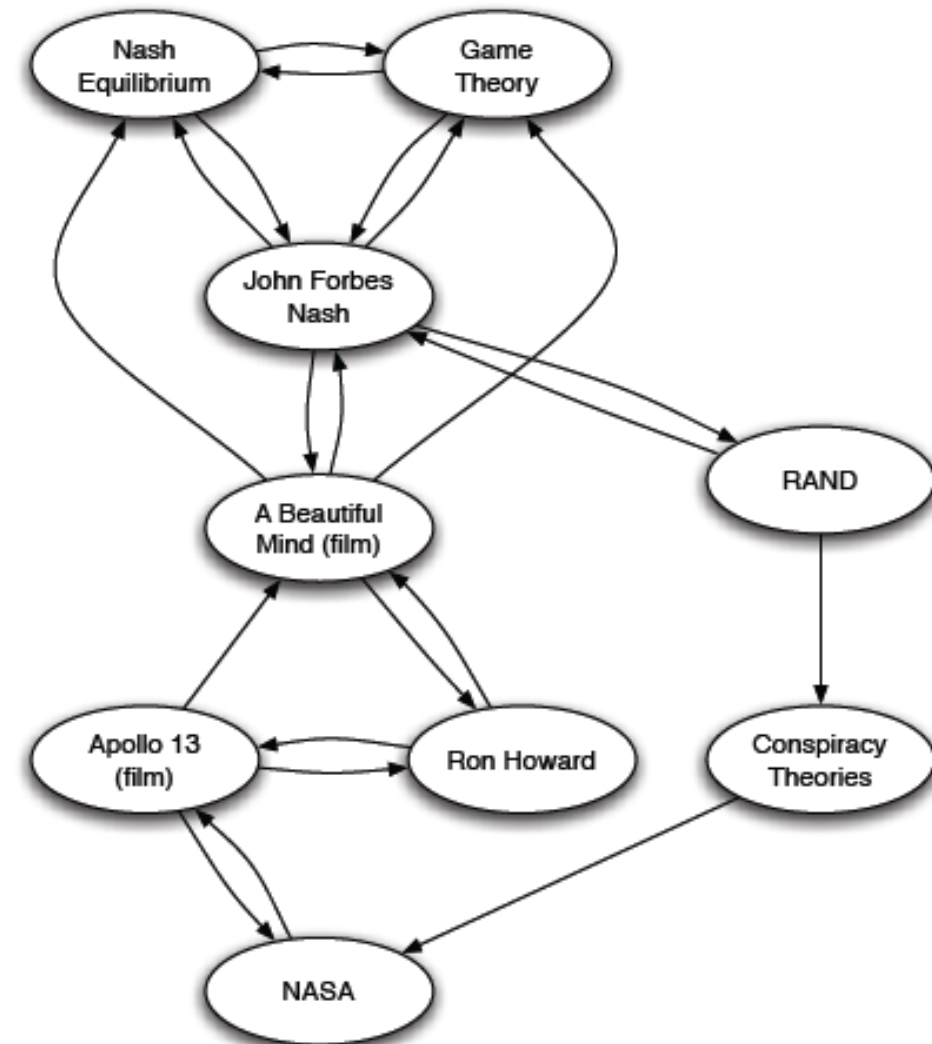
The Web as a Directed Graph



Other Information Networks

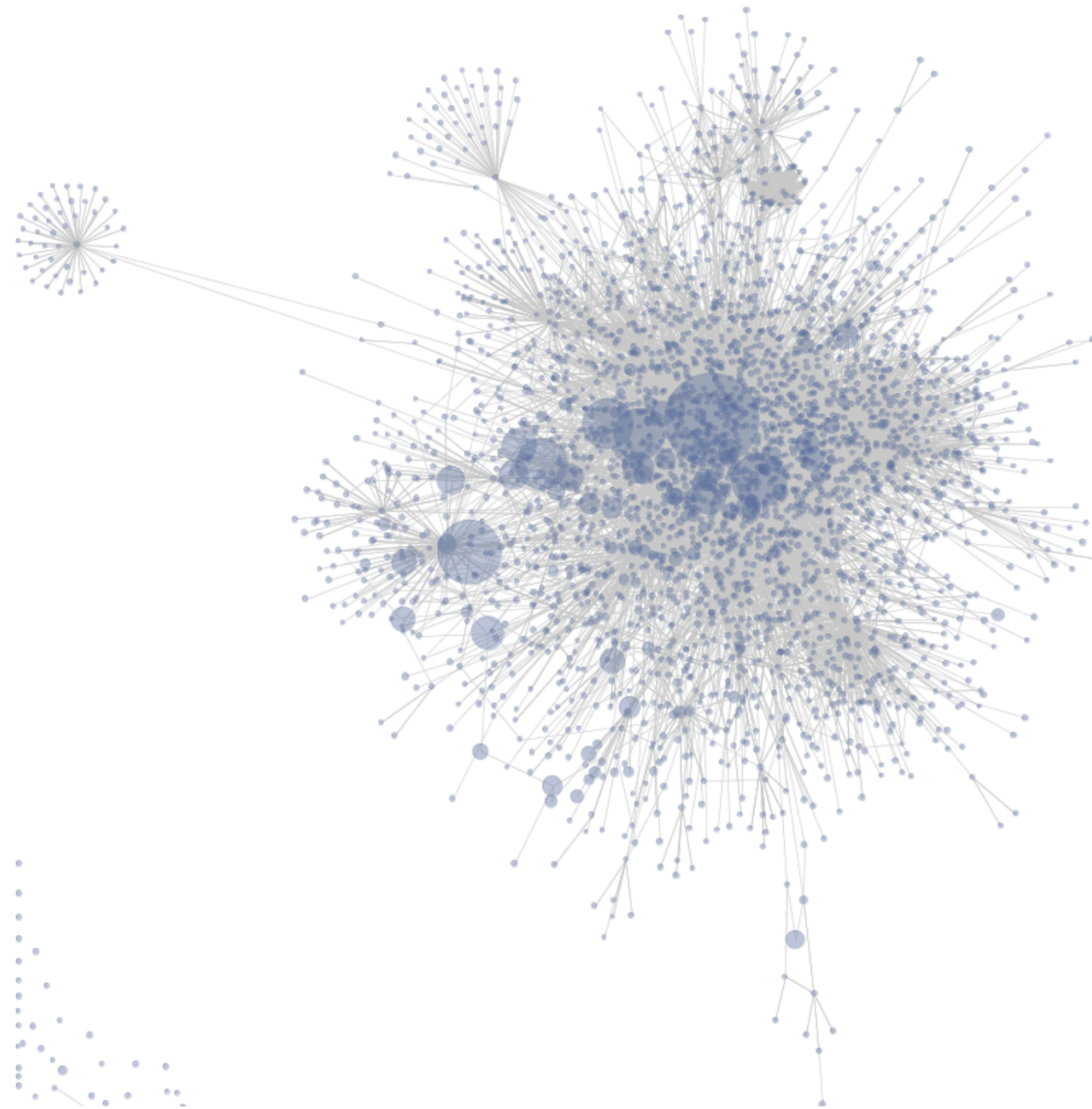


Citations



References in an encyclopedia

Other Information Networks



References between pages in a part of Wikipedia

What Does the Web Look Like?

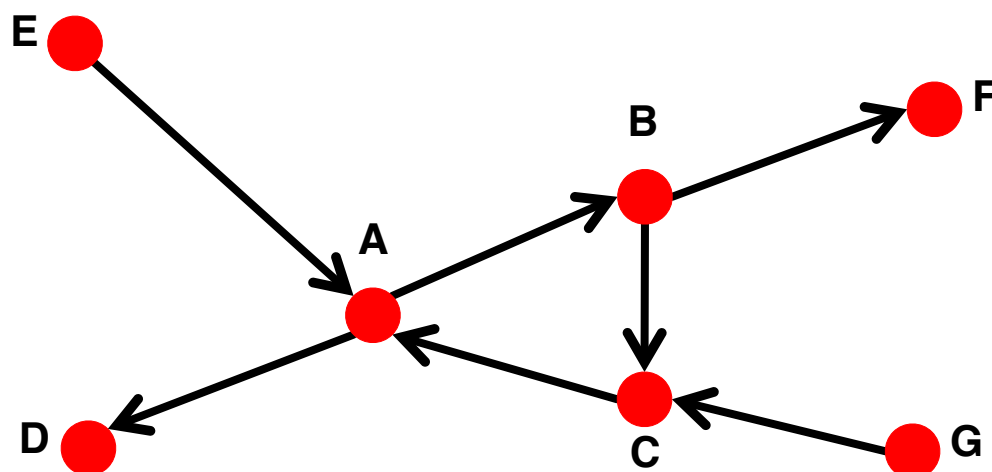
- How is the Web linked?
- What is the “map” of the Web?

What Does the Web Look Like?

- How is the Web linked?
- What is the “map” of the Web?

Web as a directed graph [Broder et al. 2000]:

- Given node v , what can v reach?
- What other nodes can reach v ?



$$In(v) = \{w \mid w \text{ can reach } v\}$$
$$Out(v) = \{w \mid v \text{ can reach } w\}$$

For example:

$In(A) = \{?\}$

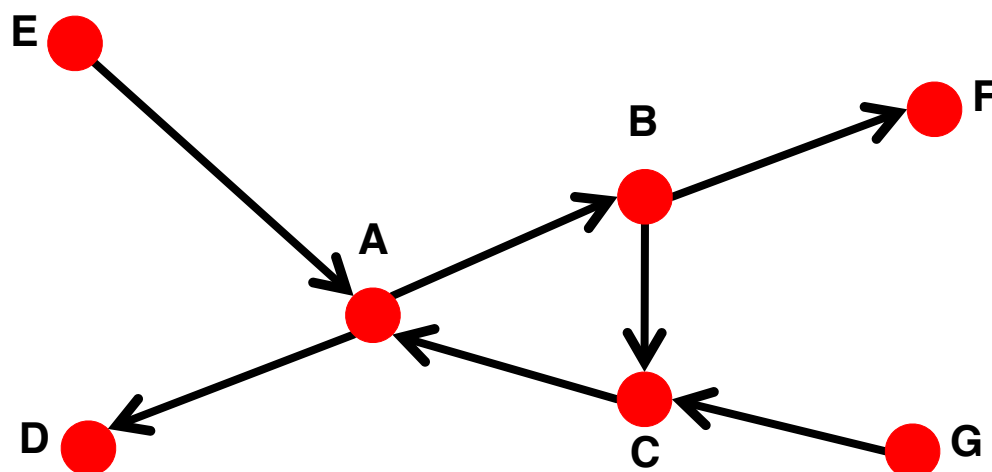
$Out(A) = \{?\}$

What Does the Web Look Like?

- How is the Web linked?
- What is the “map” of the Web?

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For example:

$In(A) = \{A, B, C, E, G\}$
 $Out(A) = \{A, B, C, D, F\}$

Directed Graphs

■ Two types of directed graphs:

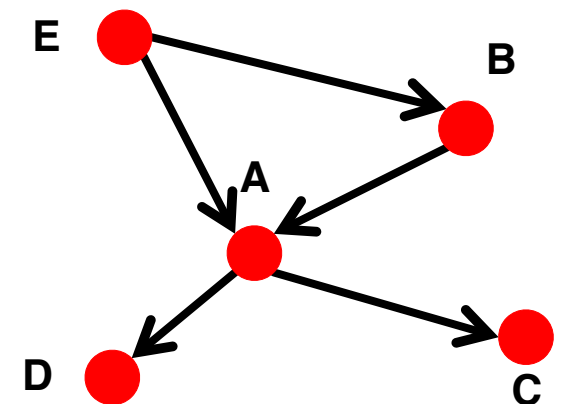
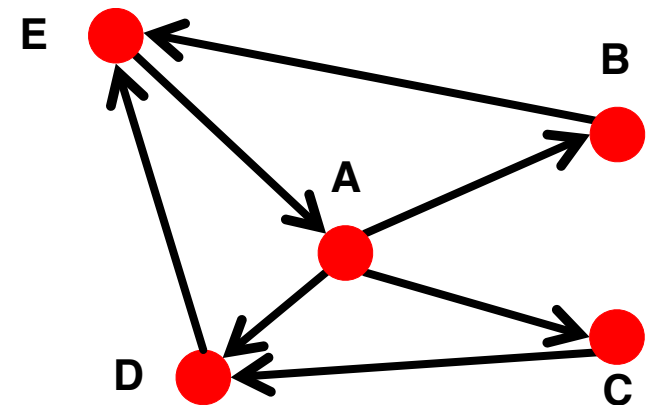
■ Strongly connected graph:

- Any node can reach any node via a directed path

$$In(A)=Out(A)=\{A,B,C,D,E\}$$

■ DAG – Directed Acyclic Graph:

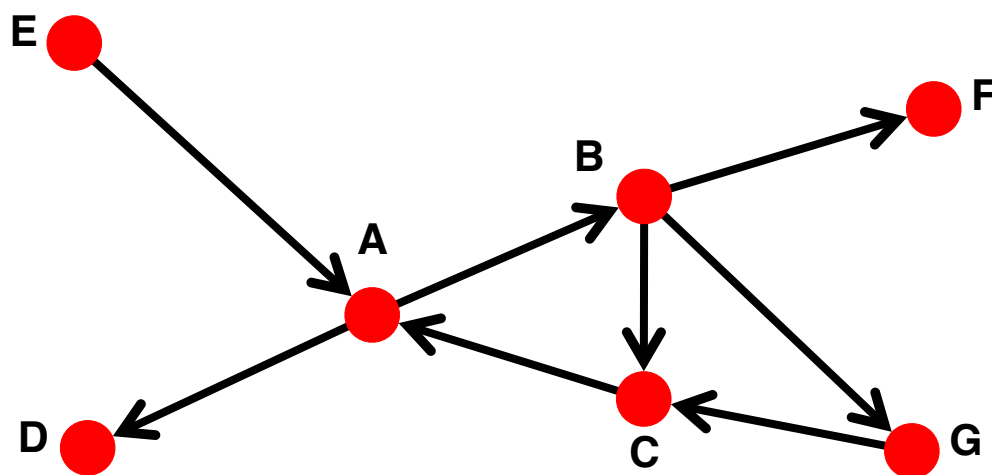
- Has no cycles: if u can reach v , then v can not reach u



Any directed graph can be expressed in terms of these two types!

Strongly Connected Component

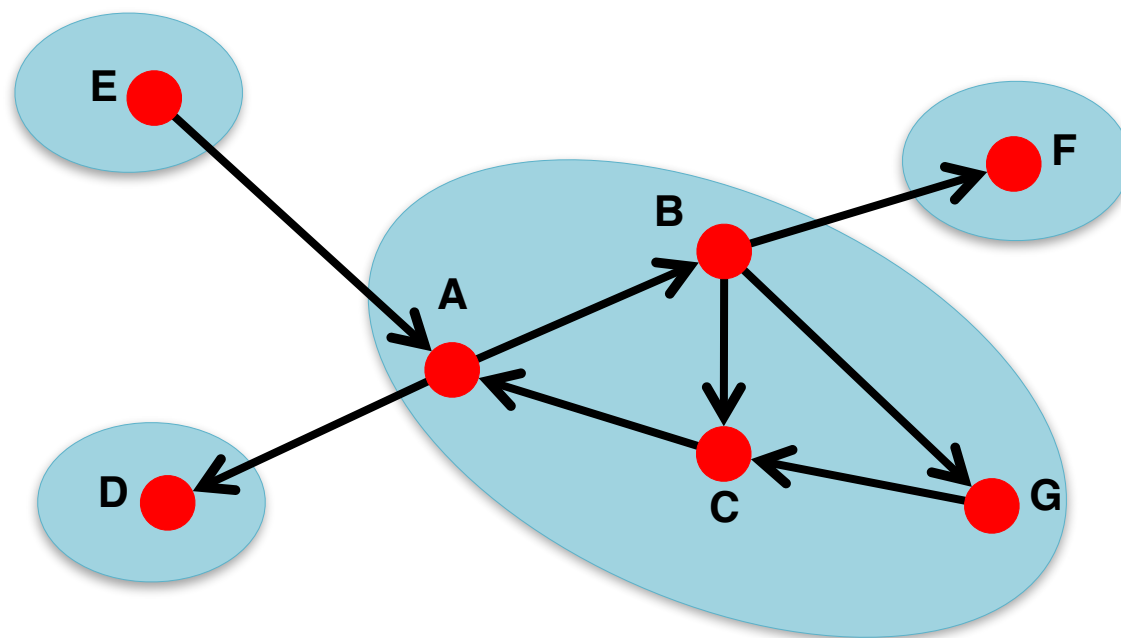
- **Strongly connected component (SCC)**
is a set of nodes S so that:
 - Every pair of nodes in S can reach each other
 - There is no larger set containing S with this property



What are the strongly connected components of this graph?

Strongly Connected Component

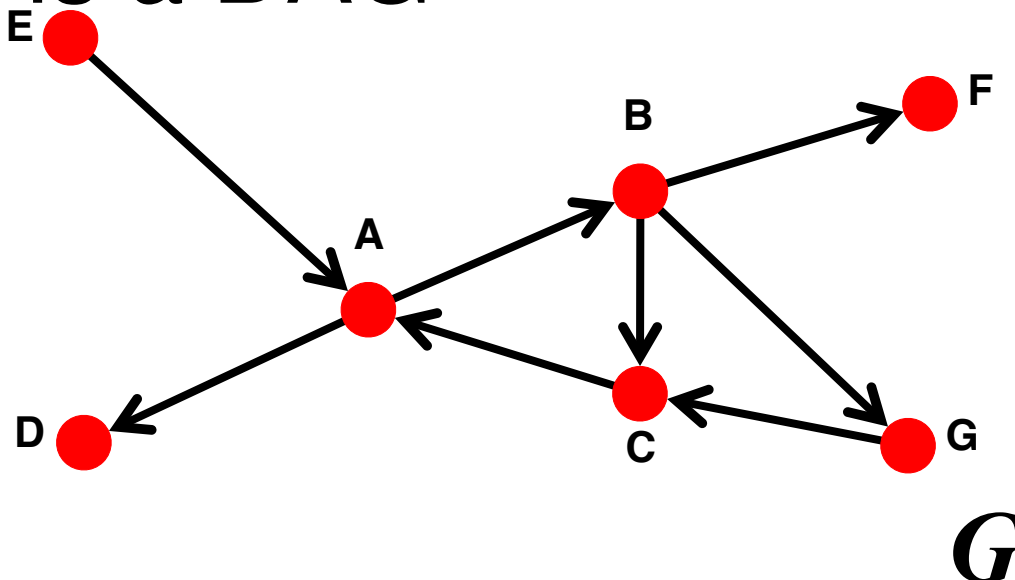
- **Strongly connected component (SCC)**
is a set of nodes S so that:
 - Every pair of nodes in S can reach each other
 - There is no larger set containing S with this property



Strongly connected
components of the graph:
 $\{A, B, C, G\}$, $\{D\}$, $\{E\}$, $\{F\}$

Strongly Connected Component

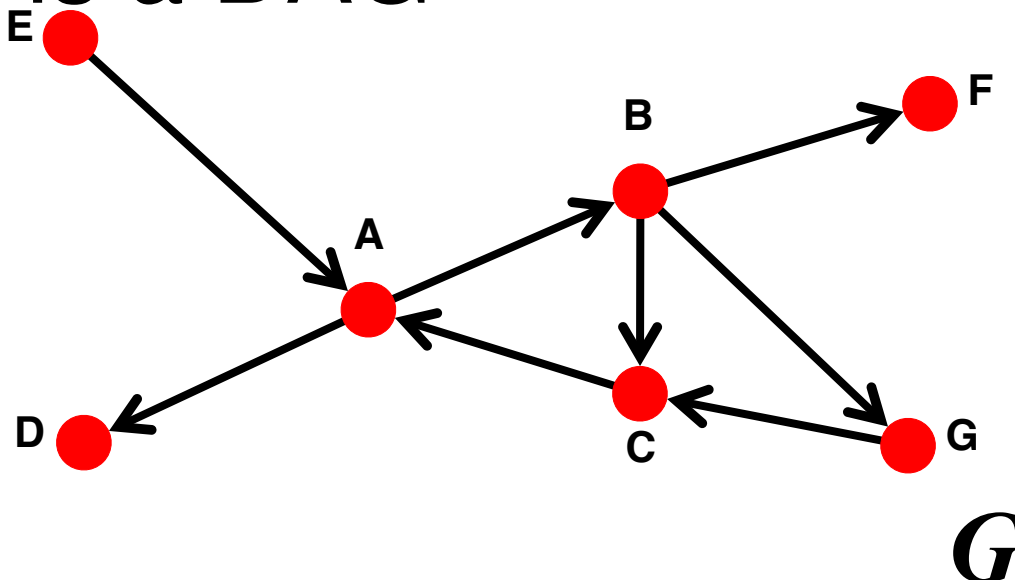
- Fact: Every directed graph is a DAG on its SCCs
- (1) SCCs partitions the nodes of G
 - That is, each node is in exactly one SCC
- (2) If we build a graph G' whose nodes are SCCs, and with an edge between nodes of G' if there is an edge between corresponding SCCs in G , then G' is a DAG



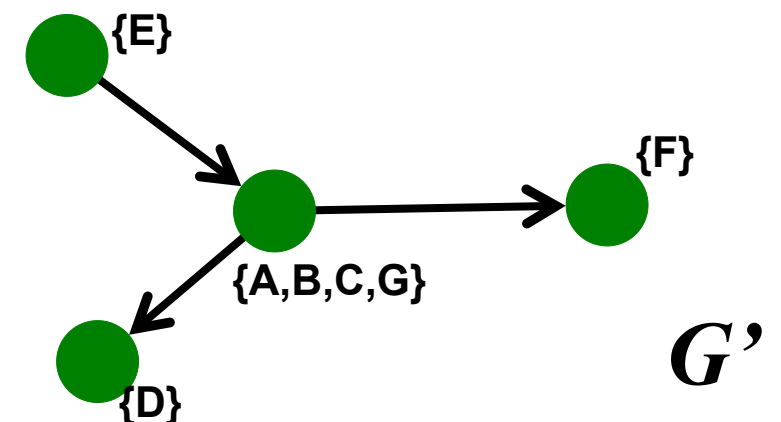
G' ?

Strongly Connected Component

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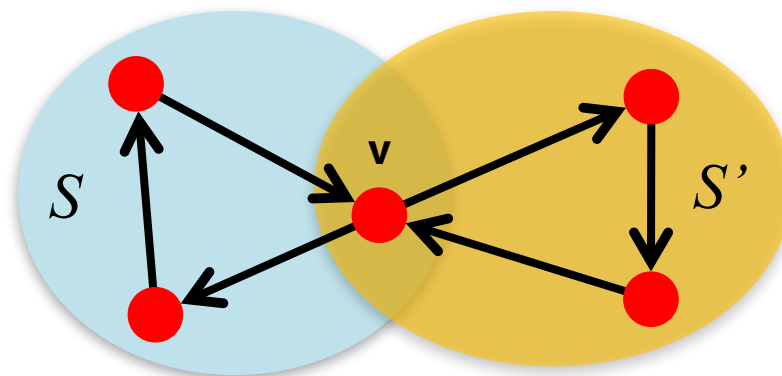


- (1) Strongly connected components of graph G : $\{A, B, C, G\}, \{D\}, \{E\}, \{F\}$
- (2) G' is a DAG:



Proof of (1)

- **Claim: SCCs partitions nodes of G .**
 - This means: Each node is member of exactly 1 SCC
- **Proof by contradiction:**
 - Suppose there exists a node v which is a member of two SCCs S and S'



- But then $S \cup S'$ is one large SCC!
 - Contradiction!

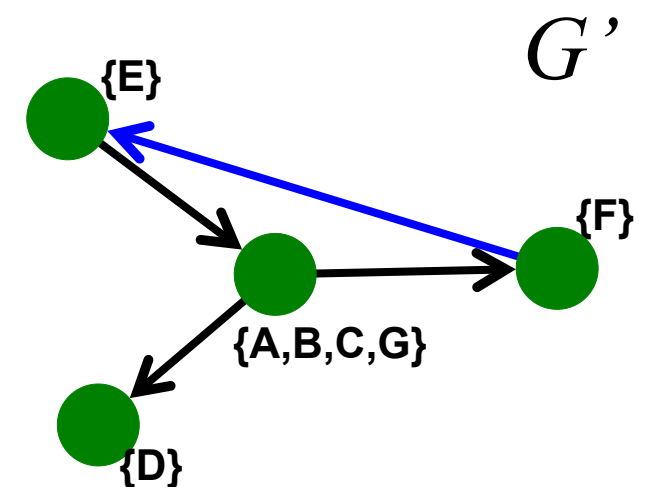
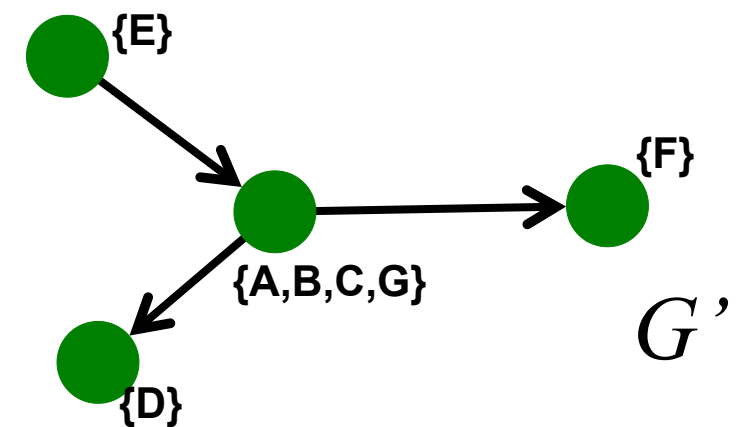
Proof of (2)

■ Claim: G' (graph of SCCs) is a DAG.

- This means: G' has no cycles

■ Proof by contradiction:

- Assume G' is not a DAG
- Then G' has a directed cycle
- Now all nodes on the cycle are mutually reachable, and all are part of the same SCC
- But then G' is not a graph of connections between SCCs (SCCs are defined as maximal sets)
 - Contradiction!



Now $\{A, B, C, G, E, F\}$ is a SCC!

Graph Structure of the Web

- **Goal:** Take a large snapshot of the Web and try to understand how its SCCs “fit together” as a DAG

- **Computational issue:**

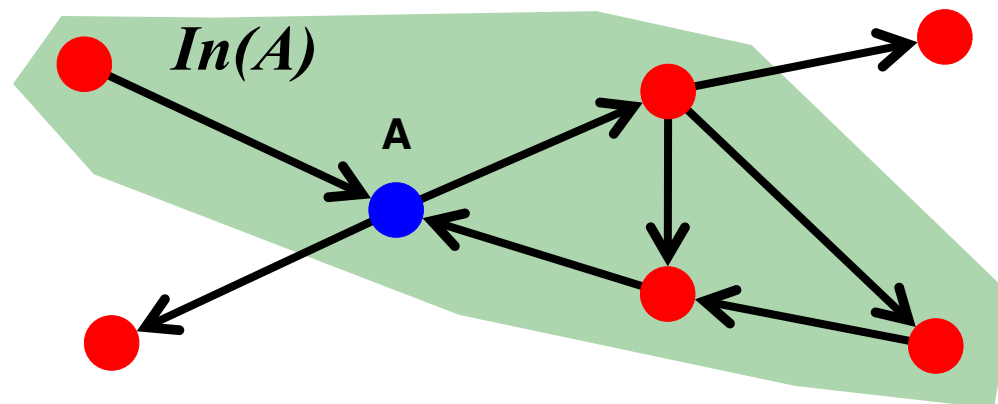
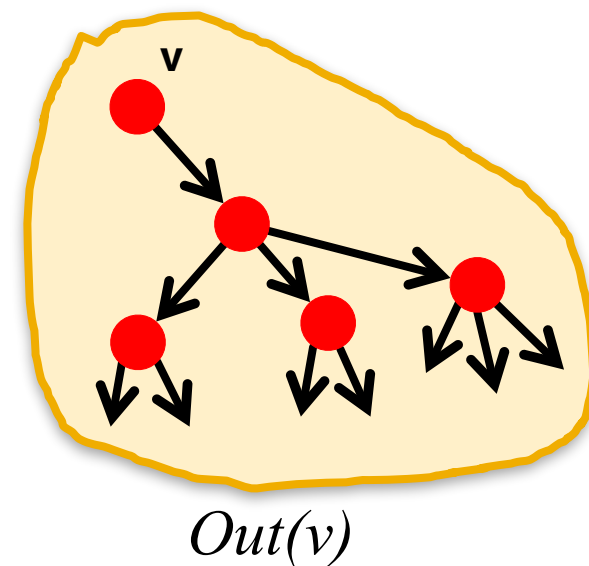
- Want to find a SCC containing node v ?

- **Observation:**

- $Out(v)$... nodes that can be reached from v

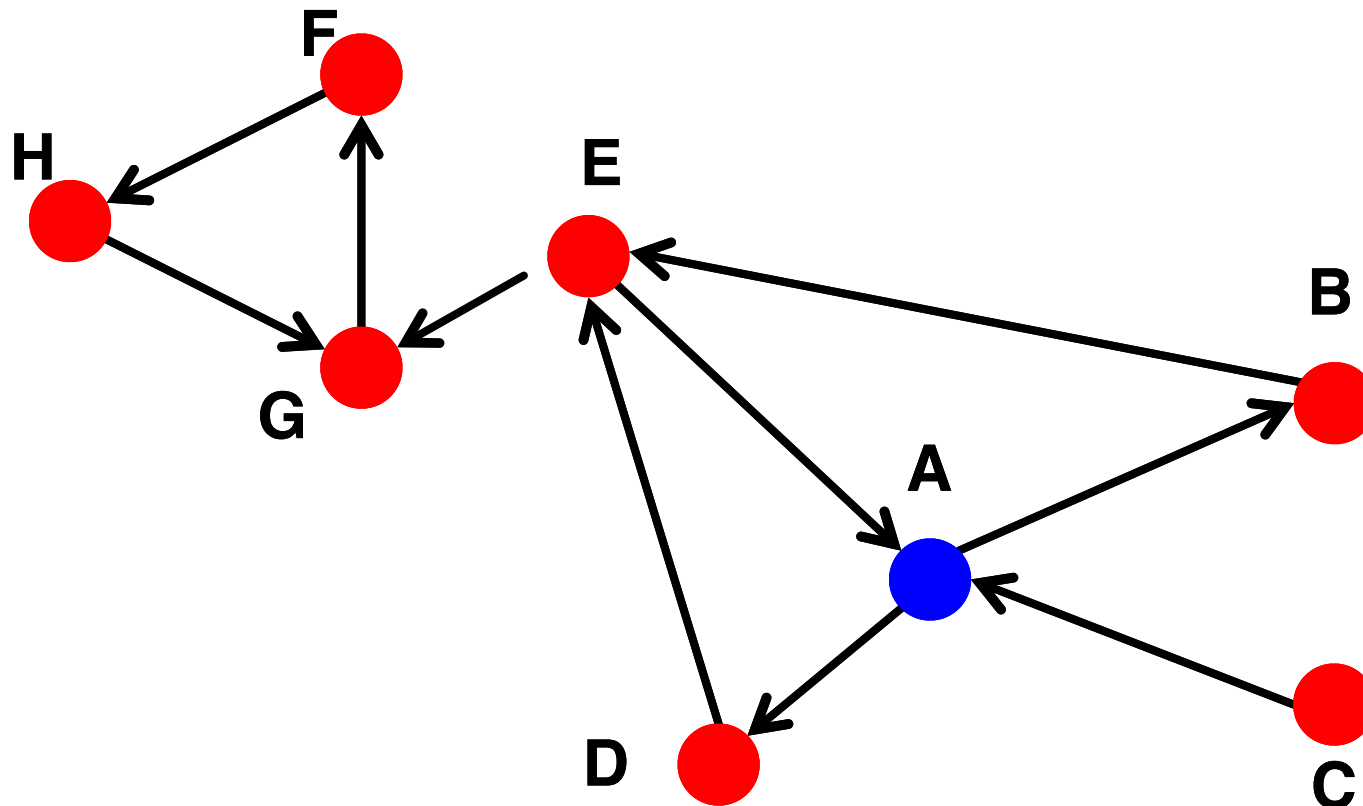
- SCC containing v is: $Out(v) \cap In(v)$

$$= Out(v, G) \cap Out(v, \bar{G}), \quad \text{where } \bar{G} \text{ is } G \text{ with all edge directions flipped}$$



$$\text{Out}(A) \cap \text{In}(A) = \text{SCC}$$

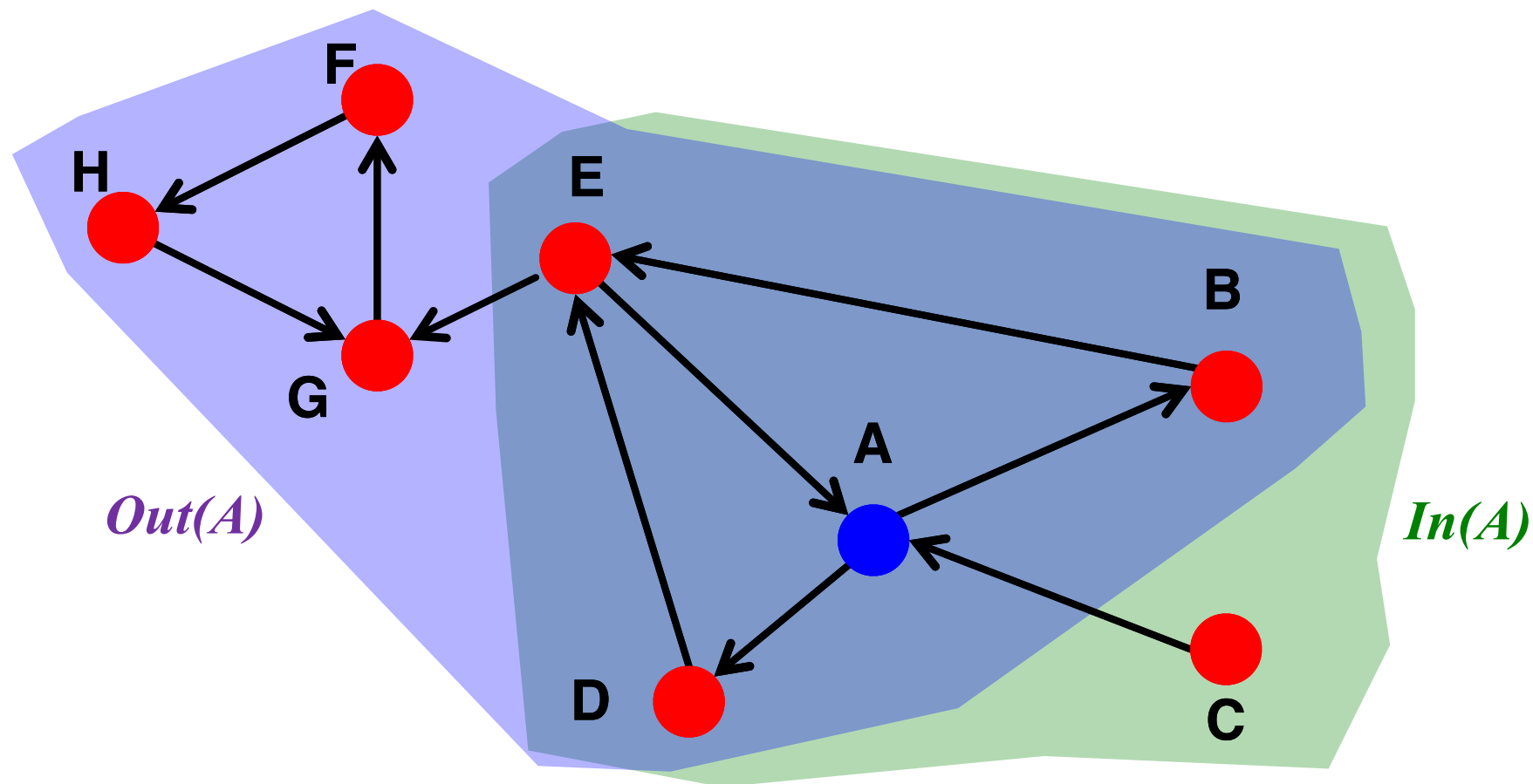
Example:



- $\text{Out}(A) = \{?\}$
- $\text{In}(A) = \{?\}$

$$\text{Out}(A) \cap \text{In}(A) = \text{SCC}$$

Example:



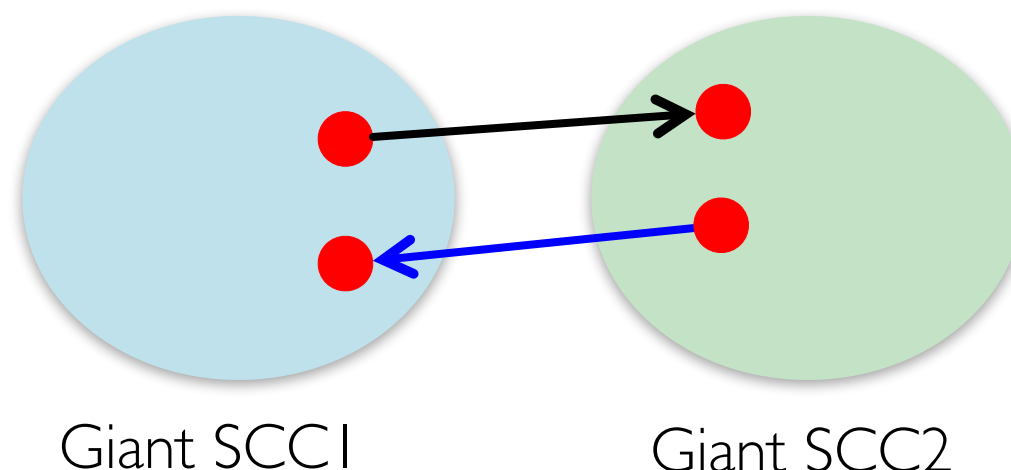
- $\text{Out}(A) = \{A, B, D, E, F, G, H\}$
- $\text{In}(A) = \{A, B, C, D, E\}$
- Therefore, $\text{SCC}(A) = \{A, B, D, E\}$

Graph Structure of the Web

- How many “big” SCCs?

Graph Structure of the Web

- **There is a single giant SCC**
 - That is, there won't be two SCCs
- **Heuristic argument:**
 - It just takes 1 page from one SCC to link to the other SCC
 - If the 2 SCCs have millions of pages the likelihood of this not happening is very very small



Structure of the Web

■ Broder et al., 2000:

- Historical Altavista crawl from October 1999
 - 203 million URLs
 - 1.5 billion links
- Computer: Server with 12GB of memory

■ Undirected version of the Web graph:

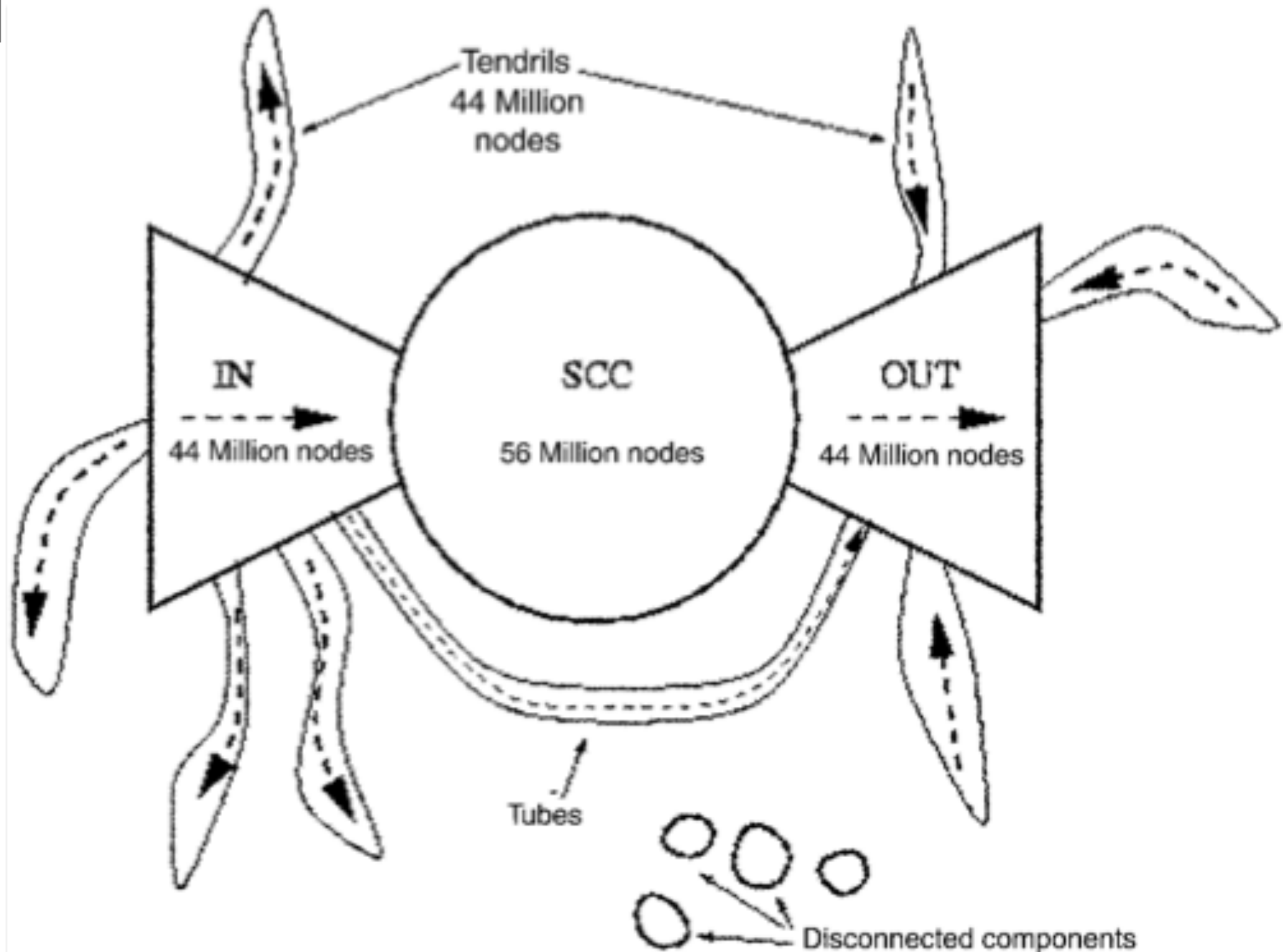
- 91% nodes in the largest weakly connected component
- Are hubs making the web graph connected?
 - Even if they deleted links to pages with in-degree >10 WCC was still $\approx 50\%$ of the graph

Structure of the Web

- **Directed version of the Web graph:**
 - Largest SCC: 28% of the nodes (56 million)
 - Taking a random node v
 - $\text{Out}(v) \approx 50\%$ (100 million)
 - $\text{In}(v) \approx 50\%$ (100 million)
- **What does this tell us about the conceptual picture of the Web graph?**

Bow-tie Structure of the Web

203 million pages, 1.5 billion links [Broder et al. 2000]



What did we do?

■ Here is what we've already done

- We took a real system (the Web)
- We represented the Web as a graph
- We used the language of graph theory to reason about the structure of the graph
- We did a computational experiment on the Web graph
- **Learned something about the structure of the Web!**

What did We Learn/Not Learn ?

■ What did we learn:

- Some conceptual organization of the Web (i.e., the bowtie)

■ What did we not learn:

■ Treats all pages as equal

- Google's homepage == my homepage

■ What are the most important pages

- How many pages have k in-links as a function of k ?

The degree distribution: $\sim k^{-2}$

- Link analysis ranking -- as done by search engines (PageRank)

■ Internal structure inside giant SCC

- Clusters, implicit communities?

■ How far apart are nodes in the giant SCC:

- Distance = # of edges in shortest path
- Avg = 16 [Broder et al.]

Recap

■ Network analysis is the language of connectedness

- Represent real-world networks from many different domains as graphs, use graph theory and algorithms to reason about them
- Social networks, information networks, knowledge networks, biological networks, etc.

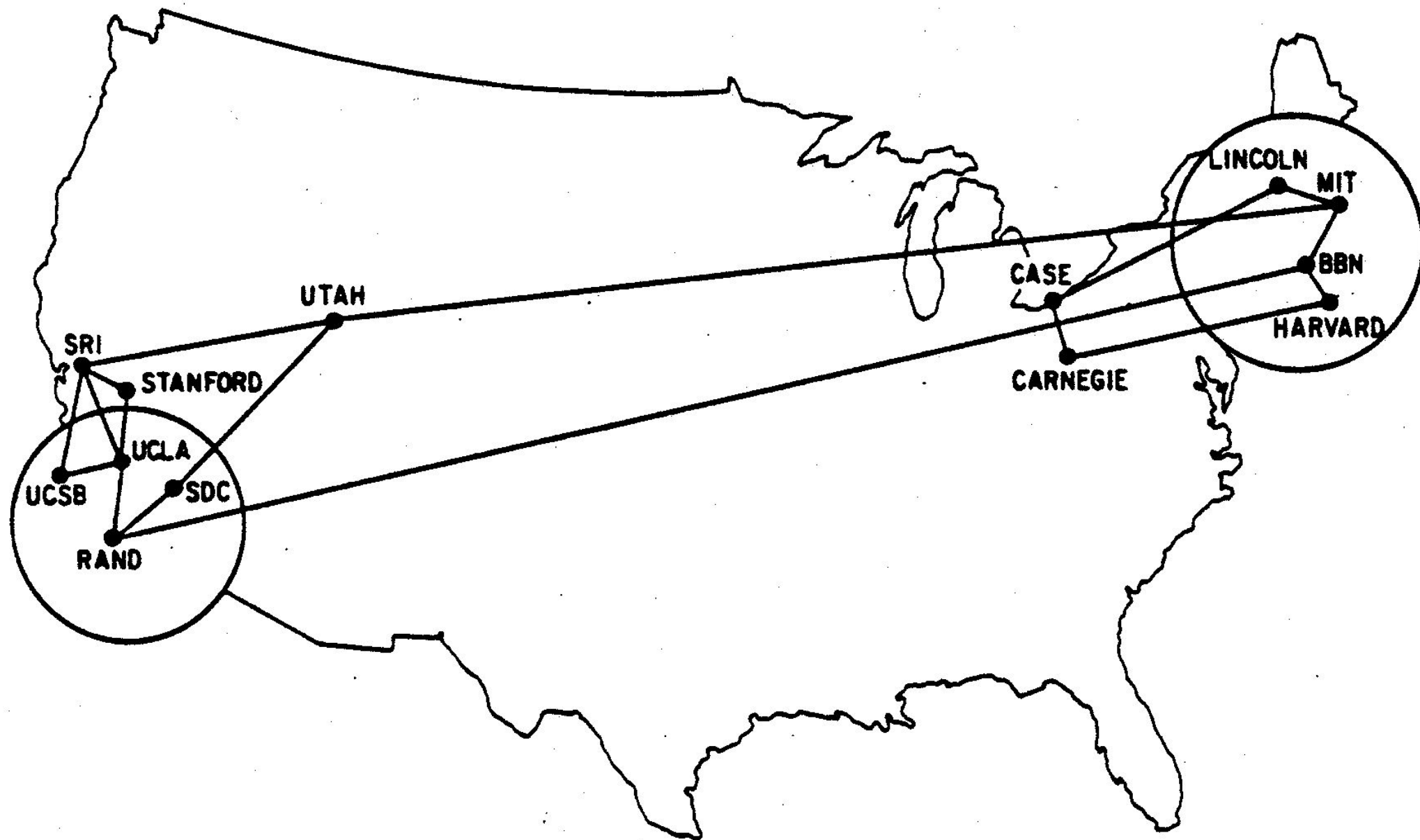
■ Network analysis fundamentals

- Nodes, edges, paths, cycles, un/directed, connected components (weak and strong)
- Choices of representation
- Every directed graph is a DAG on its SCCs

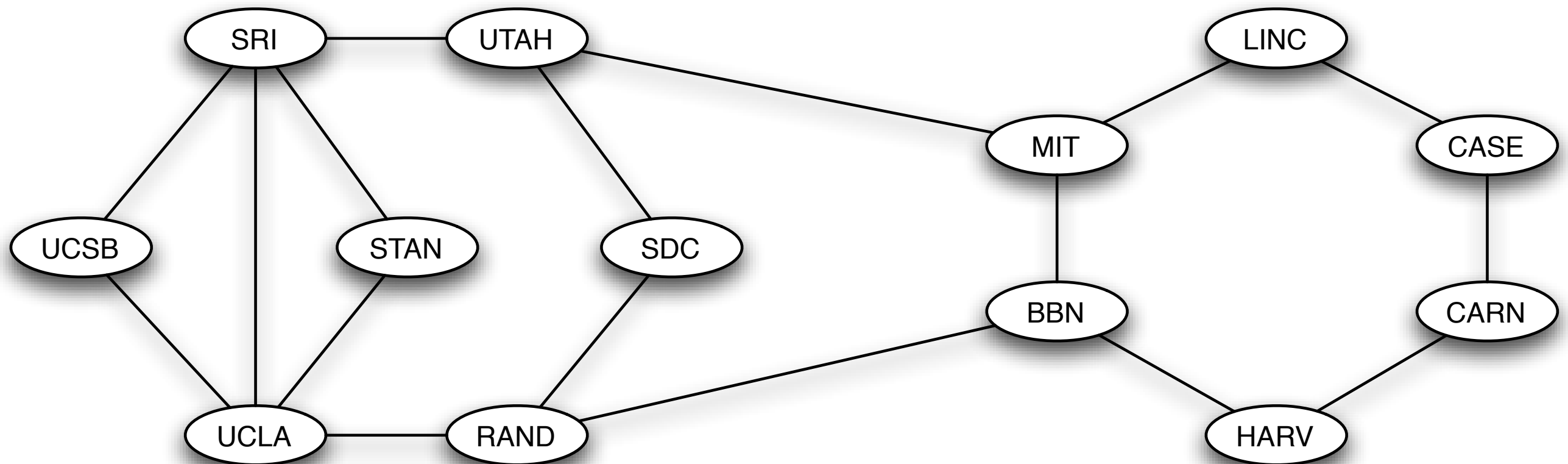
■ Structure of the Web

- Looks like a bow-tie: big giant component, IN & OUT components, tendrils, disconnected components

Arpanet, December 1970



Arpanet, December 1970



End!

This week:

- Sign up for MarkUs
- Log in to Quercus
- Read Ch. 1, 2.1-2.4, 13.1-13.4

Next week:

- Network Representations, Affiliation, Homophily, Strong and Weak Ties, Structural Holes
- Read Ch. 3.1-3.3, 4.1-4.3

Contact me: ashton@cs.toronto.edu