Tutorial 2: Turing Machines and Decidability

CSC 463

January 24, 2020

1. Show that a language L is decidable iff there is some enumerator E that prints the strings of L in lexicographic order.
2. Let A and B be decidable languages. Show that the union $A \cup B$, the intersection $A \cap B$, the concatenation $AB = \{uv | u \in A, v \in B\}$, and the complement $\bar{A}$ are also decidable.

   Which of the above closure properties remain true when decidable is replaced by semi-decidable?
3. Show that A is semi-decidable if and only if there is a mapping reduction $A \leq_m A_{TM}$. Recall that A_{TM} is the language

   $$A_{TM} = \{\langle M, w \rangle : M \text{ is a Turing machine that accepts } w\}.$$

   This exercise, combined with the fact that A_{TM} is semidecidable, shows that A_{TM} is **complete** for the class of semi-decidable problems.
4. Show that if A is semi-decidable and there is a mapping reduction $A \leq_m \bar{A}$, then A is decidable.