

# Tutorial 1: Regular Languages

CSC 463

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Regular languages are those languages which are accepted by some deterministic finite automata (or a number of equivalent models for deterministic finite automata). They provide a limited, but relatively simple model of computation to analyze. In this exercise we will investigate some questions about regular languages. It will be useful to review regular languages before moving onto more complicated models of computation such as Turing machines as it is known that Turing machines with a constant amount of memory are equivalent to finite automata.

1. Let  $k \geq 1$  be an integer. Define  $C_k \subset \{0, 1\}^*$  as the language consisting of all strings containing a 1 exactly  $k$  places from the end. Assume that  $C_k$  only contains strings of length at least  $k$ . For example,

$$C_2 = \{10, 11, 000010, 010010, 1111011, \dots\}.$$

- (a) Design a deterministic finite automata that accepts  $C_2$  and  $C_3$ . Can you generalize your construction for all  $C_k$ ?  
(b) Design a nondeterministic finite automata that accepts  $C_2$  and  $C_3$ . Can you generalize your construction for all  $C_k$ ?  
(c) Prove that any deterministic finite automata accepting  $C_k$  must have at least  $2^k$  states.  
(Hint: apply the “pigeonhole principle”.)
2. Recall that pumping lemma is a tool that is used to show that a language  $L \subseteq \{0, 1\}^*$  is not a regular language. The lemma states that for any regular language  $L$ , there is a non-negative integer  $p$  such that for any string  $s \in L$  with  $|s| \geq p$ ,  $s$  can be decomposed into substrings  $s = xyz$  such that  $|y| > 0$ ,  $|xy| \leq p$  and  $xy^i z \in L$  for any non-negative integer  $i$ <sup>1</sup>.  
(a) Prove that the language  $L = \{0^k 1^k | k \geq 0\} = \{01, 0011, 000111, \dots\}$  is not regular using the pumping lemma.  
(b) Prove that the language  $L = \{y^3 | y \in \{0, 1\}^*\} = \{000, 010101, 110110110, \dots\}$  is not regular using the pumping lemma.  
(c) Consider the language  $F = \{a^i b^j c^k | i, j, k \geq 0 \text{ and if } i = 1 \text{ then } j = k\}$ 
  - Show that  $F$  satisfies the post-condition of the pumping lemma (i.e. there is some integer  $p$  for which a decomposition satisfying the conditions is possible).
  - Show that  $F$  is not regular. (Hint: apply the “closure properties”)

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<sup>1</sup> $y^i$  is string concatenation of  $y$  for  $i$  times and  $|x|$  is the length of  $x$