Problem Set 5: Final Assignment

CSC 463

Due by April 17, 2020 at 11:59pm

This assignment, intended as a replacement for the final exam of the course, covers all of the topics we have covered so far this term. You may consult with others concerning the general approach for solving problems on assignments, but you must write up all solutions entirely on your own. Copying assignments is a serious academic offense and will be dealt with accordingly. You may cite results discussed in lecture or the course textbook. You are encouraged to write precisely and concisely; it should be possible to write your solutions to the problem set within a page per question. Submit your work online by uploading a pdf file or image of your solutions onto Crowdmark. In all problems, you can assume that you are given valid encodings as input to the problem.

1. Short answer questions: determine whether or not each of the following statements are True or False. If it is True, give a proof. If it is False, explain why the statement is false.
   (a) Let **PATH** be the problem of deciding if a directed graph G has an $s-t$ path. If $\mathbf{P} \neq \mathbf{NP}$, then **PATH** is not **NP**-Complete.
   (b) The halting problem is **NP**-Hard and not **NP**-Complete.

2. Recall on the midterm exam we showed that the language SEP of pairs of Turing machines $\langle M_1, M_2 \rangle$ satisfying $L(M_1) = \overline{L(M_2)}$ is **not** semidecidable [1]. Show that SEP is also **not co-semidecidable**. You may use the fact that

$$ALL_{TM} = \{\langle M \rangle : \text{M is a Turing machine with } L(M) = \Sigma^*\}$$

   is not semidecidable or co-semidecidable.

3. You are an epidemiologist studying COVID-19 at the University of Toronto. You have a graph G with vertices V representing people in a certain community and there is an edge if there has been possible interaction between two people. To study the spread of the disease, you would like to compute the minimum number of groups k where V can be partitioned (split) into k groups $S_1, \ldots, S_k$ where every person in a group S_i has interacted with everyone else in that group. Recall that partitioning means that every person in exactly one group.

   Your friend who is a computer science professor at York University claims that this problem **PEOPLE-PARTITION**: given a graph G and integer k, deciding if G has a partitioning with at most k groups, is **NP**-Complete. Prove this claim. You may use the fact that graph k-colouring is **NP**-Complete for any constant $k \geq 3$.

4. A CNF Boolean formula ϕ is **monotone** if it has no negated variables. Note that every monotone formula is satisfiable. Let **MIN-SAT** be the problem: given a monotone formula ϕ and an integer k, determine if ϕ has a satisfying assignment with at most k of its variables set to true. Show that **MIN-SAT** is **NP**-Complete. You may the fact that vertex cover is **NP**-Complete.

[1] Recall that $\overline{A}$ is the complement of A

5. The (m, n, k)-game is a game that generalizes the familiar game of Tic-Tac-Toe. There are two players, called "X", and "O". Each player takes turns to place their marker "X" or "O" on an $m \times n$ grid G, and the first player to get k markers consecutively in a row, column, or diagonally wins.

   Let **GT** be the problem of given deciding if Player "X" has a winning strategy on an (m, n, k)-game, given the input that it is currently Player "X"s turn on current grid G and the parameters m, n, k. Show that **GT** is in **PSPACE.** [2]

6. Recall that an undirected graph G is **bipartite** if its vertices can be divided into two sets so that every edge goes from one set to the other. Let **BIPARTITE** be the problem of testing if a graph is bipartite. Show that **BIPARTITE** is in **NL**. You may use the fact that a graph is not bipartite if and only if has a cycle with an odd number of vertices.

[2] In fact it is **PSPACE**-Complete but you do not need to show this.