Problem Set 1

CSC 463

Due by January 31, 2020, 2pm

Each problem set counts for 10% of your mark. You may consult with others concerning the general approach for solving problems on assignments, but you must write up all solutions entirely on your own. Copying assignments is a serious academic offense and will be dealt with accordingly.

You may use any results discussed in lecture, the course textbook, or tutorial. It should be possible to write your solutions to the problem set within 2-3 pages of work. Submit your work online by uploading a pdf file or image of your solutions onto Crowdmark.

1. Let F be the language $F = \{a^i b^j c^k : i + j = k, i, j, k \geq 1\}$. Show that F is not regular and furthermore, describe (in English or pseudocode) a Turing machine that accepts F.

2. A *Turing machine with left reset* is a Turing machine with its transition function replaced by

$$\delta : Q \times \Gamma \mapsto Q \times \Gamma \times \{R, RESET\}.$$

   If $\delta(q, a) = (q', b, RESET)$, then the Turing machine writes b at its current tape position, and afterwards, the tape head moves to the leftmost position on the tape and the machine goes into state q'.

   Show that the set of languages recognized by Turing machines with left reset is the same as the set of languages recognized by Turing machines with left and right moves only.

3. Prove that a language $A \subseteq \Sigma^*$ is semi-decidable if and only if there is a decidable binary relation $R \subseteq \Sigma^* \times \Sigma^*$ such that for all $x \in \Sigma^*$, $x \in A$ if and only if there is some $y \in \Sigma^*$ for which $(x, y) \in R$.

   A binary relation R is *decidable* if there is a Turing machine M where if a pair $(x, y) \in \Sigma^* \times \Sigma^*$ is given as input, then M determines whether or not $(x, y) \in R$ in finite time.

4. Show that every infinite semi-decidable language has an infinite decidable subset.