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RSA Overview

- 1. Select two very large prime numbers p and q.
- 2. Compute n = pq and $\phi(n) = (p 1)(q 1)$.
- 3. Choose an encryption key e relatively prime to $\phi(n)$.
- 4. Calculate the decryption key d such that $ed = 1 \pmod{\phi(n)}$.
- 5. Publish e and n, and keep d, p, and q secret.

RSA Overview (2)

Encryption:

$$C = M^e \mod n$$

Decryption:

$$M = C^d \mod n$$

- Modulus, n of length b bits
- Public exponent, e
- Private exponent, d
- Prime1, p, and Prime2, q

C: Ciphertext (encrypted string)

n = p*q

M: Message

RSA Overview (3)

RSA Overview (4)

- 1. Prime generation is easy it's easy to find a random prime number, even large ones
- 2. Multiplication is easy given p and q, it's easy to calculate n = pq
- **3.** Modulo inverse is easy given e and $\varphi(n)$, it's easy to calculate d s.t. ed mod $\varphi(n) = 1$
- 4. Modular exponentiation is easy given *n*, *m* and *e*, it's easy to compute *c* = *m*[^]e mod *n*
- 5. Prime factorization is hard given *n* it's hard to find primes *p* and *q* such that *pq* = *n*
- 6. Modular root extraction is hard given n, e and c, it's difficult to recover m such that c = m^e mod n, without knowing p or q (or d).

RSA Overview (5)

- Often e is picked first, then n = pq is chosen such that $gcd(e, \varphi(n)) = 1$
- The default value of e in OpenSSL is 65,537
- This is a **Fermat prime**, with the form $2^{(2^k)} + 1$, where k = 4
- This allows calculating *m*^e to be faster you square *m*, *k*+1 times, and then multiply the result by *m*.





Using OpenSSL to securely communicate between two parties

Demo: Create Alice's private key

openssl genpkey -algorithm RSA -pkeyopt rsa_keygen_bits:2048
-pkeyopt rsa_keygen_pubexp:3 -out privkey-A.pem

This command creates a generates a private key (**genpkey**) with a specified algorithm (**RSA**) we provide the size of **n** in bits and and the value of **e** and the output file (**privkey-A.pem**)

Demo: Create Alice's public key

openssl pkey -in privkey-A.pem -pubout -out pubkey-A.pem

This command takes in a private key (**pkey -in**) with a specified name (**privkey-A.pem**) and the outputs a public key (**-pubout**) with specified name (**pubkey-A.pem**)

Demo: How to view private keys (as text)...

openssl pkey -in privkey.pem -text -noout | less

This command takes in a private key (**privkey.pem** in example above). We want to view as **-text** and we pipe into **less** for easy sequential viewing. The **-noout** prevents the base64 encoding from being printed as well.

Demo: How to view public keys (as text)...

openssl pkey -in pubkey.pem -pubin -text -noout | less

This command takes in a public key (using **-pubin**, **pubkey.pem** in example above). We want to view as **-text** and we pipe into **less** for easy sequential viewing.

Demo: Alice creates a signing request

openssl req -new -key privkey-A.pem -out A-req.csr

Here you specify details for a certificate you want to create. It will prompt you for your country, organization, email, etc... This request will be processed by a certificate authority, who will generate a certificate for Alice.

Demo: Generate the certificate

openssl x509 -req -in A-req.csr -CA root.crt -CAkey root.key
-CAcreateserial -out Alice.crt -days 500 -sha256

The Certificate Authority generates a certificate using the **x509** utility. It takes in a request (**-req -in A-req.csr**) and outputs to **A.crt**. We specify the length of its validity (**-days 500**). **-CAcreateserial** creates a serial file and assigns a serial number to the certificate. A fingerprint is created using the **sha256** algorithm

Demo: View the certificate

openssl x509 -in Alice.crt -text -noout

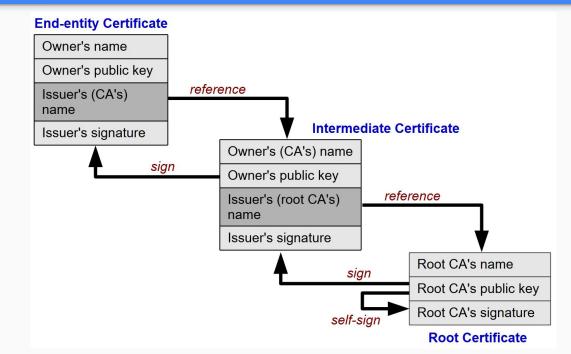
This command allows you to view the certificate in text. The **-noout** option is so the base64 encoding of the certificate does not also get printed. Using this you can view details such as it's expiry date, serial number, issuer, etc...

Demo: Verify the certificate against the CA

openssl verify -CAfile root.crt Bob.crt

The **verify** utility can be used to check if Bob's certificate was signed by the certificate authority's certificate (**-CAfile root.crt**). You should get a **B.crt: OK** if the certificate can be trusted.

Chain of Trust



Demo: Extract a public key from a certificate

openssl x509 -pubkey -in Bob.crt -noout > pubkey-B.pem

Extract the public key using the **-pubkey** option and put it in **pubkey-B.pem** Alice now has Bob's public key, and can use it to encrypt files to send to Bob.

Demo: Alice encrypts her message

openssl pkeyutl -encrypt -in largefile.txt -pubin -inkey
pubkey-B.pem -out ciphertext.bin

Using utilities (**pkeyutl**) Alice will encrypt (**-encrpyt**) the message.txt file using a public key (**-pubin**). She will use Bob's public key (**-inkey pubkey-B.pem**) so only Bob can decrypt the file. Alice stores the output in ciphertext.bin.

Demo: Error!

Public Key operation error - data too large for key size:rsa_pk1.c:153:

RSA can only encrypt data smaller than the key length- it is not for encrypting arbitrarily large files! The solution here is to use symmetric key encryption-Alice can generate a symmetric key and share it with Bob using RSA.

Demo: Alice generates a random key

openssl rand -base64 32 -out symkey.pem

The rand command is a pseudo-random byte generator. It is seeded using the \$HOME/.rnd file, and can take in additional seed sources using the -rand flag. Alice outputs 32 random bytes to **symkey.pem** and encodes it in base64.

Demo: Alice encrypts her symkey

openssl pkeyutl -encrypt -in symkey.pem -pubin -inkey
pubkey-B.pem -out symkey.enc.pem

Using utilities (**pkeyutl**) Alice will encrypt (**-encrpyt**) the symkey.pem file using a public key (**-pubin**). She will use Bob's public key (**-inkey pubkey-B.pem**) so only Bob can decrypt it. Alice stores the output in symkey.enc.pem.

Demo: Alice's encrypted signature

openssl dgst -sha1 -sign privkey-A.pem -out signature.bin
symkey.pem

Alice will hash the symkey using the sha256 algorithm (**dgst -sha256**). She then encrypts the hash with her private key (**-sign privkey-A.pem**). The output is **signature.bin**

Demo: Alice transmits her encrypted message and signature

```
cp signature.bin ../Bob
```

cp symkey.enc.pem ../Bob

We want to send our (Alice's) encrypted message (**ciphertext**) and signature (**signature.bin**) to Bob. For the demo, we'll simply copy the files over to Bob's folder - but you can imagine this happening over a network via some transfer protocol.

Demo: Bob decrypts Alice's message

openssl pkeyutl -decrypt -in symkey.enc.pem -inkey privkey-B.pem
-out symkey.pem

Using utilities (**pkeyutl**), Bob can decrypt (**-decrypt**) taking in (**-in**) the file to decrypt (**ciphertext.bin**) using a provided key (**-inkey privkey-B.pem**). Finally, the decrypted text is outputted (**-out**) to (**symkey.pem**)

Demo: Bob verifies message is from Alice

openssl dgst -sha1 -verify pubkey-A.pem -signature signature.bin
symkey.pem

For Bob to verify the message, he applies the same hash that Alice used for her signature (**dgst -sha1**) and uses her public key (**-pubkey-A.pem**) to verify (**-verify**) the provided signature (**-signature signature.bin**) by decrypting it and comparing the result to the hashed, decrypted message (**symkey.pem**)

Demo: Alice encrypts the largefile using AES

openssl enc -aes-256-cbc -pass file:symkey.pem -p -md sha256 -in largefile.txt -out ciphertext.bin

The **enc** utility is used for symmetric key encryption, with **aes-256-cbc** specified as the algorithm. The **key derivation function** uses **sha256**, and the **-p** flag will print the key, salt and initialization vector to the screen.

Demo: Alice sends the ciphertext to Bob

cp ciphertext.bin ../Bob

Again, we'll simply copy it over for the demo, but this can be done over a network just as easily.

Demo: Bob decrypts the ciphertext

openssl enc -aes-256-cbc -d -pass file:symkey.pem -p -md sha256
-in ciphertext.bin -out largefile_received.txt

Bob will decrypt the ciphertext using a similar command that Alice used to encrypt it. Note: Bob needs to know what encryption Alice used for this to work. He will use the **-d** flag to specify decryption and put it in **largefile_received.txt**.

Demo: A summary - what happened?

- 1. Alice generated public and private keys
- 2. Alice generated a certificate signing request- the CA took it and made her a certificate
- 3. Alice extracted Bob's public key from Bob's certificate
- 4. Alice generated a symmetric key and encrypted it with Bob's public key (symkey.enc.pem)
- 5. Alice hashed the symmetric key, then encrypted her hash with her private key (signature.bin)
- 6. Bob decrypted symkey.enc.pem with his private key to get symkey.pem
- 7. Bob decrypted signature.bin with Alice's public key, hashed symkey.pem and compared them
- 8. Alice used the symkey to encrypt largefile.txt
- 9. Bob used the symkey to decrypt largefile.txt

References

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https://gist.github.com/fntlnz/cf14feb5a46b2eda428e000157447309

Extra: Root CA makes self-signed certificate

openssl req -x509 -new -nodes -key rootCA.key -sha256 -days 1024
-out rootCA.crt

Here is a command to self-sign a certificate, we used this for our pseudo root CA (the trusted third-party).