Symbolic Trajectory Evaluation - A Survey

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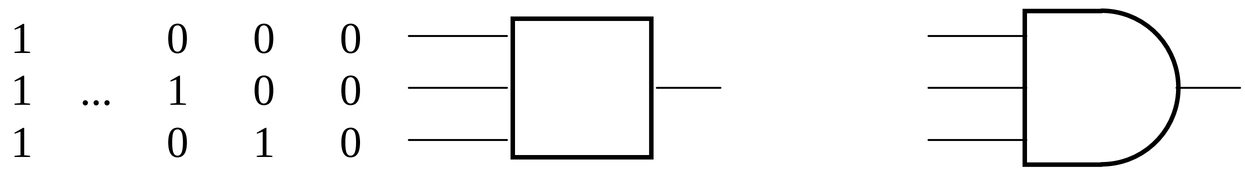
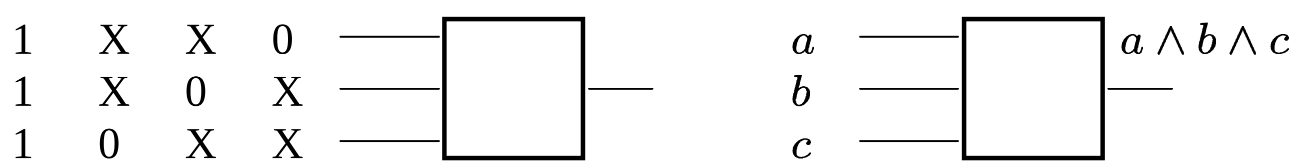
Motivation

- Simulation *vs.* verification



- Multi-valued *vs.* symbolic simulation



- Symbolic Trajectory Evaluation (STE) - a multi-valued symbolic verification method based on simulation
- STE *vs.* model-checking

Basic STE Theory

Model

- Complete lattice $(\mathcal{S}, \leq)$ of states
- Monotonic next-state function $Y : \mathcal{S} \rightarrow \mathcal{S}$

Behaviors

- Infinite *sequences* $\sigma_1,\ \sigma_2,\ \ldots \in \mathcal{S}^\omega$
- Infinite *trajectories* - sequences obeying next-state function,

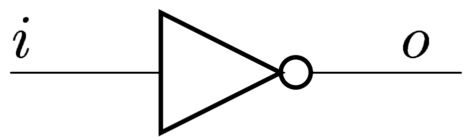
$$Y(\sigma_i) \leq \sigma_{i+1}, \quad \text{for all } i$$

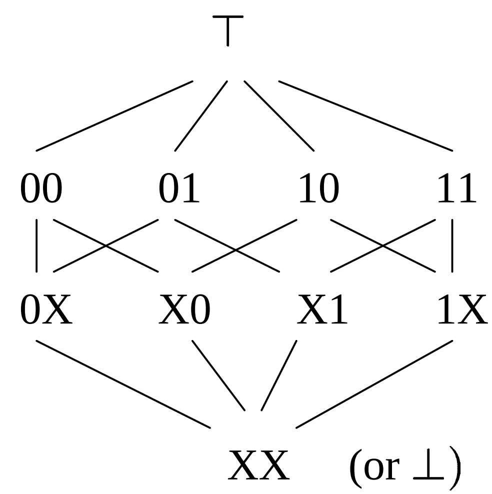
- Lattice order extended to sequences and trajectories pointwise,

$$\sigma_1,\ \sigma_2,\ \ldots \quad \leq \quad \gamma_1,\ \gamma_2,\ \ldots \quad \text{iff} \quad \sigma_i \leq \gamma_i, \quad \text{for all } i$$

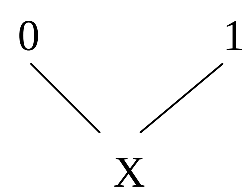
Example (I)

Inverter circuit





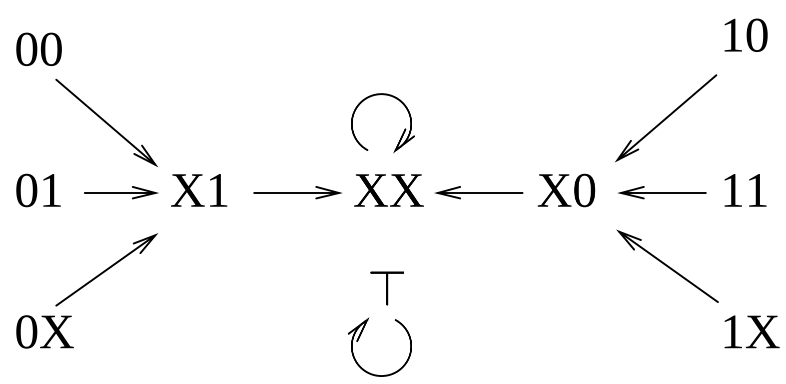
Information partial order



Example (II)

- Each circuit node has *excitation*, constraint on its next value

  for all states $s_1 s_2$, $\quad y_i(s_1 s_2) = X$, $\quad y_o(s_1 s_2) = \neg s_1$, $\quad$ so $Y = y_i\, y_o$



- Sequence $\quad 1X,\ X0,\ \bot,\ \bot,\ \ldots \quad$ *is* a trajectory
- Sequence $\quad 00,\ \bot,\ \bot,\ \ldots \quad$ *is not*

Logic

Trajectory formulas

- Atom - *simple state predicate*, monotonic, and having unique lowest state, *defining state*, where true

  e.g., $(i \text{ is } 1)$ with defining state $1X$

  - true of a trajectory iff true of its initial state

- Conjunction of trajectory formulas $\varphi \wedge \psi$ - usual semantics

- Next-time formula $\mathbf{N}\phi$ - true of trajectory $\sigma_1, \sigma_2, \ldots$ iff ϕ true of $\sigma_2, \sigma_3, \ldots$

- Nothing else

Exemple for inverter: $(i \text{ is } 1) \wedge \mathbf{N}(o \text{ is } 0)$

Specification

Assertions

$$A \Rightarrow C$$

with A, C trajectory formulas

- True of a model iff for every trajectory σ

$$\text{if} \quad \sigma \models A \quad \text{then} \quad \sigma \models C$$

- Better: set of trajectories satisfying A contained in that of those satisfying C

Inverter specification:

$$(i \text{ is } 1) \;\Rightarrow\; \mathbf{N}(o \text{ is } 0)$$

$$(i \text{ is } 0) \;\Rightarrow\; \mathbf{N}(o \text{ is } 1)$$

In LTL: $\quad (i \rightarrow \circ\neg o) \;\wedge\; (\neg i \rightarrow \circ\, o)$

Verification (I)

To check $(i \text{ is } 1) \Rightarrow \mathbf{N}(o \text{ is } 0)$

- Consider *defining sequence*

$$1X,\ \bot,\ \bot,\ \ldots$$

  for $(i \text{ is } 1)$

  all trajectories satisfying $(i \text{ is } 1)$ are those above this sequence

- Make it into *defining trajectory*

$$\tau = 1X,\ X0,\ \bot,\ \bot,\ \ldots$$

  the lowest trajectory satisfying $(i \text{ is } 1)$

- Consider defining sequence

$$\sigma = \bot,\ X0,\ \bot,\ \bot,\ \ldots$$

  for $\mathbf{N}(o \text{ is } 0)$

- Check $\sigma \leq \tau$

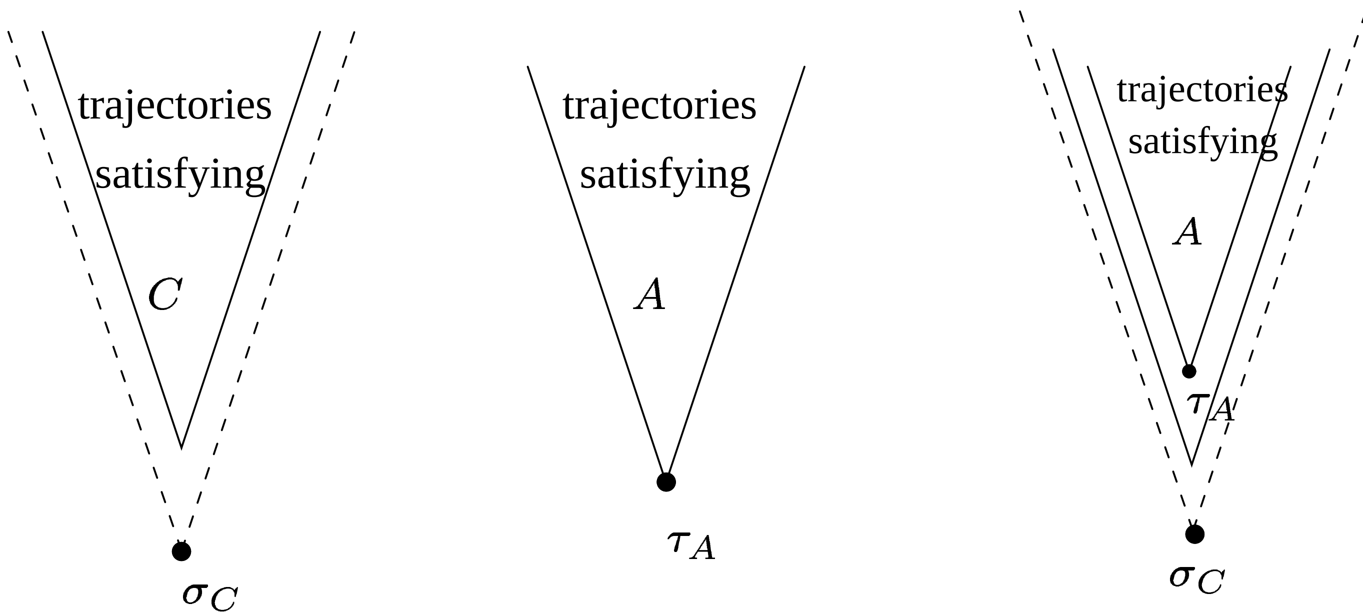
Verification (II)

In general, to check $A \Rightarrow C$

check $\sigma_C \leq \tau_A$

where $\sigma_C,\ \tau_A$ are the defining sequence for C and defining trajectory for A, respectively

Justification



Symbolic Version

Allow Boolean variables

- Inverter specification becomes

$$(i \text{ is } x) \;\Rightarrow\; \mathbf{N}(o \text{ is } \neg x)$$

- Checked by

$$\bot,\ X\neg x,\ \bot,\ \bot,\ \ldots \quad \leq \quad xX,\ X\neg x,\ \bot,\ \bot,\ \ldots$$

- True iff inequality holds for all possible interpretations of the variables
- There are symbolic states, sequences, trajectories, formulas
- Symbolic means *parameterized* by Boolean variables

Implemented using BDDs!

A Few Points

- $\mathbf{N}(o \text{ is } 0) \;\Rightarrow\; (i \text{ is } 1)$ fails:

  $$1X,\ \bot,\ \bot,\ \ldots \quad \not\leq \quad \bot,\ X0,\ \bot,\ \ldots$$

  simulation only works forward

+ $(i \text{ is } 0) \wedge (i \text{ is } 1) \;\Rightarrow\; (o \text{ is } 0)$ succeeds, but vacuity detected

  defining sequence for $(i \text{ is } 0)$: $0X,\ \bot,\ \bot,\ \ldots$

  defining sequence for $(i \text{ is } 1)$: $1X,\ \bot,\ \bot,\ \ldots$

  defining trajectory for their conjunction $\top,\ \bot,\ \bot,\ \ldots$

  by pointwise lub

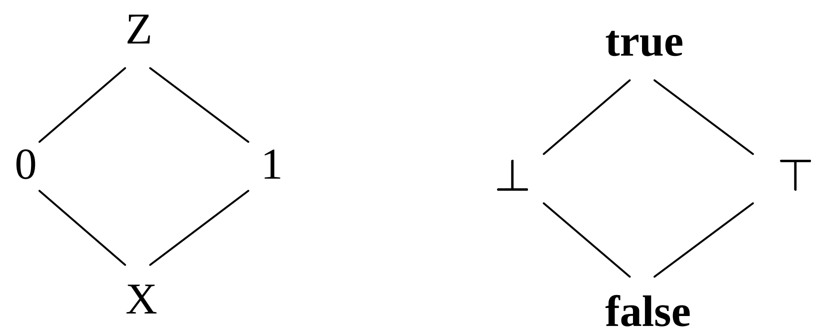
+ Verification does not depend on the size of the state space

- Four-valued state space, but two-valued verification answer

- Very restricted verification capabilities - only over finite sequences, cannot reason about eventuality, or support disjunction, etc.

Beyond Basics

- Fixpoint computations for checking assertions of type $(A \Rightarrow C)^*; G$
- Using enriched syntax and a four-valued information + truth lattice



- Generalized STE for checking *assertion graphs* representing all ω-regular properties

Forte Tool

- Forte is a formal verification environment implementing STE, used at Intel
- First and current restricted academic version released January 2003
- Essentially performs symbolic simulation, not verification
- Example of STE invocation

```
let ant = [(T, "out[1]", T, 0, 1),
           (T, "out[0]", T, 0, 1),
           (T, "c", F, 0, 1),
           (T, "c", T, 1, 2)];
let cons = [(T, "out[1]", F, 1, 2),
            (T, "out[0]", F, 1, 2)];
STE "" model [] ant cons trace;
```

Summary and Open Problems

- STE is a special-purpose model-checking method
- Successfully used in industry (Intel, IBM, Motorola) to verify large memories and datapth circuits
- Relationship to standard model-checking still unclear
- Formally shown to be a form of data-flow analysis and its multi-valued models of circuits to be over-approximations of concrete ones
- To do: prove a direct relationship with multi-valued abstraction and model-checking as we know them
- To do: see how standard model-checking can benefit from STE