# MIDTERM TEST -- CSC 354S

University of Toronto
Spring 1995

50 marks -- 50 minutes

Test aids allowed: one 8.5" X 11" (double-sided) fact sheet, non-programmable scientific/statistical calculator, ruler

Notes to students: State all assumptions where appropriate. In general, show your intermediate work.

1. **[20 marks total]**

Consider the function $f(x) = 20x(1-x)^3$ , $0 < x < 1$ .

a) **[6 marks]** Show that $f$ is indeed a density function.

b) **[4 marks]** Give the formula for the distribution function $F$ that corresponds to the density function $f$ .

c) **[10 marks]** Using the acceptance-rejection method, show how to generate efficiently random variates having the density function $f$ . Be explicit; show details of your work.

2. **[8 marks]**

Explain succinctly in a few sentences what the basic idea (purpose, principle) of maximum-likelihood estimators is.

[Content, meaning correctness, is of course important in this answer, but style, meaning pithiness, is as well. State it correctly and state it simply.]

3. **[10 marks]**

In Problem Set 2, question 2, you were to implement an *M/M/*1 queue to calculate delay times in queue, based on a formula. Specifically, there were IID interarrival times $A_1, A_2, \cdots$ and IID service times $S_1, S_2, \cdots$. Customers were served in a FIFO manner. This gave rise to a sequence of delays in queue $D_1, D_2, \cdots$. Justify the formula that related $D_{i+1}$ to quantities in the sequences $\{A\}$, $\{S\}$, and $\{D\}$, and to certain constants.

4. **[12 marks total]**

Suppose we have a system that admits to a terminating simulation. We make 5 independent replications and construct a point estimate and 95 percent confidence interval for the mean throughput rate, based on the obtained throughput values $X_1 = 7221$, $X_2 = 7155$, $X_3 = 7250$, $X_4 = 7260$, and $X_5 = 7260$. Note that page 2 of this test contains a useful statistical table.

a) **[2 marks]** Calculate the point estimate and the confidence interval.

b) **[5 marks]** Approximately how many replications in total would be required to obtain an absolute error of 50?

c) **[5 marks]** Approximately how many replications in total would be required to obtain a relative error of 5 percent?

**End of test** (but remember about page 2)

**Total Marks = 50**

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