Discovering Hidden Structure of House Prices with Relational Latent Manifold Model

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Relational Learning

- Traditional Learning
  - Samples are i.i.d. from an unknown distribution D
- Relational Learning
  - Samples are no longer i.i.d
  - Related to each other in complex ways: values of unknown variables depends on each other
  - Dependencies could be direct or indirect (hidden)
- Examples
  - Web page classification
  - Scientific document classification
- Need a form of collective prediction

Predicting House Price

- “Location Location Location”

Predicting House Price

- “Location Location Location”

Predicting House Price

- "Location Location Location"

Predicting House Price

- "Location Location Location"

- Price is a function of quality/desirability of neighborhood
- Which in turn is a function of quality/desirability of other houses
- This is the relational aspect of the price

Predicting House Price

- However there is more to it
  - The Non-Relational aspect of price

Predicting House Price

- However there is more to it
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1 bedroom, 1 bathroom

Predicting House Price

- However there is more to it
  - The Non-Relational aspect of price

1 bedroom, 1 bathroom

4 bedroom, 3 bathroom

Predicting House Price

- However there is more to it
  - The Non-Relational aspect of price

1 bedroom, 1 bathroom

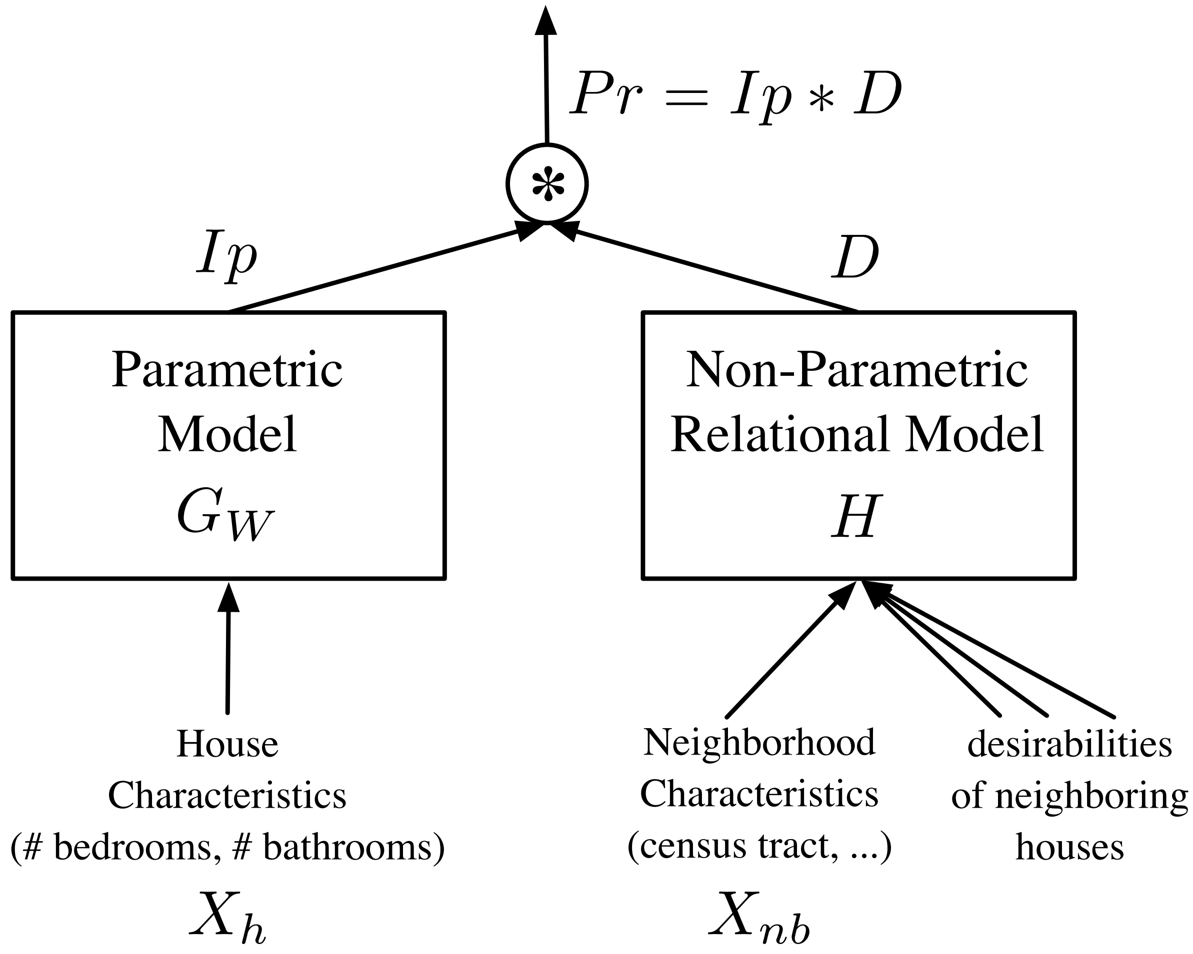
4 bedroom, 3 bathroom

- One can view this as the "intrinsic price"

The Idea

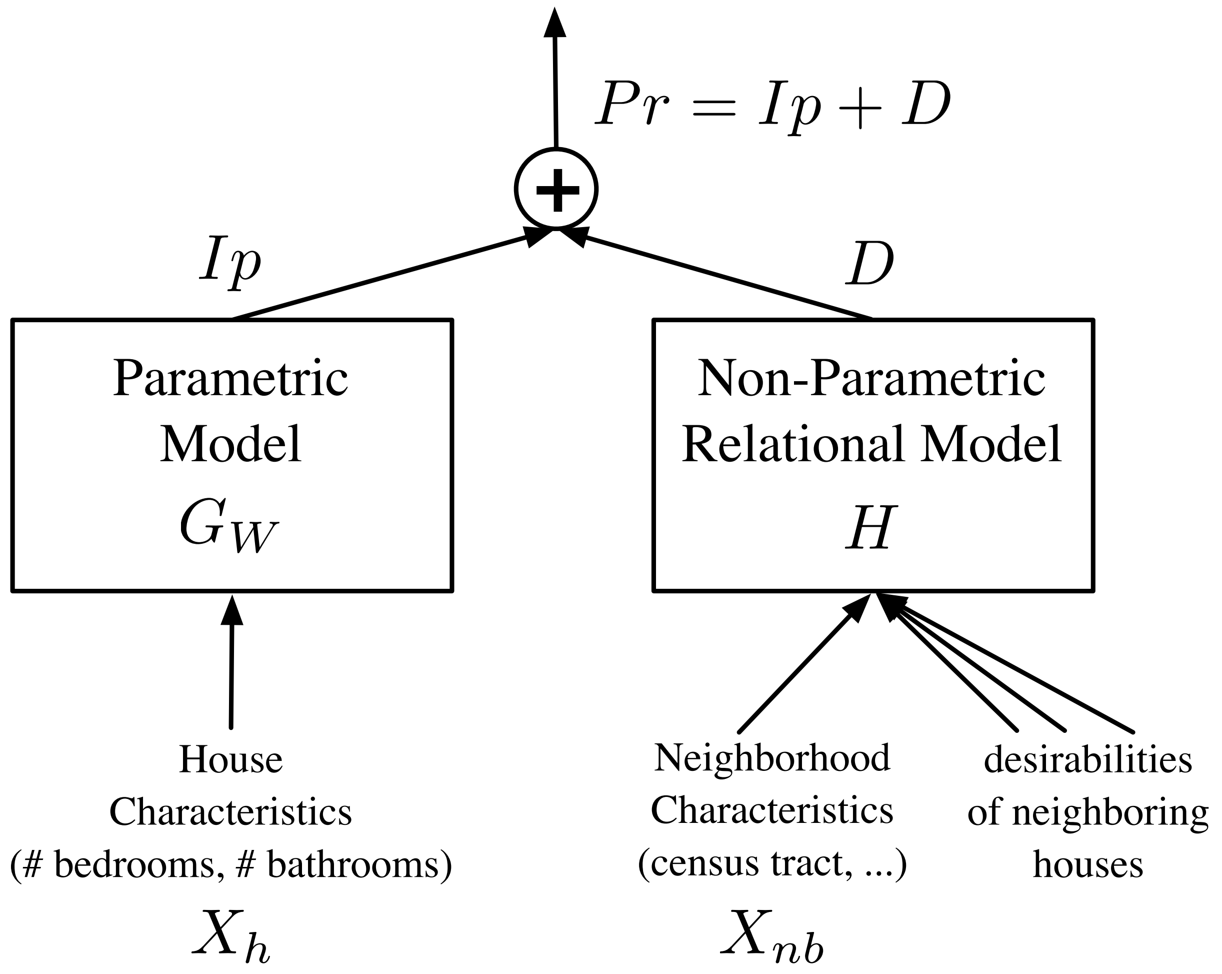
- Price of a house is modeled as

$$price = (intrinsic\, price) * (desirability\, of\, its\, location)$$



The Idea

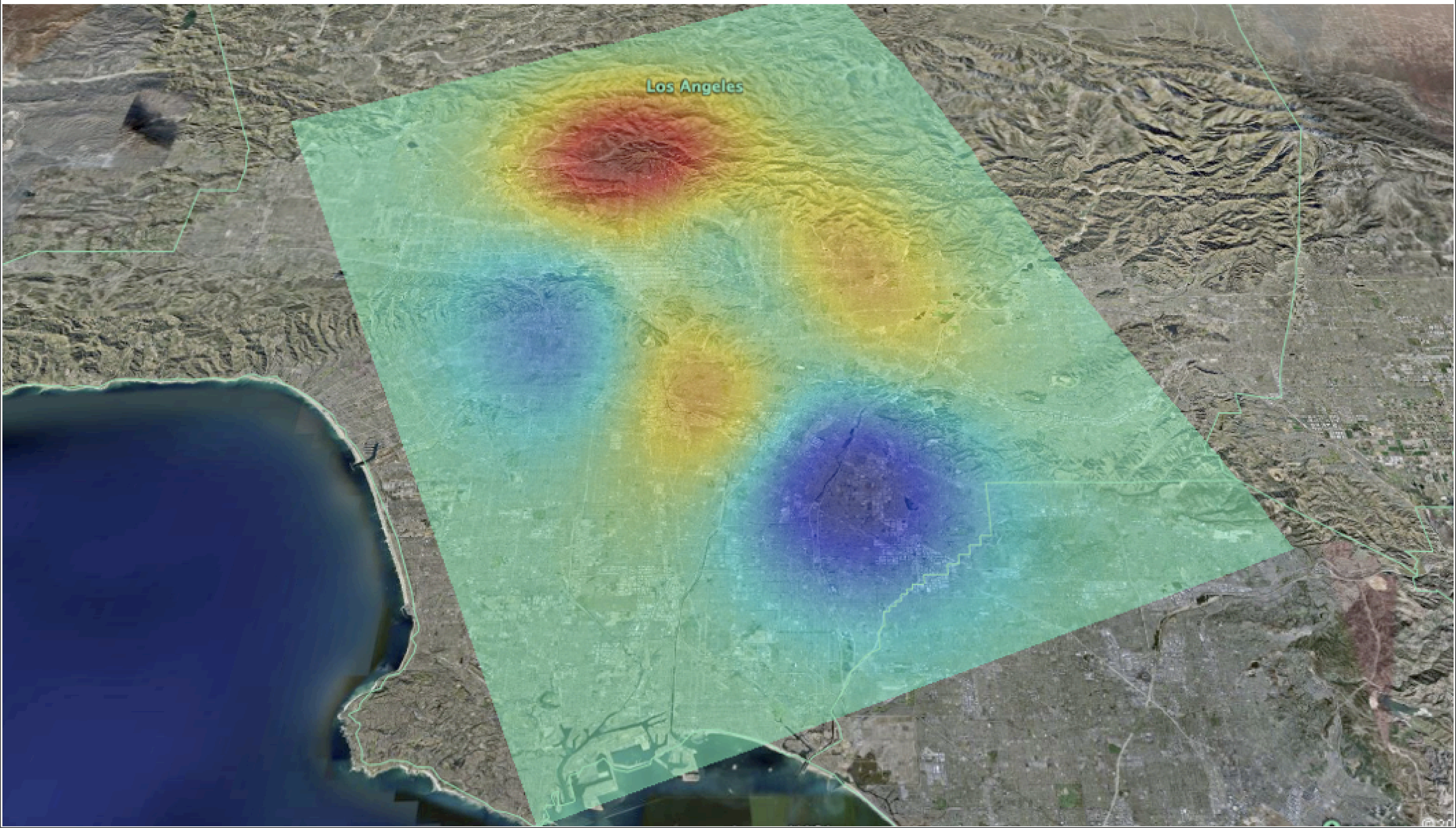
- In fact we compute the log of the price



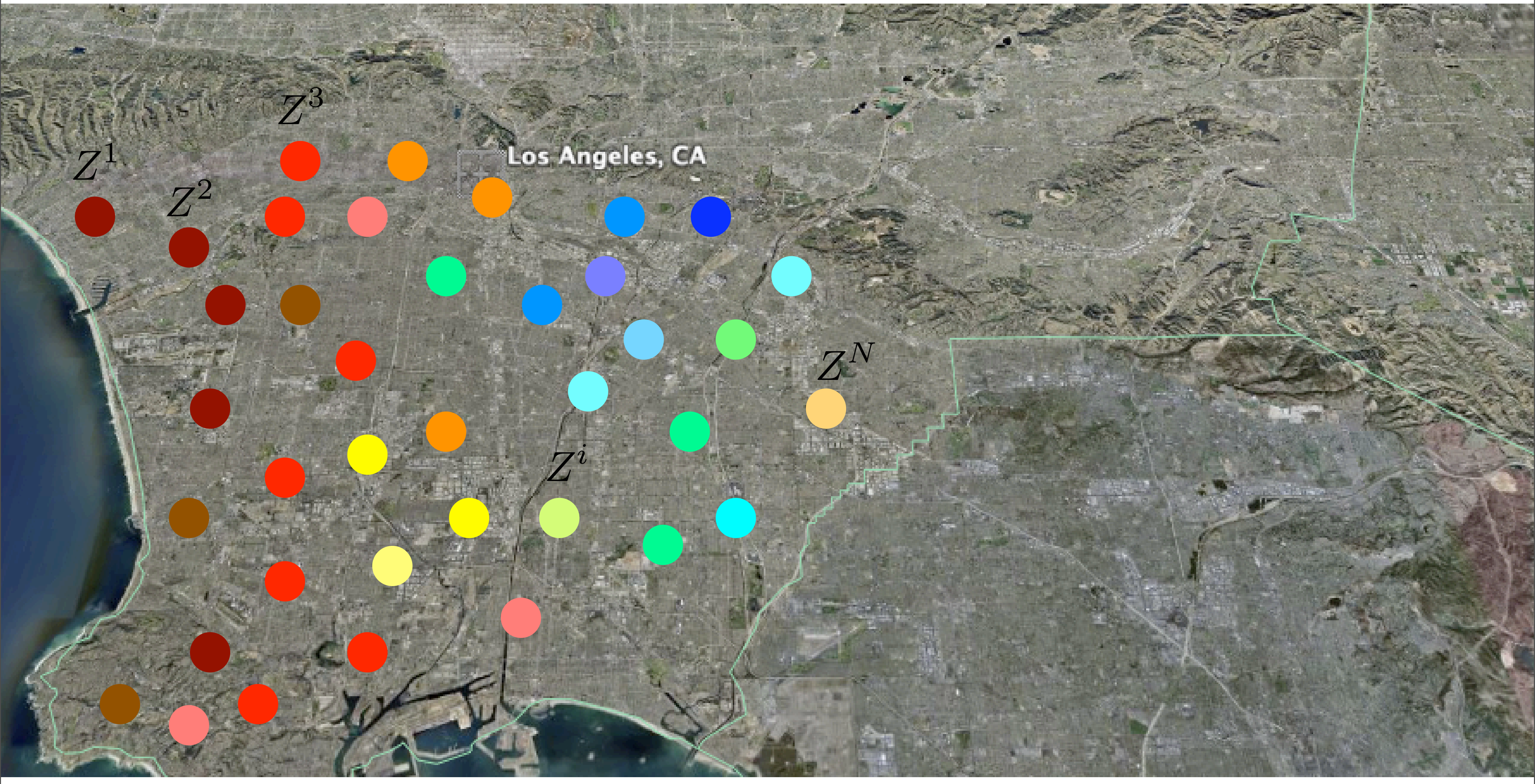
Our Contribution

- A novel technique for relational regression problems
- Allows relationships via the hidden variables
- Allows non-linear likelihood functions
- Propose efficient training and inference algorithm
- Apply it to the house price prediction problem

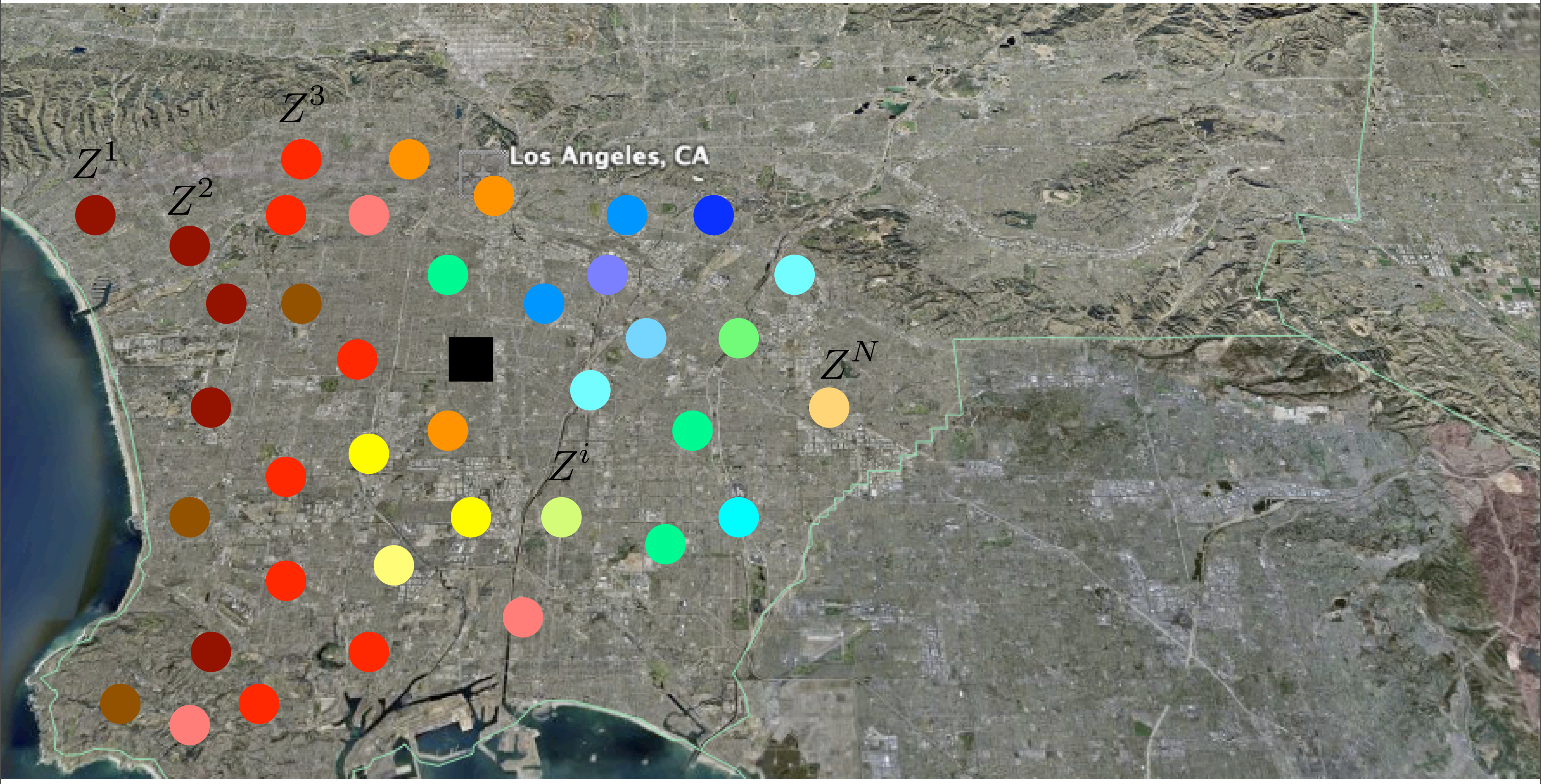
Estimating Desirabilities



Estimating Desirabilities



Estimating Desirabilities



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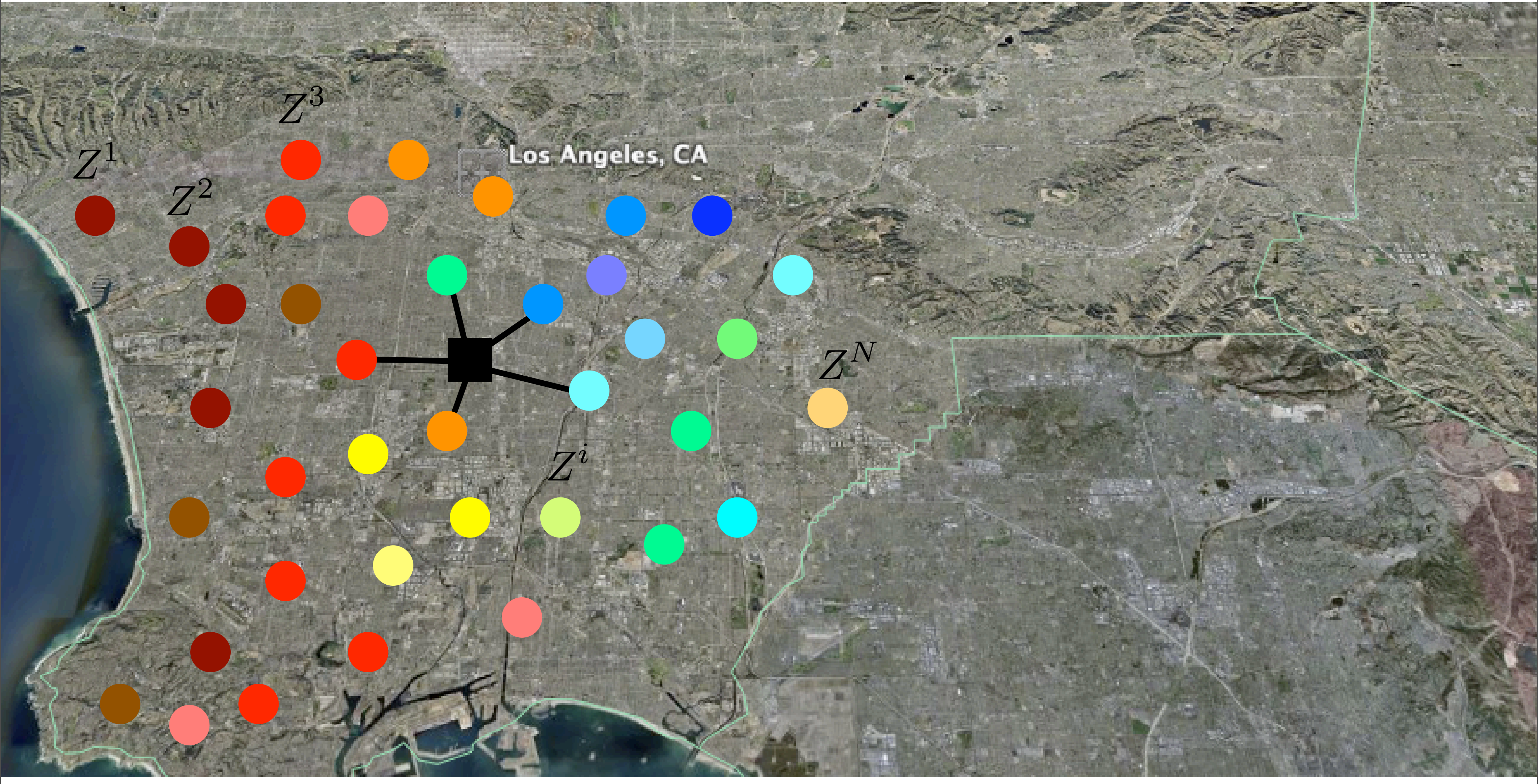


Estimating Desirabilities

- Any form of smooth interpolation is good
- Kernel Interpolation

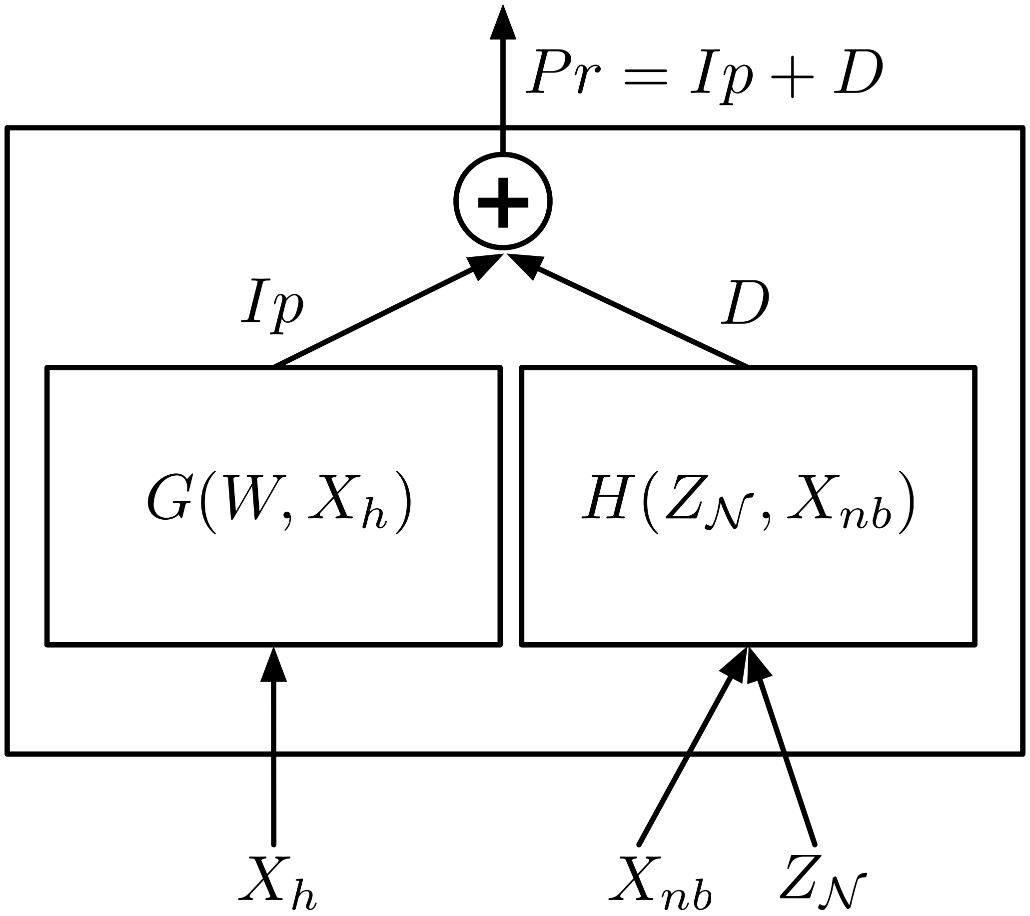
$$H(Z_{\mathcal{N}^i}, X^i_{nb}) = \sum_{j \in \mathcal{N}^i} Ker(X^i_{nb}, X^j_{nb}) Z^j$$

- Local Weighted Linear Regression

$$(\beta^*_{Z_{\mathcal{N}^i}}, \alpha^*_{Z_{\mathcal{N}^i}}) = \mathrm{argmin}_{\beta,\alpha} \sum_{j \in \mathcal{N}^i} (Z^j - (\beta + \alpha X^j))^2 Ker(X^i_{nb}, X^j_{nb})$$

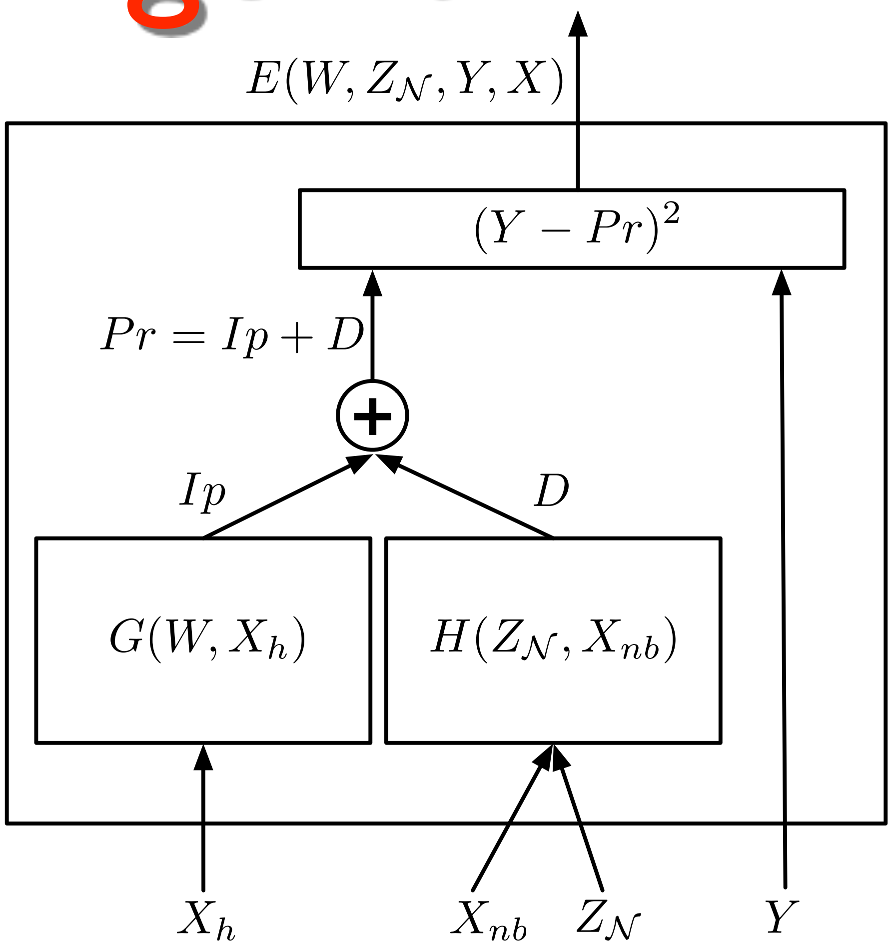
$$\begin{aligned} H(Z_{\mathcal{N}^i}, X^i_{nb}) &= \beta^*_{Z_{\mathcal{N}^i}} + \alpha^*_{Z_{\mathcal{N}_i}} X^i \\ &= \sum_k U^{ik} Z^k \end{aligned}$$

The Inference Algorithm



The Learning Algorithm

- Done by maximizing the likelihood of the data
- Achieved by minimizing the negative log-likelihood function wrt W, $\mathbf{Z}$
- Boils down to minimizing energy loss



$$\mathcal{L}(W, \mathbf{Z}) = \frac{1}{2} \sum_{i=1}^{N} (Y^i - (G(W, X_h^i) + H(Z_{\mathcal{N}^i}, X_{nb}^i))^2 + R(\mathbf{Z})$$

The Learning Algorithm

$$\mathcal{L}(W,\mathbf{Z}) = \frac{1}{2}\sum_{i=1}^{N}(Y^i - (G(W,X_h^i) + H(Z_{\mathcal{N}^i}, X_{nb}^i))^2 + \frac{r}{2}||Z||^2)$$

- Two phase: generalized EM type
- Phase I:
  - Keep W fixed and optimize with respect to $\mathbf{Z}$
  - The above loss reduces to

$$\mathcal{L}(\mathbf{Z}) = \frac{r}{2}||\mathbf{Z}||^2 + \frac{1}{2}\sum_{i=1}^{N}(Y^i - (G(W,X_h^i) + U^i \cdot \mathbf{Z}))^2$$

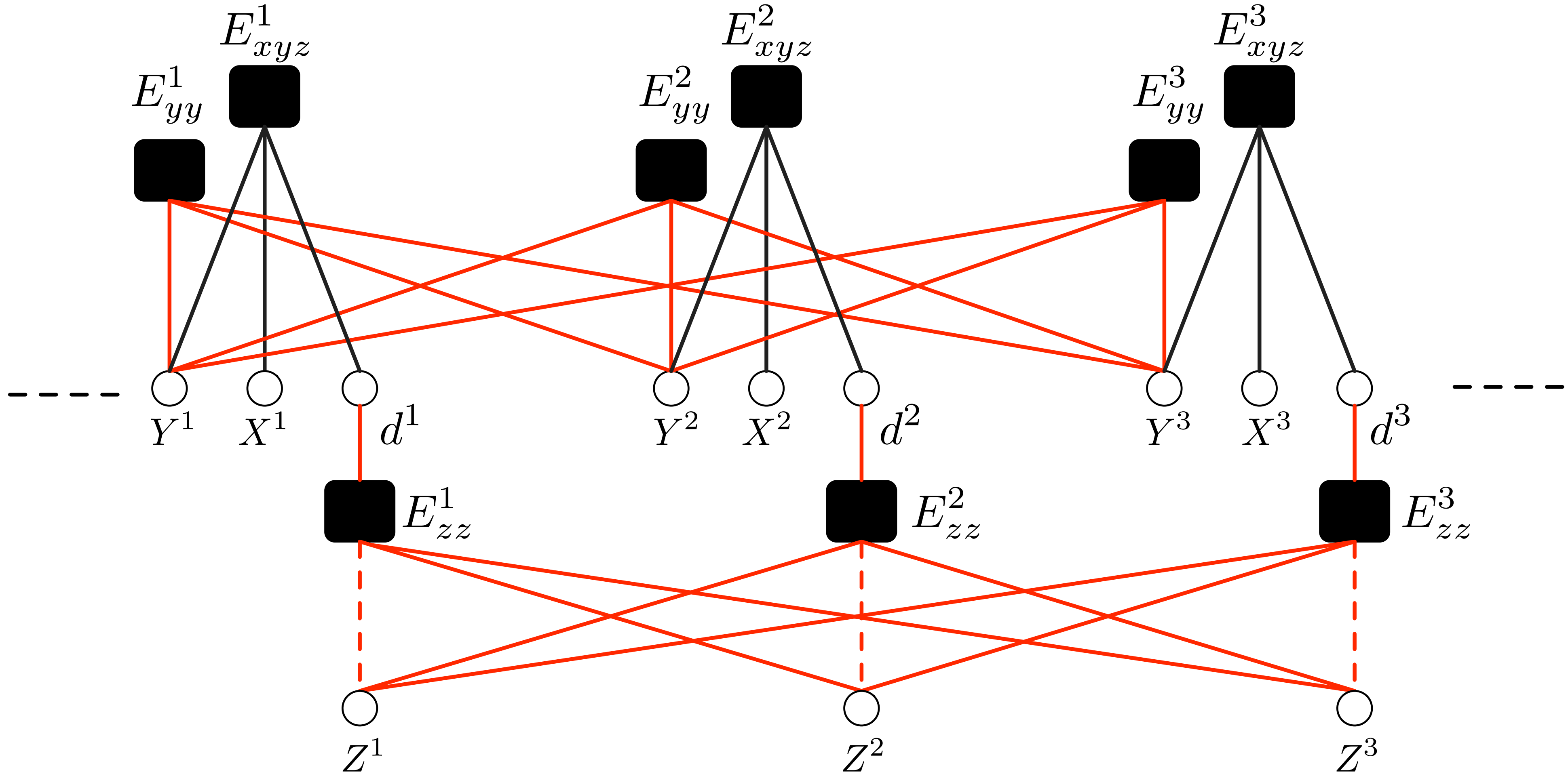
  - Sparse linear system: was solved using conjugate gradient

The Learning Algorithm

$$\mathcal{L}(W,\mathbf{Z}) = \frac{1}{2}\sum_{i=1}^{N}(Y^i - (G(W,X_h^i) + H(Z_{\mathcal{N}^i},X_{nb}^i))^2 + R(\mathbf{Z})$$

- Phase II:
  - Keep $\mathbf{Z}$ fixed and optimize with respect to W
  - The parameters are shared among samples
  - Neural network was optimized using simple gradient decent

The General Framework



$$E(W, \mathbf{Z}, \mathbf{Y}, \mathbf{X}) = \sum_{i=1}^{N} E^i_{xyz}(W_{xyz}, X^i, Y^i, D^i) + E^i_{yy}(W_{yy}, Y^i, \mathbf{Y}) + E^i_{zz}(W_{zz}, D^i, \mathbf{Z})$$

Experiments

- Dataset
  - Houses from Los Angeles Country transacted only in 2004
  - They span 1754 census tracts and 28 school district
  - A total of around 70,000 samples
  - We used a total of 19 features in X_h
    - living area, year built, # bedrooms, # bathrooms, pool, prior sale price, parking spaces, parking types, lot acerage, land value, improvement value, % improvement, new construction, foundation, roof type, heat type, site influence, and gps coordinates
  - We used 6 features as part of X_{nb}
    - median house hold income, average time of commute to work, proportion of units owner occupied, and academic performance index

Experiments

- Dataset
  - All variables containing any form of price/area/income information were mapped into log space
  - Non-numeric discrete variables were coded using a 1-of-K coding scheme
  - Only Single Family Residences were estimated
  - A total of 42025 complete labeled samples
  - Training set
    - 37822 (90%)
  - Test set
    - 4203 (10%)

Baseline Methods

- Nearest Neighbor
- Linear Regression
- Locally Weighted Linear Regression
- Fully Connected Neural Network

Results

- Absolute Relative Forecasting error is computed

$$error^i = \frac{|Pr^i - A^i|}{A^i}$$

Model Class	*Model*	*< 5%*	*< 10%*	*< 15%*
Non-Parametric	K - Nearest Neighbor	25.41	47.44	64.72
Parametric	Linear Regression	26.58	48.11	65.12
Non-Parametric	Locally Weighted Regression	32.98	58.46	75.21
Parametric	Fully Connected Neural Network	33.79	60.55	76.47
Hybrid	Relational Factor Graph	39.47	65.76	81.04

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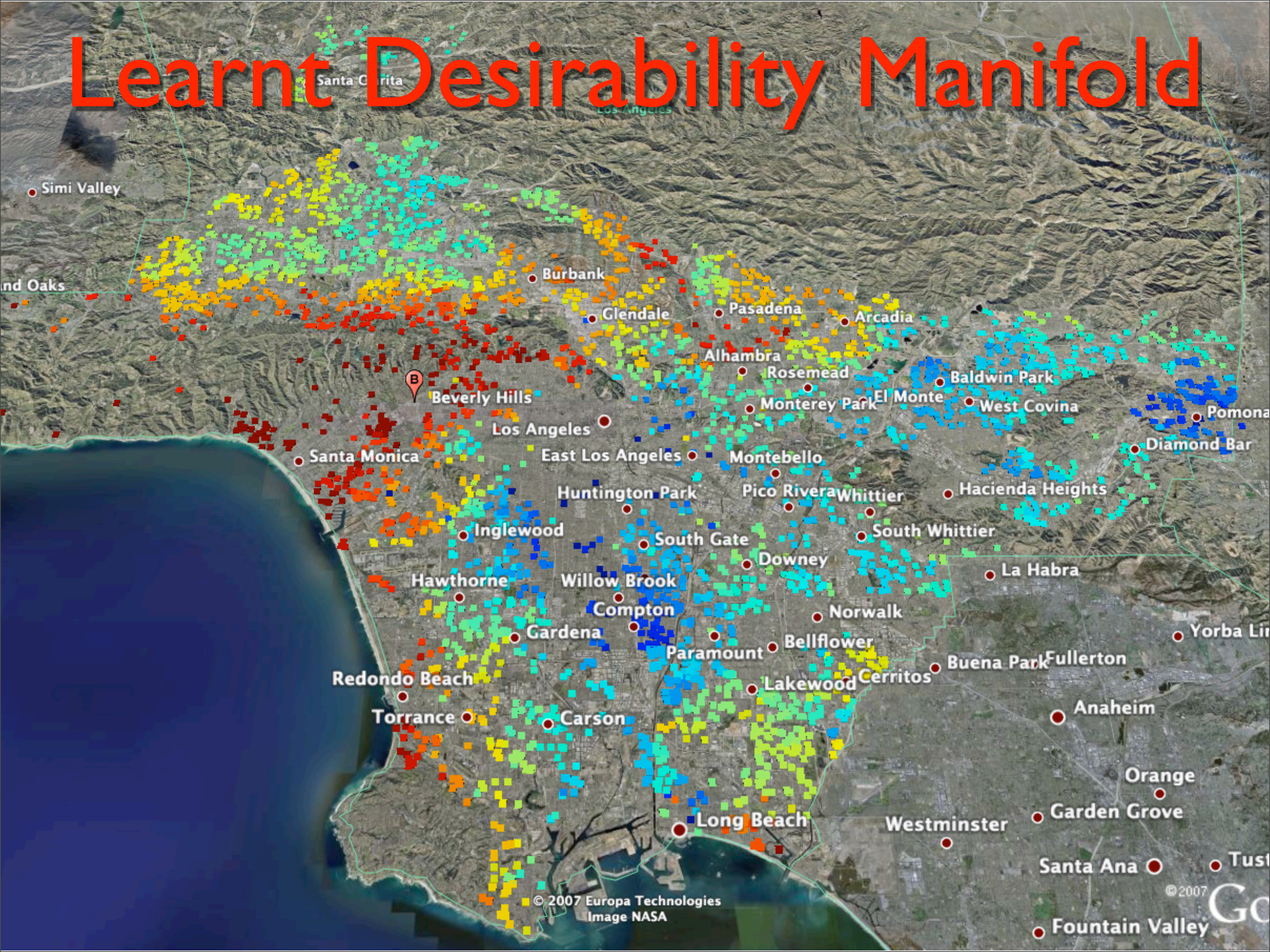
Learnt Desirability Manifold



Thank You Very Much!!!

Real Estate Price Prediction

- Direct dependencies between Y is not captured
- First factor
  - Non-relational: captures dependencies between individual variables and the price
    $$E^i_{xyz}(Y^i, X^i, D^i) = (Y^i - (G(W_{xyz}, X^i_h) + D^i))^2$$
- Second factor
  - Relational: captures the influence on the price of a house from other (related houses) via the hidden variables
    $$E^i_{zz}(D^i, H(X^i_{nb}, Z_{\mathcal{N}^i})) = (D^i - H(X^i_{nb}, Z_{\mathcal{N}^i}))^2$$
  - $H(X^i_{nb}, Z_{\mathcal{N}^i})$ non-parametric estimate of desirability of the location of the house, obtained from related houses

Real Estate Price Prediction

- $H(X_{nb}^i, Z_{\mathcal{N}^i})$ could take any smooth form
- Kernel Interpolation

$$H(X_{nb}^i, Z_{\mathcal{N}^i}) = \sum_{j \in \mathcal{N}^i} Ker(X_{nb}, X_{nb}^j) Z^j$$

- Local Weighted Linear Regression

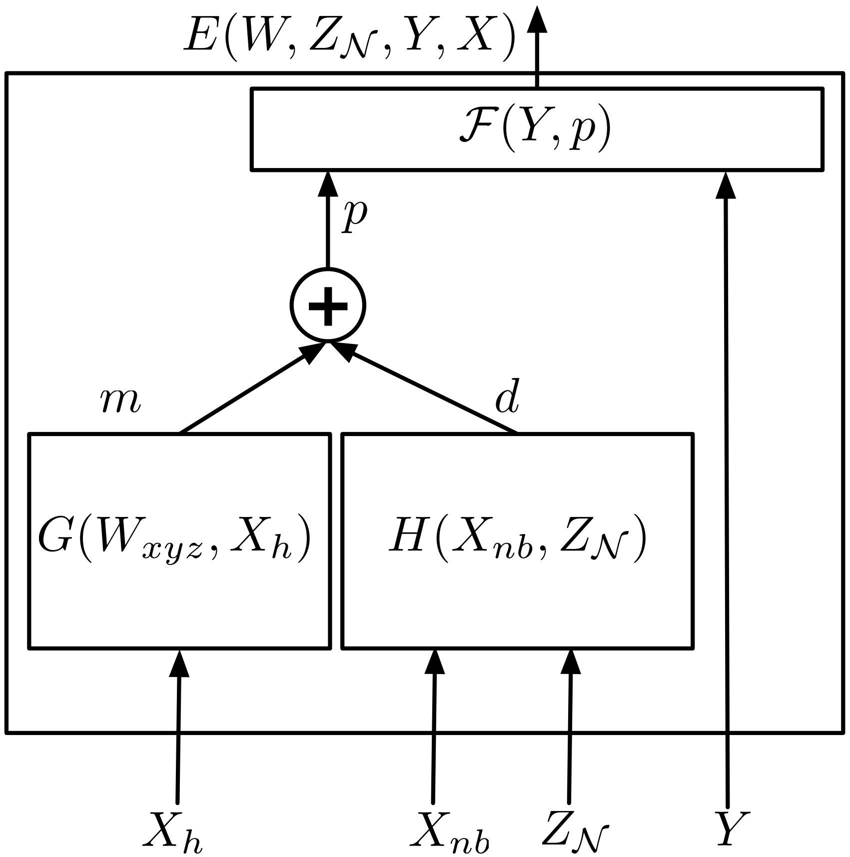
$$(\beta^*, \alpha^*) = \arg\min_{\beta,\alpha} \sum_{j \in \mathcal{N}^i} (Z^j - (\beta + \alpha X^j))^2 Ker(X^i, X^j)$$

$$\begin{aligned} H(X_{nb}^i, Z_{\mathcal{N}^i}) &= \beta^* + \alpha^* X \\ &= \sum_{j \in \mathcal{N}^i} a^j Z^j \end{aligned}$$

Real Estate Price Prediction

- The total energy associated with a single sample is

$$E^i_{xyz}(Y^i, X^i, D^i) + E^i_{zz}(D^i, H(X^i_{nb}, Z_{\mathcal{N}^i}) = (Y^i - (G(W_{xyz}, X^i_h) + D^i))^2 + (D^i - H(X^i_{nb}, Z_{\mathcal{N}^i}))^2$$



$$E^i(Y^i, X^i) = (Y^i - (G(W_{xyz}, X^i_h) + H(X^i_{nb}, Z_{\mathcal{N}^i}))^2$$

Real Estate Price Prediction

- The factor graph is