1. A complete binary tree is a binary tree in which every node has two children, except for the leaves (which have no child), and every leaf is at the same depth (same distance from the root).

a) Give a recursive definition for the set of all complete binary trees.

b) Use structural induction on the definition of part a) to prove that, for all complete binary trees T, if h is the height of T, then T contains exactly $2^{h+1} - 1$ nodes.

2. Let E be the set of well-formed algebraic expressions. i.e.,

i $x, y, z \in E$

ii If $e_1, e_2 \in E$, then $(e_1 + e_2)$, $(e_1 - e_2)$, $(e_1 * e_2)$, and $(e_1 \div e_2)$ also belong to E

iii E contains nothing else.

For any expression $e \in E$, let $vr(e)$ and $op(e)$ denote the number of variable and operator occurrences in e. Prove by structural induction over E that $\forall e \in E$, $vr(e) = op(e) + 1$.

3. Give a recursive definition for the set $\{x \in \mathbb{Z} : \ x \text{ is even}\}$ (the set of all even integers). Show that the aforementioned set is uniquely defined by the recursive definition.