1. A "prime factorization" of an integer n is a sequence of prime numbers whose product equals n, e.g., $84 = 2 \times 2 \times 3 \times 7$. Prove all integers $n \geq 2$ have a prime factorization.

2. What amounts of postages can be made exactly using only 3¢ and 5¢ stamps? Prove your claim by complete induction.

3. Define

$$P(n): \text{ In every set of } n \text{ kids, all kids have the same eye colour}$$

The following proof tries to use simple induction to show $\forall n \in \mathbb{N}, P(n)$. Can you explain why the proof is wrong?

Base case: $P(0)$ is vacuously true: in every set of 0 kids, all kids have the same eye colour.

Induction step: Let $i \in \mathbb{N}$ and suppose $P(i)$ holds. Let S be a set of $i+1$ kids, say $S = \{h_1, h_2, ..., h_{i+1}\}$. Consider $\{h_1, h_2, ..., h_i\}$: this is a set of i kids, so by IH, all kids in that set have the same eye colour, say A. Now, consider $\{h_2, ..., h_i, h_{i+1}\}$: this is also a set of i kids, so by IH, all kids in that set have the same eye colour, say B. Since kids $h_2, \cdots, h_i$ belong to both sets and cannot have two different eye colours, it must be that $A = B$. This means every kid in S has the same eye colour.