Consider the following algorithm.

```
1: function MYSTERY(A,s,f)
2:     # A is a list and s,f are indices such that 0 ≤ s ≤ f + 1 ≤ length(A)
3:     if s > f then
4:         return 0
5:     end if
6:     m = ⌊(f−s+1)/4⌋
7:     res = Mystery(A, s, s + m − 1)
8:     # loop precondition goes here...
9:     for i = s + m, ··· , f − m do
10:        res = res + A[i]
11:    end for
12:    # loop postcondition goes here...
13:    res = res + Mystery(A, f − m + 1, f)
14:    return res
15: end function
```

1. State clear and precise preconditions for this algorithm.

Ans: If you look at the comment at the start of the algorithm, you can find a partial answer. Moreover line 10 suggests that elements of A must be numeric. We can experss these formally as

1. A is a list of numbers
2. s, f are integers that satisfy $0 \leq s \leq f + 1 \leq length(A)$

□

2. State clear and precise postconditions for this algorithm.

Ans: The indices of lines 7, 9 and 13 are consecutive values and we do sum values in these lines. Hence, we intuitively expect the algorithm to add all the elements of A together. More formally:

1. Mystery(A,s,f) returns $A[s] + \cdots + A[f]$

□

3. Prove the correctness of this algorithm.
Note: You may assume that the loop is correct without proof, as long as you state clear preconditions and postconditions specifically for the loop where indicated by comments.

Ans:
Loop precondition: $res = A[s] + \cdots + A[s + m - 1]$ ($res = 0$ if $m = 0$)

Loop postcondition: $res = A[s] + \cdots + A[f - m]$
By induction on $n = f - s + 1$, we prove that Mystery(A,s,f) terminates and returns $A[s] + ... + A[f]$ for all lists of numbers A and integers s,f such that $0 \leq s \leq f + 1 \leq length(A)$.

Base Case: Suppose A is a list of numbers and s,f are integers such that $0 \leq s = f + 1 \leq length(A)$. Then, the call Mystery(A,s,f) executes only line 3 and returns 0, and 0 is the sum of empty list. Hence, Mystery(A,s,f) terminates and returns $A[s] + \cdots + A[f]$.

Induction Step: Suppose $n \geq 1$ and, for all integers s,f such that $0 \leq f - s + 1 < n$, Mystery(A,s,f) terminates and returns $A[s] + \cdots + A[f]$ for all lists of numbers A such that $0 \leq s \leq f + 1 \leq length(A)$ (IH). Suppose $f - s + 1 = n$ and A is a list of numbers such that $0 \leq s \leq f + 1 \leq length(A)$. Then, Mystery($A$,$s$,$f$) executes the test on line 1 and fails since $n \geq 1$ implies $f \geq s$. Then it executes lines 6 and 7 and hence make recursive call on an input that satisfies the preconditions $0 \le s \le s+m-1+1 = s+m \le length(A)$. The input size of the recursive call is exactly $s+m-1-s+1 = m = \lfloor \frac{n}{4} \rfloor < n$ so by IH, we know that this call will terminate and return $A[s] + \cdots + A[s+m-1]$. This means that the loop on line 9 is entered with its preconditions satisfied. Hence, when the loop terminates, the postconditions are also satisified. In other words, $res = A[s] + ... + A[f-m]$ upon termination of the loop. When the recursive call on line 13 is made, the arguments meet the precondition $0 \le f-m+1 \le f+1 \le len(A)$ (since $m \ge 0$), and the input size is again exactly $\lfloor \frac{n}{4} \rfloor$. By the IH, this call terminates and returns $A[s-m+1] + \cdots + A[f]$. Therefore, the current call also terminates and returns

$$A[s] + \cdots + A[f-m] + A[f-m+1] + \cdots + A[f] = A[s] + \cdots + A[f]$$

Hence, for all s,f such that $f-s+1 = n$, Mystery(A,s,f) terminates and returns $A[s] + \cdots + A[f]$, for all lists of numbers A such that $0 \le s \le f+1 \le length(A)$. In conclusion, by induction, Mystery(A,s,f) terminates and returns $A[s] + \cdots + A[f]$ for all lists of numbers A such that $0 \le s \le f+1 \le length(A)$. □