1. Prove that for all \( n \in \mathbb{N} \), every set of size \( n \) has exactly \( 2^n \) subsets.

   Ans: The precise definition of predicate is
   
   \[ P(n) : \text{all sets of size } n \text{ have } 2^n \text{ subsets} \]
   
   **Base case:** An empty set (set of size 0) has only one subset which is the empty set itself. So \( P(0) \) is true.

   **Induction step:** Assume for \( n \in \mathbb{N} \), \( P(n) \) is true (IH). In other words, assume all sets of size \( n \) have \( 2^n \) subsets. Suppose \( S = \{a_1, a_2, \ldots, a_{n+1}\} \) is a set of size \( n+1 \). Subsets of \( S \) can be split into those that contain the element \( a_{n+1} \) and those that don’t. Any subset in the former can be converted to a unique subset in the latter by removing the element \( a_{n+1} \) and any subset in the latter can be converted to a unique subset in the former by adding the element \( a_{n+1} \). Therefore, there are an equal number of subsets in both categories. Now, each subset of \( S \) that does not contain \( a_{n+1} \) is a subset of \( S' = S - \{a_{n+1}\} = \{a_1, a_2, \ldots, a_n\} \). Moreover, any subset of \( S' \) is in fact a subset of \( S \) that does not contain the element \( a_{n+1} \). \( S' \) is a set of size \( n \) and by IH has \( 2^n \) subsets. Therefore, both categories of subsets of \( S \) have a size of \( 2^n \) and hence \( S \) has \( 2 \times 2^n = 2^{n+1} \) subsets.

   Given the proofs for the base case and the induction step, we can conclude, by induction, that \( P(n) \) is true for all \( n \in \mathbb{N} \).

2. Prove that \( \forall n \geq 13, n^2 > 12n + 5 \).

   Ans: This examples requires the use of simple induction with a non-zero base. Because of that, we simply define the predicate as
   
   \[ P(n) : n^2 > 12n + 5 \]
   
   and try to prove, as our base case, that \( P(13) \) holds (since \( n \geq 13 \)).

   **Base case:**
   
   \[ 13^2 = 169 \]
   
   \[ > 161 = 156 + 5 = 12 \times 13 + 5 \]

   Therefore, \( P(13) \) is true.

   **Induction step:** Assuming that for \( n \in \mathbb{N} \), \( P(n) \) is true (IH). We will prove that \( P(n + 1) \) is also true.

   \[ (n + 1)^2 = n^2 + 2n + 1 \]
   
   \[ > 12n + 5 + 2n + 1 \quad \text{(by the IH)} \]
   
   \[ \geq 12n + 5 + 26 + 1 \quad (n \geq 13) \]
   
   \[ > 12n + 12 + 5 = 12(n + 1) + 5 \]

   Now, we have proved that \( P(n + 1) \) is true, so we can conclude by induction that \( \forall n \geq 13, n^2 > 12n + 5 \).