## Principles of Computer Networks <br> Tutorial 4

## Problem 1

The total amount of time to get the IP address is

$$
R T T_{1}+R T T_{2}+\cdots+R T T_{n}
$$

Once the IP address is known, $R T T_{o}$ elapses to set up the TCP connection and another $R T T_{O}$ elapses to request and receive the small object. The total response time is

$$
2 R T T_{o}+R T T_{1}+R T T_{2}+\cdots+R T T_{n}
$$

## Problem 2

a) Consider a distribution scheme in which the server sends the file to each client, in parallel, at a rate of a rate of $u_{s} / N$. Note that this rate is less than each of the client's download rate, since by assumption $u_{s} / N \leq d_{\text {min }}$. Thus each client can also receive at rate $u_{s} / N$. Since each client receives at rate $u_{s} / N$, the time for each client to receive the entire file is $F /\left(u_{s} I N\right)=N F / u_{s}$. Since all the clients receive the file in $N F / u_{s}$, the overall distribution time is also $N F / u_{s}$.
b) Consider a distribution scheme in which the server sends the file to each client, in parallel, at a rate of $d_{\text {min }}$. Note that the aggregate rate, $N d_{\min }$, is less than the server's link rate $u_{s}$, since by assumption $u_{s} I_{-} N \geq d_{\text {min }}$. Since each client receives at rate $d_{\text {min }}$, the time for each client to receive the entire file is F/ $d_{\text {min }}$. Since all the clients receive the file in this time, the overall distribution time is also $\mathrm{F} / d_{\text {min }}$.
c) From Section 2.6 we know that

$$
D_{C S} \geq \max \left\{N F / u_{s}, F / d_{\min }\right\} \quad(\text { Equation } 1)
$$

Suppose that $u_{s} / N \leq d_{\min }$. Then from Equation 1 we have $D_{C S} \geq N F / u_{s}$. But from (a) we have $D_{C S} \leq N F / u_{s}$. Combining these two gives:

$$
D_{C S}=N F / u_{s} \text { when } u_{s} / N \leq d_{\min } .(\text { Equation } 2)
$$

We can similarly show that:
$D_{C S}=F / d_{\text {min }}$ when $u_{s} / N \geq d_{\text {min }}$ (Equation 3).
Combining Equation 2 and Equation 3 gives the desired result.

## Problem 3

a) Define $u=u_{1}+u_{2}+\ldots .+u_{N}$. By assumption

$$
u_{s} \leq\left(u_{s}+u\right) / N \quad \text { Equation } 1
$$

Divide the file into $N$ parts, with the $i^{\text {th }}$ part having size $\left(u_{i} / u\right) F$. The server transmits the $\mathrm{i}^{\text {th }}$ part to peer i at rate $r_{\mathrm{i}}=\left(u_{\mathrm{i}} / u\right) u_{\mathrm{s}}$. Note that $r_{1}+r_{2}+\ldots . .+r_{\mathrm{N}}=u_{\mathrm{s}}$, so that the aggregate server rate does not exceed the link rate of the server. Also have each peer $i$ forward the bits it receives to each of the $N-1$ peers at rate $r_{\mathrm{i}}$. The aggregate forwarding rate by peer $i$ is $(N-1) r_{i}$. We have

$$
(N-1) r_{\mathrm{i}}=(N-1)\left(u_{\mathrm{s}} u_{\mathrm{i}}\right) / u<=u_{\mathrm{i}},
$$

where the last inequality follows from Equation 1. Thus the aggregate forwarding rate of peer $i$ is less than its link rate $u_{i}$.

In this distribution scheme, peer $i$ receives bits at an aggregate rate of

$$
r_{i}+\sum_{j<i} r_{j}=u_{s}
$$

Thus each peer receives the file in $\mathrm{F} / \mathrm{u}_{\mathrm{s}}$.
b) Again define $u=u_{1}+u_{2}+\ldots .+u_{N}$. By assumption

$$
u_{\mathrm{s}}>=\left(u_{\mathrm{s}}+u\right) / N \quad \text { Equation } 2
$$

Let $r_{i}=u_{\mathrm{i}} /(N-1)$ and

$$
r_{N+1}=\left(u_{\mathrm{s}}-u /(N-1)\right) / N
$$

In this distribution scheme, the file is broken into $N+1$ parts. The server sends bits from the $\mathrm{i}^{\text {th }}$ part to the $\mathrm{i}^{\text {th }}$ peer $(\mathrm{i}=1, \ldots ., N)$ at rate $\mathrm{r}_{\mathrm{i}}$. Each peer i forwards the bits arriving at rate $r_{i}$ to each of the other N-1 peers. Additionally, the server sends bits from the $(N+1)^{\text {st }}$ part at rate $\mathrm{r}_{\mathrm{N}+1}$ to each of the $N$ peers. The peers do not forward the bits from the $(N+1)^{\text {st }}$ part.

The aggregate send rate of the server is

$$
r_{1}+\ldots .+r_{\mathrm{N}}+N r_{\mathrm{N}+1}=u /(N-1)+u_{\mathrm{s}}-u /(N-1)=u_{\mathrm{s}}
$$

Thus, the server's send rate does not exceed its link rate. The aggregate send rate of peer $i$ is

$$
(N-1) r_{\mathrm{i}}=u_{\mathrm{i}}
$$

Thus, each peer's send rate does not exceed its link rate.
In this distribution scheme, peer i receives bits at an aggregate rate of

$$
r_{i}+r_{N+1}+\sum_{j \gg} r j=u /(N-1)+\left(u_{s}-u /(N-1)\right) / N=\left(u_{s}+u\right) / N
$$

Thus each peer receives the file in $N F /\left(u_{s}+u\right)$.
(For simplicity, we neglected to specify the size of the file part for $i=1, \ldots, N+1$. We now provide that here. Let $\Delta=\left(u_{s}+u\right) / N$ be the distribution time. For $i=1, \ldots, N$, the $i^{\text {th }}$ file
part is $F_{i}=r_{i} \Delta$ bits. The $(N+1)^{\text {st }}$ file part is $F_{N+1}=r_{N+1} \Delta$ bits. It is straightforward to show that $F_{1}+\ldots . .+F_{N+1}=F$.)
c) The solution to this part is similar to the previous question. We know from Section 2.6 that

$$
D_{P 2 P}>=\max _{\left\{F / u_{s}, N F /\left(u_{s}+u\right)\right\}}
$$

Combining this with $a$ ) and b) gives the desired result.

