Principles of Computer Networks

Tutorial 3

Problem 1

a)

$P_n = \binom{N}{n}(\lambda\Delta t)^n(1-\lambda\Delta t)^{N-n}$ n=0, 1, …., N

b)

$$P_n = \binom{N}{n}(\lambda\Delta t)^n(1-\lambda\Delta t)^{N-n}$$

For T fixed, as $N \to \infty$, we have $\Delta_t = T/N \to 0$

- $\lim_{\Delta t\to 0}(1-\lambda\Delta t)^N = \lim_{\Delta t\to 0}(1-\lambda\Delta t)^{T/\Delta t} = e^{-\lambda T}$
- $\lim_{\Delta t\to 0}(1-\lambda\Delta t)^{-n} = 1$
- $\lim_{N\to\infty}\frac{N(N-1)\ldots(N-(n-1))}{N^n} = \lim_{N\to\infty} 1\left(1-\frac{1}{n}\right)\left(1-\frac{2}{n}\right)\ldots(1-\frac{n-1}{n}) = 1$

We then obtain that

- $P_n = \frac{(\lambda T)^n}{n!}e^{-\lambda T}$ n=0, 1, …., N

which is the Poisson distribution.

c) Let T be the inter-arrival time. Then using (b), we have that

$P(T \le t) = P$(at least one arrival in the interval [0,t))

$= 1 - e^{-\lambda t}$, $t > 0$

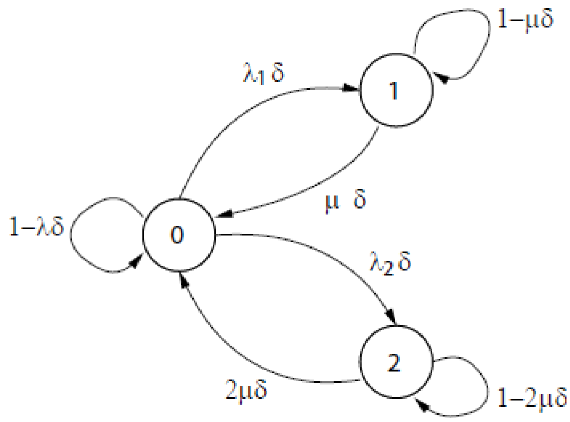
i.e. the inter-arrival time is given by an exponential distribution.

Problem 2

a) $\frac{\lambda_1}{\lambda_2+\lambda_1}$

b) $\frac{\lambda_2}{\lambda_2+\lambda_1}$

c) Define $\lambda = \lambda_2 + \lambda_1$. The state transition diagram is shown below



d) Define $\rho_1 = \frac{\lambda_1}{\mu}$ and $\rho_2 = \frac{\lambda_2}{2\mu}$. It follows that

$$P_1 = (1 - \mu\delta)P_1 + \lambda_1\delta P_0$$

$$P_2 = (1 - 2\mu\delta)P_2 + \lambda_2\delta P_0$$

Which leads to,

$$P_1 = \frac{\lambda_1}{\mu} P_0$$

$$P_2 = \frac{\lambda_2}{2\mu} P_0$$

Using the condition that $P_0 + P_1 + P_2 = 1$, we obtain that

$$P_0 \left(1 + \frac{\lambda_1}{\mu} + \frac{\lambda_2}{2\mu}\right) = 1$$

Define $\bar{\rho} = \frac{2\lambda_1+\lambda_2}{2\mu}$, then we get

$$P_0 = \frac{1}{1+\bar{\rho}}$$

$$P_1 = \overline{\rho_1}\frac{1}{1+\bar{\rho}}$$

$$P_2 = \overline{\rho_2}\frac{1}{1+\bar{\rho}}$$

Problem 3

(a)

$$P = \lambda_1 \Delta t$$

(b)

$$P = \lambda_2 \Delta t$$

(c)

$$P = \lambda_1 \Delta t(1 - \lambda_2 \Delta t) \approx \lambda_1 \Delta t$$

(d)

$$P = \lambda_2 \Delta t(1 - \lambda_1 \Delta t) \approx \lambda_2 \Delta t$$

(e)

$$P = (1 - \lambda_1 \Delta t)(1 - \lambda_2 \Delta t) = 1 - (\lambda_1 + \lambda_2)\Delta t + \lambda_1 \lambda_2 (\Delta t)^2 \approx 1 - (\lambda_1 + \lambda_2)\Delta t$$

(f)

$$P = \lambda_1 \Delta t(1 - \lambda_2 \Delta t) + \lambda_2 \Delta t(1 - \lambda_1 \Delta t) = (\lambda_1 + \lambda_2)\Delta t - 2\lambda_1 \lambda_2 (\Delta t)^2 \approx (\lambda_1 + \lambda_2)\Delta t$$

(g)

$$P = \lambda_1 \lambda_2 (\Delta t)^2 \approx 0$$

(h)

$$P = \frac{\lambda_1 \Delta t(1 - \lambda_2 \Delta t)}{\lambda_1 \Delta t(1 - \lambda_2 \Delta t) + \lambda_2 \Delta t(1 - \lambda_1 \Delta t)} \approx \frac{\lambda_1}{\lambda_1 + \lambda_2}$$

(i)

$$P = \frac{\lambda_2 \Delta t(1 - \lambda_1 \Delta t)}{\lambda_1 \Delta t(1 - \lambda_2 \Delta t) + \lambda_2 \Delta t(1 - \lambda_1 \Delta t)} \approx \frac{\lambda_2}{\lambda_1 + \lambda_2}$$

(j) Let E_1 be the event that the link OUT_1 transmits a packet in a time-slot of length Δt, and let A be the event that we have an arrival at the switch. Then we have that

$$P(E_1) = P(E_1 \mid A)P(A) = p(\lambda_1 + \lambda_2)\Delta t$$

(k) Let E_2 be the event that the link OUT_2 transmits a packet in a time-slot of length Δt, and let A be the event that we have an arrival at the switch. Then we have that

$$P(E_2) = P(E_2 \mid A)P(A) = (1 - p)(\lambda_1 + \lambda_2)\Delta t$$