Principles of Computer Networks

Tutorial 3

Problem 1

In Tutorial 2, we saw that the probability of having \( n \) arrivals in the \([0, k\Delta t]\) time interval of the discrete-time system was a binomial distribution:

\[
\binom{k}{n} (\lambda\Delta t)^n (1 - \lambda\Delta t)^{k-n}
\]

where \( 0 \leq n \leq k \), and \( \lambda\Delta t \) is the probability of one packet arriving during one time-slot \( \Delta t \) where the arrival rate is \( \lambda \).

Assume \( \Delta t \) approaches 0, and a packet is still processed instantly (no queueing). Consider a time interval of fixed length \( T \) which is divided into \( N \) slots of equal length \( \Delta t = T/N \). In each time slot, exactly one packet arrives with probability \( \lambda\Delta t \), and no packet arrives with probability \( 1 - \lambda\Delta t \). Therefore, the probability that two or more packets arrive in one time slot is 0.

a) What is the probability \( P_n \) that \( n, n = 0, 1, ..., N \), packets arrive in the time interval \([0, T]\)?

b) Find the probability \( P_n \) as the number of time slots \( N \) approaches infinity (\( N \to \infty \)) (and the interval \( \Delta t \) approaches 0, \( \Delta t \to 0 \)). Hint: You may want to use

\[
\lim_{x \to 0} (1 + ax)^k = e^{ak} \text{ and, for } N \text{ very large, } N! \approx \frac{(N/e)^N}{\sqrt{2\pi N}}
\]

c) Assuming that \( \Delta t \to 0 \), what is the distribution of the time between two successive packet arrivals?

Problem 2

Consider a transmission system (queue) that can hold at most one packet (the packet that is in service), i.e. there is no buffer and a new packet either goes directly into service or is dropped. The system receives Poisson packet traffic from two other nodes, 1 and 2, at rates \( \lambda_1 \) and \( \lambda_2 \), respectively. The service times of the packets are independently, exponentially distributed with a mean \( \frac{1}{\mu} \) for packets from node 1, and \( \frac{1}{2\mu} \) for packets from source 2.
Define $P_0$ as the steady-state probability that system does not serve packets, $P_1$ as the steady-state probability that system serves a packet from node 1, and $P_2$ as the steady-state probability that system serves a packet from node 2.

Give answers to the following questions.

a) What is the probability that a packet that gets accepted into service is a packet from node 1?

b) What is the probability that a packet that gets accepted into service is a packet from node 2?

c) Let the states 0, 1, and 2 indicate the case that we find in the system no packet, one packet of source 1, and one packet of source 2, respectively. Draw the state-transition diagram of the system.

d) Compute the steady-state probabilities $P_0$, $P_1$ and $P_2$.

Problem 3

Consider a switch with two incoming and two outgoing links. The links are all synchronized, and the switch sends and receives in time slots of length $\Delta t$. The switch immediately forwards incoming packets to one of the outgoing links. An incoming packet is routed to link OUT$_1$ with probability $p$ and routed to link OUT$_2$ with probability $1-p$. In each time slot, the link receives exactly one new packet on link IN$_1$ with probability $\lambda_1\Delta t$, and receives no packet with probability $1 - \lambda_1\Delta t$. Similarly, the link receives exactly one new packet on link IN$_2$ with probability $\lambda_2\Delta t$, and receives no packet with probability $1 - \lambda_2\Delta t$.

To simplify the analysis, we assume that the events (arrivals) on link IN$_1$ and IN$_2$ are independent. Further, $\Delta t$ is assumed to be very small; consequently, $\Delta t^2 \ll \Delta t$, and the approximation $\Delta t^2 \approx 0$ can be made.
Give answers to the following questions.

a) Find the probability that the switch receives only one packet from link IN1 in a time slot.
b) Find the probability that the switch receives only one packet from link IN2 in a time slot.
c) Find the probability that the switch receives only one packet from IN1 and no packet from IN2 in a time slot.
d) Find the probability that the switch receives only one packet from IN2 and no packet from IN1 in a time slot.
e) Find the probability that the switch receives no packets in a time slot.
f) Find the probability that the switch receives exactly one packet (which could be either from link IN1 or IN2) in a time slot.
g) Find the probability that the switch receives exactly two packets in a time slot.
h) Given that the switch receives exactly one packet in a given time slot, what is the probability that this packet is from link IN1?
i) Given that the switch receives exactly one packet in a given time slot, what is the probability that this packet is from link IN2?
j) Find the probability that the switch sends in a time slot exactly one packet on link OUT1.
k) Find the probability that the switch sends in a time slot exactly one packet on link OUT2.