Principles of Computer Networks

Tutorial 3

Problem 1

In Tutorial 2, we saw that the probability of having n arrivals in the $[0, k\Delta t]$ time interval of the discrete-time system was a binomial distribution:

$$\binom{k}{n}(\lambda\Delta t)^n(1-\lambda\Delta t)^{k-n}$$

where $0 \le n \le k$, and $\lambda\Delta t$ is the probability of one packet arriving during one time-slot Δt where the arrival rate is λ.

Assume Δt approaches 0, and a packet is still processed instantly (no queueing). Consider a time interval of fixed length T which is divided into N slots of equal length $\Delta t = T/N$. In each time slot, exactly one packet arrives with probability $\lambda\Delta t$, and no packet arrives with probability $1 - \lambda\Delta t$. Therefore, the probability that two or more packets arrive in one time slot is 0.

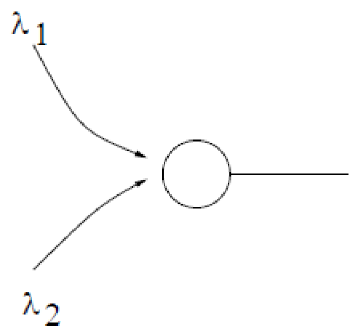
a) What is the probability P_n that n, $n = 0, 1, ..., N$, packets arrive in the time interval $[0, T]$?

b) Find the probability P_n as the number of time slots N approaches infinity ($N \to \infty$) (and the interval Δt approaches 0, $\Delta t \to 0$). Hint: You may want to use $\lim_{x\to 0}(1+ax)^{\frac{k}{x}} = e^{ak}$ and, for N very large, $N! \approx \frac{(N/e)^N}{\sqrt{2N\pi}}$

c) Assuming that $\Delta t \to 0$, what is the distribution of the time between two successive packet arrivals?

Problem 2

Consider a transmission system (queue) that can hold at most one packet (the packet that is in service), i.e. there is no buffer and a new packet either goes directly into service or is dropped. The system receives Poisson packet traffic from two other nodes, 1 and 2, at rates λ_1 and λ_2, respectively. The service times of the packets are independently, exponentially distributed with a mean $\frac{1}{\mu}$ for packets from node 1, and $\frac{1}{2\mu}$ for packets from source 2.



Define P_0 as the steady-state probability that system does not serve packets, P_1 as the steady-state probability that system serves a packet from node 1, and P_2 as the steady-state probability that system serves a packet from node 2.

Give answers to the following questions.

a) What is the probability that a packet that gets accepted into service is a packet from node 1?

b) What is the probability that a packet that gets accepted into service is a packet from node 2?

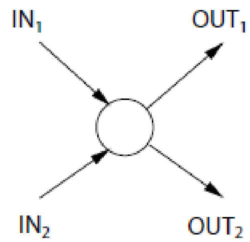
c) Let the states 0, 1, and 2 indicate the case that we find in the system no packet, one packet of source 1, and one packet of source 2, respectively. Draw the state-transition diagram of the system.

d) Compute the steady-state probabilities P_0, P_1 and P_2.

Problem 3

Consider a switch with two incoming and two outgoing links. The links are all synchronized, and the switch sends and receives in time slots of length Δt. The switch immediately forwards incoming packets to one of the outgoing links. An incoming packet is routed to link OUT_1 with probability p and routed to link OUT_2 with probability $1-p$. In each time slot, the link receives exactly one new packet on link IN_1 with probability $\lambda_1 \Delta t$, and receives no packet with probability $1- \lambda_1 \Delta t$. Similarly, the link receives exactly one new packet on link IN_2 with probability $\lambda_2 \Delta t$, and receives no packet with probability $1- \lambda_2 \Delta t$.

To simplify the analysis, we assume that the events (arrivals) on link IN_1 and IN_2 are independent. Further, Δt is assumed to be very small; consequently, $\Delta t^2 << \Delta t$, and the approximation $\Delta t^2 \approx 0$ can be made.



Give answers to the following questions.

a) Find the probability that the switch receives only one packet from link IN_1 in a time slot.

b) Find the probability that the switch receives only one packet from link IN_2 in a time slot.

c) Find the probability that the switch receives only one packet from IN_1 and no packet from IN_2 in a time slot.

d) Find the probability that the switch receives only one packet from IN_2 and no packet from IN_1 in a time slot.

e) Find the probability that the switch receives no packets in a time slot.

f) Find the probability that the switch receives exactly one packet (which could be either from link IN_1 or IN_2) in a time slot.

g) Find the probability that the switch receives exactly two packets in a time slot.

h) Given that the switch receives exactly one packet in a given time slot, what is the probability that this packet is from link IN_1?

i) Given that the switch receives exactly one packet in a given time slot, what is the probability that this packet is from link IN_2?

j) Find the probability that the switch sends in a time slot exactly one packet on link OUT_1.

k) Find the probability that the switch sends in a time slot exactly one packet on link OUT_2.