# Principles of Computer Networks <br> Tutorial 1 

## Problem 1

Consider a transceiver unit which can send and receive data over a wireless communication link. The transceiver can be in one out of two possible states: state $S_{\text {off }}$ (when it is not sending/receiving data) and $S_{o n}$ (when it is sending/receiving data).

Assume that the transceiver changes states at discrete time steps $k=0,1,2,3, \ldots \ldots$ When the state of the transceiver is equal to $S_{\text {off }}$ at time $k$, then at time $(k+1)$ it will be in the state $S_{o f f}$ with probability $2 / 3$ and be in the state $S_{o n}$ with probability $1 / 3$. Similarly, when at time $k$ the transceiver is in state $S_{o n}$, then at time $(k+1)$ it is in state $S_{\text {off }}$ with probability $1 / 3$ and in state $S_{o n}$ with probability $2 / 3$. The state diagram of the transceiver is shown as:

(a) Assume that at time $k=0$ the transceiver is in state $S_{o f f}$. What is the probability that the transceiver is still in state $S_{o f f}$ at time $k=1$ ?
(b) Assume that at time $k=0$ the transceiver is in state $S_{o f f}$. Using (a), what is the probability that the transceiver is in state $S_{o n}$ at time $k=2,3,4$ ?
(c) Assume that the transceiver is at time $k=0$ in state $S_{o f f}$ with probability $P_{o f f}$ and in state $S_{o n}$ with probability $P_{o n}$. Express the probability that the transceiver is at time $k=1$ in state $S_{o f f}$, and state $S_{o n}$, as a function of $P_{o f f}$ and $P_{o n}$.
(d) Find initial probabilities $P_{o f f}, P_{o n}$ such that the transceiver is at time $k=1$ in state $S_{o f f}$, and $S_{o n}$, again with probability $P_{o f f}$, and $P_{o n}$, respectively.
(e) Assume that at time $k=0$ the transceiver is in state $S_{o f f}$. Using the result of (d), try to guess that probabilities that the buffer is in state $S_{o n}$ and $S_{o f f}$ after a very long time, i.e. as $k$ approaches infinity.

## Problem 2

The length of data packets can vary in a wide range (some packets are very short and some packets are very long). To capture this, we model packet lengths as a random variable with a geometric distribution. That is, the probability that a packet is $L$ bits long is given by.

$$
P(L=l)=\alpha(1-\alpha)^{l-1}, \quad l \geq 1 .
$$

(a) Derive the average packet length, i.e. derive $E[L]$.
(b) Consider a specific packet. Assume that we know that the length of this packet is larger than $l_{0}$. Find the probability that the packet is $l$ bits long, $l>l_{0}$.
(c) Derive the expected packet length when we know that $L>l_{0}$, i.e. derive $E\left[L \mid L>l_{0}\right]$.

