Principles of Computer Networks
Tutorial 1

Problem 1

Consider a transceiver unit which can send and receive data over a wireless communication link. The transceiver can be in one out of two possible states: state $S_{off}$ (when it is not sending/receiving data) and $S_{on}$ (when it is sending/receiving data).

Assume that the transceiver changes states at discrete time steps $k = 0, 1, 2, 3, \ldots$. When the state of the transceiver is equal to $S_{off}$ at time $k$, then at time $(k + 1)$ it will be in the state $S_{off}$ with probability $2/3$ and be in the state $S_{on}$ with probability $1/3$. Similarly, when at time $k$ the transceiver is in state $S_{on}$, then at time $(k + 1)$ it is in state $S_{off}$ with probability $1/3$ and in state $S_{on}$ with probability $2/3$. The state diagram of the transceiver is shown as:

(a) Assume that at time $k = 0$ the transceiver is in state $S_{off}$. What is the probability that the transceiver is still in state $S_{off}$ at time $k = 1$?

(b) Assume that at time $k = 0$ the transceiver is in state $S_{off}$. Using (a), what is the probability that the transceiver is in state $S_{on}$ at time $k = 2, 3, 4$?

(c) Assume that the transceiver is at time $k = 0$ in state $S_{off}$ with probability $P_{off}$ and in state $S_{on}$ with probability $P_{on}$. Express the probability that the transceiver is at time $k = 1$ in state $S_{off}$, and state $S_{on}$, as a function of $P_{off}$ and $P_{on}$.

(d) Find initial probabilities $P_{off}$, $P_{on}$ such that the transceiver is at time $k = 1$ in state $S_{off}$, and $S_{on}$, again with probability $P_{off}$, and $P_{on}$, respectively.

(e) Assume that at time $k = 0$ the transceiver is in state $S_{off}$. Using the result of (d), try to guess that probabilities that the buffer is in state $S_{on}$ and $S_{off}$ after a very long time, i.e. as $k$ approaches infinity.

Problem 2

The length of data packets can vary in a wide range (some packets are very short and some packets are very long). To capture this, we model packet lengths as a random variable with a geometric distribution. That is, the probability that a packet is $L$ bits long is given by:

$$P(L = l) = \alpha(1 - \alpha)^{l-1}, \quad l \geq 1.$$
(a) Derive the average packet length, i.e. derive $E[L]$.

(b) Consider a specific packet. Assume that we know that the length of this packet is larger than $l_0$. Find the probability that the packet is $l$ bits long, $l > l_0$.

(c) Derive the expected packet length when we know that $L > l_0$, i.e. derive $E[L \mid L > l_0]$. 