## Assigment 1

## 1

Suppose that a 2 Mbps link is being shared among a pool of users. Furthermore, assume that each user needs 200 kbps when transmitting, but only does so $1 \%$ of the time.

1. How many users can the link handle if we assume that circuit switching is used?
2. Now assume packet switching is used and suppose there are 50 users in the pool. Find the probability that at any given time, exactly 5 users are transmitting at the same time.
3. What is the probability there are 21 or more users transmitting simultaneously

## 2

Suppose two hosts are connected by direct link that is $5,000 \mathrm{~km}$ long and has a bandwidth of $R=1 \mathrm{Mbps}$. Suppose the propagation speed over the link is $2.7 * 10^{8} \mathrm{~m} / \mathrm{s}$.

1. What is the bandwidth-delay product $R \times d_{d r o p}$ ?
2. Consider sending a file of 50,000 bits from Host A to Host B. Suppose the file is sent continuously as one large message. What is the maximum number of bits that will be in the link at any given time?
3. What is the width (in meters) of a bit in the link?
4. Derive a general expression for the width of a bit in terms of the propagation speed s , the transmission rate R , and the length of the link m

## 3

Consider a communication link which connects A with B , and suppose that A sends to B a file with N bits. During the transmission, each bit is being corrupted independently with a bit error probability p. Assume that B can detect perfectly whether a bit got corrupted during the transmission. You can utilize $\sum_{k=1}^{\infty}(1-x)^{k-1}=\frac{1}{x}$. Give answers to the following questions.

- What is the probability $P_{0}$ that the file is received without an error at $B$ (i.e., no bit is getting corrupted during the transmission)?
- What is the probability $P_{m}$ that the file is received with $m$ bit errors?
- What is the probability $P_{t r}$ that the file is received with at most $m$ bit errors?
- Suppose that $B$ accepts the file when there are $m$ or less bit errors, and otherwise asks $A$ to resend the file. What is the probability $P_{k}$ that $A$ has to send the file $k$ times to get it accepted by B?
- What is the expected number of times that $A$ has to send the file to get it accepted at $B$ ?


## 4

Consider a buffer which can hold up to 2 packets. Assume that time is divided into time slots of length $\Delta t$ and consider the sequence of time slots indexed by $k=0,1,2, \ldots$. In each time slot, the buffer is in one of three possible states: $S_{0}$ (empty), $S_{1}$ (holds 1 packet), $S_{2}$ (holds 2 packets). In each time slot, one new packets arrive with probability $p_{a}$. When the buffer holds at least one packet, then one packet leaves the buffer during one time slot with probability $p_{l}$.

To simplify the analysis, we assume the following:

- In each time slot, only one of the following events can happen: (a) one packet arrives, (b) one packet leaves, or (c) no packet arrives or leaves (i.e., you can ignore the events that two or more packets arrive (leave) in one time slot, or that there is a packet arrival and departure occurring in one time slot).
- When the buffer is in state $S_{2}$ and a new packet arrives in a time slot, then this packet will be dropped (and the buffer stays in state $S_{2}$ ).
- The buffer is empty at time $k=0$.

Give the answers to the following questions as functions of $p_{a}$ and $p_{l}$.
(a) Draw the state transition diagram of the system. The diagram should show the states (as circles), the possible transitions between states (as arrows) and the probability of each transition (as functions of $p_{a}, p_{l}$ ).
(b) Find the probability that at time $k=1$ the buffer is still empty.
(c) Find the probability that at time $k=1$ the buffer holds one packet.
(d) Find the probability that at time $k=1$ the buffer holds two packets.
(e) Assume that we can not predict exactly the state of the buffer at time $k=0$, but we only know that the buffer is in state $S_{0}, S_{1}$, and $S_{2}$ with probability $P_{0}, P_{1}, P_{2}$, respectively. Express the probability that the buffer is in state $S_{0}, S_{1}$ and $S_{2}$ at time $k=1$ as a function of $P_{0}, P_{1}$, and $P_{2}$
(f) Again, assume that we can not exactly predict the state the buffer at time $k=0$, but we only know that the buffer is in state $S_{0}, S_{1}, S_{2}$ with probability $P_{0}, P_{1}, P_{2}$ respectively. Find initial probabilities $P_{0}, P_{1}, P_{2}$, such that the probability that the buffer is at time $k=1$ in state $S_{0}, S_{1}, S_{2}$, is again equal to $P_{0}, P_{1}, P_{2}$, respectively. Express the solution as a function of $\rho=\frac{p_{a}}{p_{l}}$. Note that in this case the buffer is in steady state, i.e. the probability of being in a certain state does not change as time evolves.
(g) Assume that at time $k=0$, the buffer is empty. Using the result of ( f ), try to guess the probability that the buffer is in state $S_{0}, S_{1}, S_{2}$, after a very long time (i.e., as $k$ approaches infinity).

Condiser the following MAN between Toronto and Ottawa. There is a gient and multiplexable link between the two switches repsenting Toronto and Ottawa respecitvely. Each city has N clients sharing this MAN.


- Assume it's Christmas Day, and client i from Toronto wants to send a "Merry Chrismas" email to client j from Ottawa. List all the kinds of delays this email would encounter, and specify which of the delays are constant and which are variable.
- Suppose the network in the picture is a digital telephone network. The data propagated through the network are framed into 1byte packages. The transmission rate between any client and the city switch is 64 Kbps and the transmission rate between the two cities is 192 Mbps .

1. How many frames can a client contribute to the network in 1s?
2. How many frames can be send between two switches in 1s?
3. Conclude what is the largest number of $N$ such that a client won't feel any delay, when he/she makes a phone call to any one from the other city.
4. What delay is omitted in the our discussion.
