**CSC148 Intro. to Computer Science**

**Lecture 6: Recursion**

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**Test #1 Average**

- 66%
- Sample solution: available in the course page
- Remark requests are accepted until June 20
- Some of you may not have had the best day
  - 50% vs 150%

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**Test #2 Preparation**

- Carefully reading previous terms solution?
- Carefully reading other problems solutions?
- Watching tutorials, videos, online lessons?
- Nothing helps as much as

  getting involved in solving problems prior to see their solution

- Take most advantage of Peer Instructions
  - Its optional
  - No re-mark option
  - Provides bonus points, and most importantly opportunity to grasp

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**Example 1: sum of a list**

```python
>>> L1 = [1, 9, 8, 15]
>>> sum(L1)
33
```

```python
>>> L2 = [[1, 5], [9, 8], [1, 2, 3, 4]]
>>> sum(L2)
?  # reuse: isinstance, sum, sum_list()
```

```python
>>> L3 = [[1, 5], 9, [8, [1, 2], 3, 4]]
```

**How can we sum L3?**

In general, how can we sum any list?

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**Review**

- Last lectures
  - Linked lists
  - Wrappers and helpers
- Today
  - Quick review of linked lists
  - Introduction to recursion
- Recall
  - Utilize office hours, forum, CS help centre
    - in addition to lectures and labs

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```python
def sum_list(L):
    '''(list or int) -> int
    Return L if it’s an int, or sum of the numbers
    in possibly nested list L
    >>> sum_list(17)
    17
    >>> sum_list([1, 2, 3])
    6
    >>> sum_list([1, [2, 3], [4], 5])
    12
    ... # reuse: isinstance, sum, sum_list()
    if isinstance(L, list):
        return sum(sum_list(x) for x in L)
    else:
        return L
```

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![Diagram](image-url)
To understand recursion, trace from simple to complex:

Trace sum_list(17)

Remember how the built-in sum works.

Immediately replace calls you've already traced by their value.

Example 2: depth of a list
Define the depth of L as follows.

If L is a list, 1 plus the maximum depth of L's elements, otherwise 0.
Recursion

Example 2: depth of a list

```python
>>> L1 = [1, 9, 8, 15]
>>> depth(L1)
3
>>> L2 = [[1, 5], [9, 8], [1, 2, 3, 4]]
>>> depth(L2)
3
>>> depth(12)
1
>>> L3 = [[1, 5], 9, [8, [1, 2], 3, 4]]
How can we calculate depth of L3?
How can we calculate depth of any list?
```

Tracing depth()

- Trace in increasing complexity; at each step fill in values for recursive calls that have (basically) already been traced
  - Trace depth([])

Tracing depth()

- Trace in increasing complexity; at each step fill in values for recursive calls that have (basically) already been traced
  - Trace depth(17)

Tracing depth()

- Trace in increasing complexity; at each step fill in values for recursive calls that have (basically) already been traced
  - Trace depth([3, 17, 1])

Tracing depth()

- Trace in increasing complexity; at each step fill in values for recursive calls that have (basically) already been traced
  - Trace depth([5, [3, 17, 1], [2, 4], 6])

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def depth(L):
    ''' (list or int) -> int
    Return 0 if it’s empty or an int, otherwise 1 + max of L’s elements
    >>> depth(17)
    0
    >>> depth([17])
    1
    >>> depth([1, [2, 3, [4]], 5])
    3
    # reuse:
    if isinstance(L, list):
        if len(L) == 0:
            return 0
        else:
            return 1 + max([depth(x) for x in L])
    else:
        return 0
Tracing depth()

- Trace in increasing complexity; at each step fill in values for recursive calls that have (basically) already been traced
- Trace depth([14, 7, [5, [3, 17, 1], [2, 4], 6], 9])

Example 3: find maximum in nested list

- how would you find the max of non-nested list?
  >>> max(...)

- how would you build that list using a comprehension?
  >>> max([...])

- what should you do with list items that were themselves lists?
  >>> max([max_list(x) ...])

- get some intuition by tracing through at lists, lists nested one deep, then two deep...

max_list()

```python
def max_list(L):
    ...
    if isinstance(L, list):
        return max([max_list(x) for x in L])
    else: # L is an int
        return L
```

Tracing max_list()

- Trace in increasing complexity; at each step fill in values for recursive calls that have (basically) already been traced
- Trace max_list([3, 5, 1, 3, 4, 7])

Tracing max_list()

- Trace in increasing complexity; at each step fill in values for recursive calls that have (basically) already been traced
- Trace max_list([4, 2, [3, 5, 1, 3, 4, 7], 8])
Tracing `max_list()`

- Trace in increasing complexity; at each step fill in values for recursive calls that have (basically) already been traced
- Trace `max_list([6, [4, 2, [3, 5, 1, 3, 4, 7], 8], 5])`

Example 4: get some turtles to draw

- Spawn some turtles, point them in different directions, get them to draw a little and then spawn again…
- Try out `tree_burst.py` from the course page
- Notice that `tree_burst` returns `NoneType`: we use it for its side-effect (drawing on a canvas) rather than returning some value.