UNIVERSITY OF TORONTO

Faculty of Arts and Science

MAY 2008 EXAMINATIONS

CSC 418/2504: Computer Graphics

Duration: 3 hours

Aids allowed: calculator

There are 17 pages

First name: ________________________________

Last name: ________________________________

Student number: ________________________________

Question	Marks
1	/24
2	/14
3	/10
4	/20
5	/7
6	/15
7	/10
Total	/100

1. [*6×4 marks*] Determine whether the following statements are *True* or *False*.

A. Every 3D vectors a, b, c satisfy $a \times (b \times c) = (a \times b) \times c$ ________________

B. Consider three straight lines under perspective projection. If the lines have the same vanishing point in the image, then the three 3D lines must be co-planar (in the same 3D plane) ________________

C. The image of a 3D square under orthographic projection is always a parallelogram (assuming the square is not projected to a line) ________________

D. Specifying the eye point, lookat direction, and up vector completely determines a projective camera transformation ________________

E. If a surface is transformed by a rotation, transforming its normal vector by the same rotation will leave it perpendicular to the surface ________________

F. An animation system based on forward kinematics provides the animator more control than a system based on inverse kinematics ________________

2.

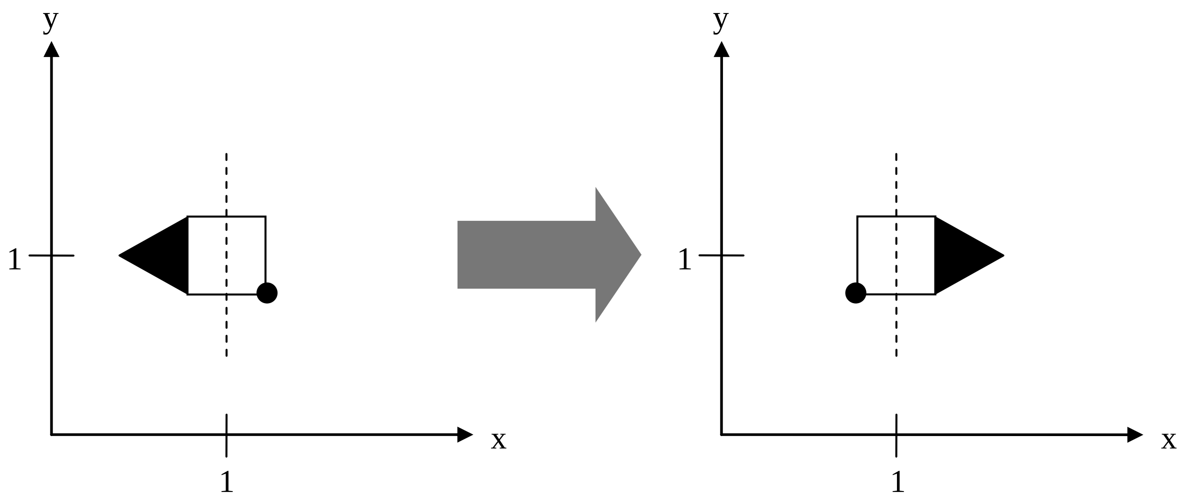
A. [*6 marks*] Throughout the course we encountered the concept of *interpolation* in various situations. Name three different types of interpolation methods and the topic (i.e. the application) where they were used.

Type of interpolation	Topic
______________________	______________________
______________________	______________________
______________________	______________________

B. [*4 marks*] If we draw black lines on a white background using Bresenham's algorithm with no anti-aliasing, does a 45° (diagonal) line look brighter, the same, or dimmer than one drawn horizontally?

C. [*4 marks*] What is roughly the memory overhead of using mipmaps for texture mapping, assuming the size of the texture image is K bytes? In deriving your estimate you are allowed to make any reasonable assumption. State your assumptions.

3. [*10 marks*] Find a 3×3 transformation matrix for transforming homogeneous coordinates in the plane that performs the transformation shown below.



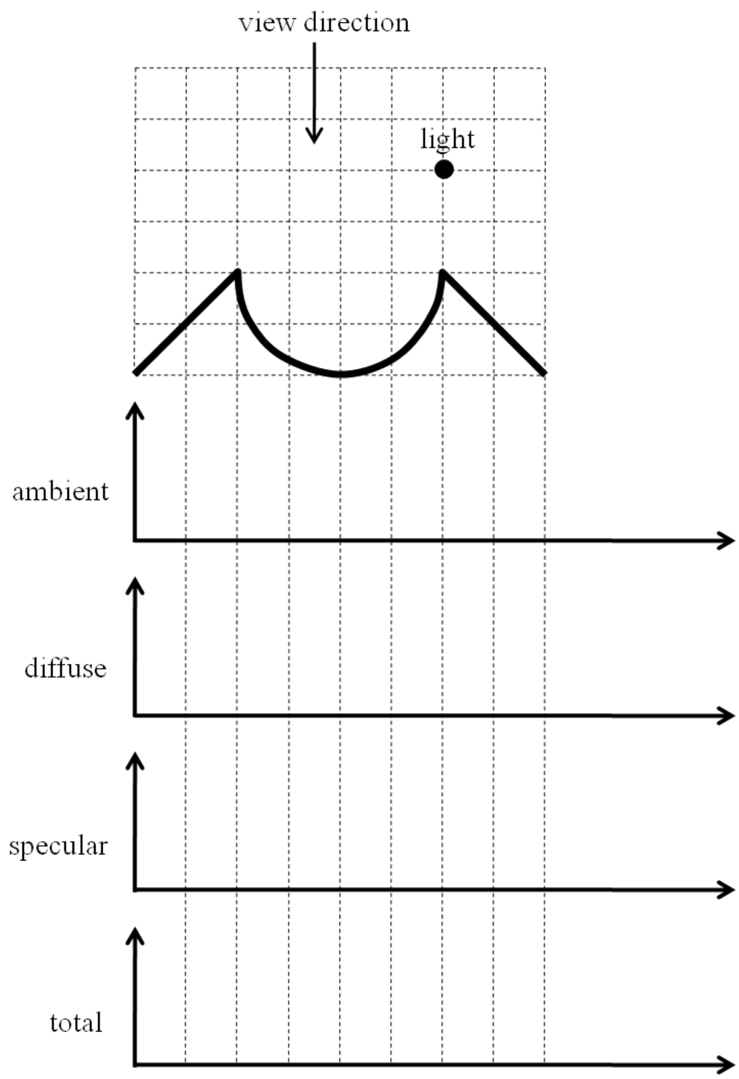
4. [*20 marks*] This question is about the Phong illumination model. For simplicity assume a 1D surface in a 2D world, as shown below. The viewpoint is far above the scene looking straight down (as in orthographic projection), and the position of the light source is specified in the diagram. Sketch three graphs showing the ambient, diffuse and specular components of the Phong model as applied to the scene. In a fourth graph, show the total intensity that would be computed (the graphs should show the intensity as a function of x).

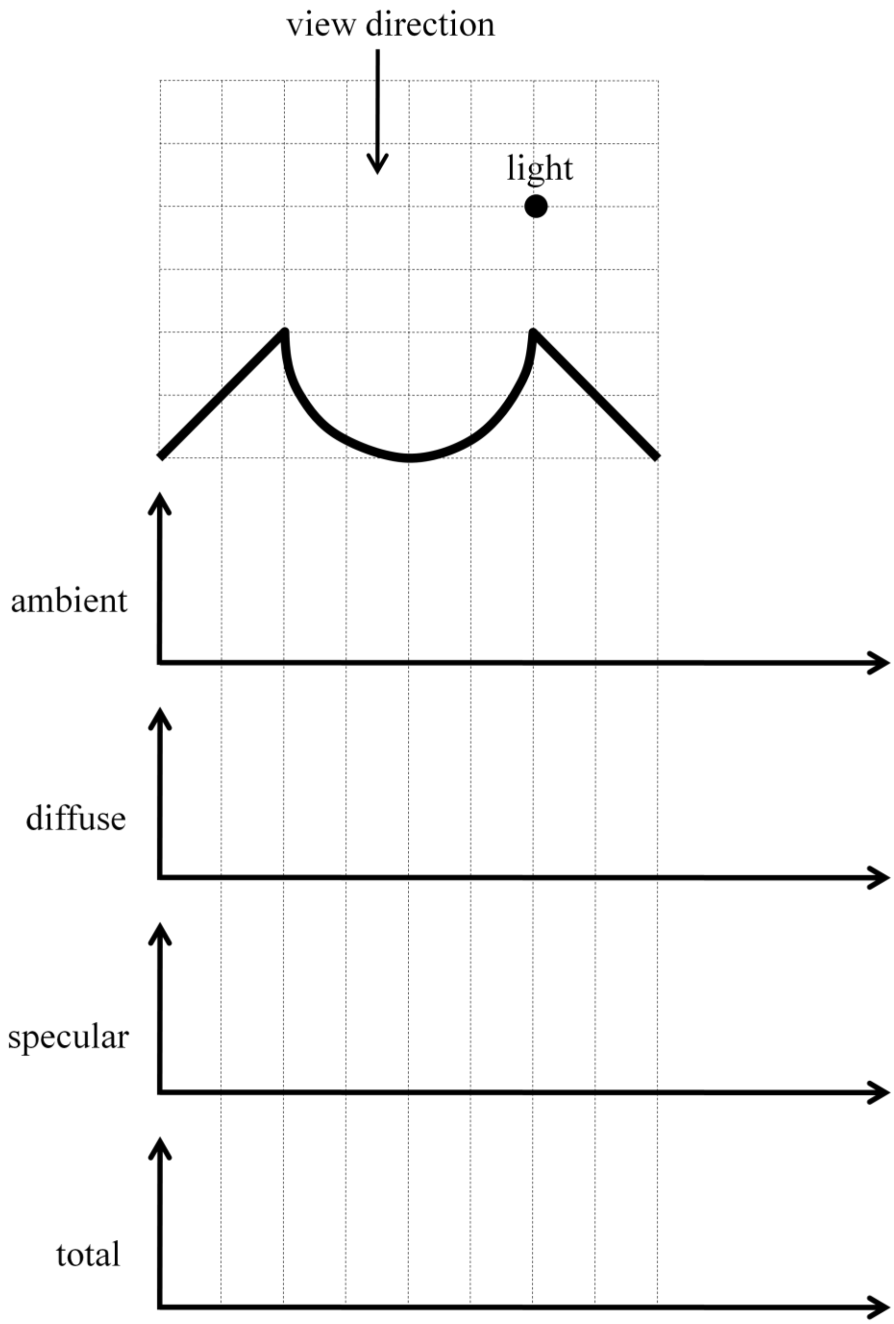
For your convenience two sets of graphs are provided. Use the first for a draft and draw the final answer as accurate as you can on the second set on the next page.

The parameters for this scene are:

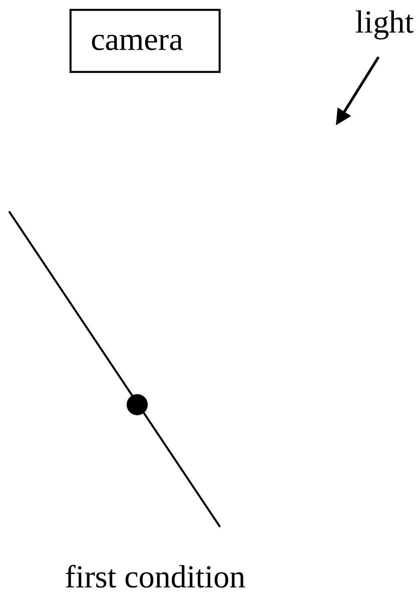
$I = k_a + k_d(N \cdot L) + K_s(R \cdot V)^n$

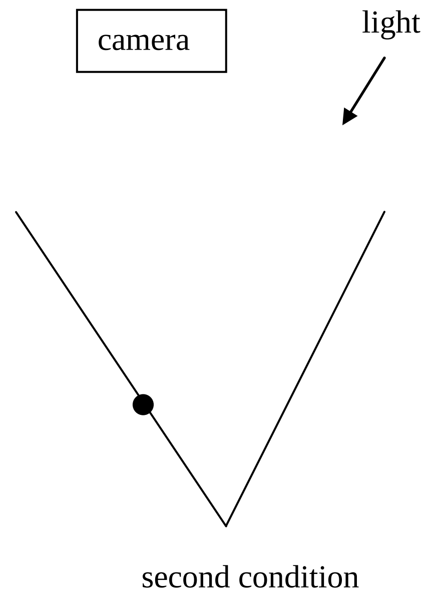
$k_a = 0.2, \quad k_d = 0.6, \quad k_s = 0.6, \quad n = 100$




view direction
light
ambient
diffuse
specular
total

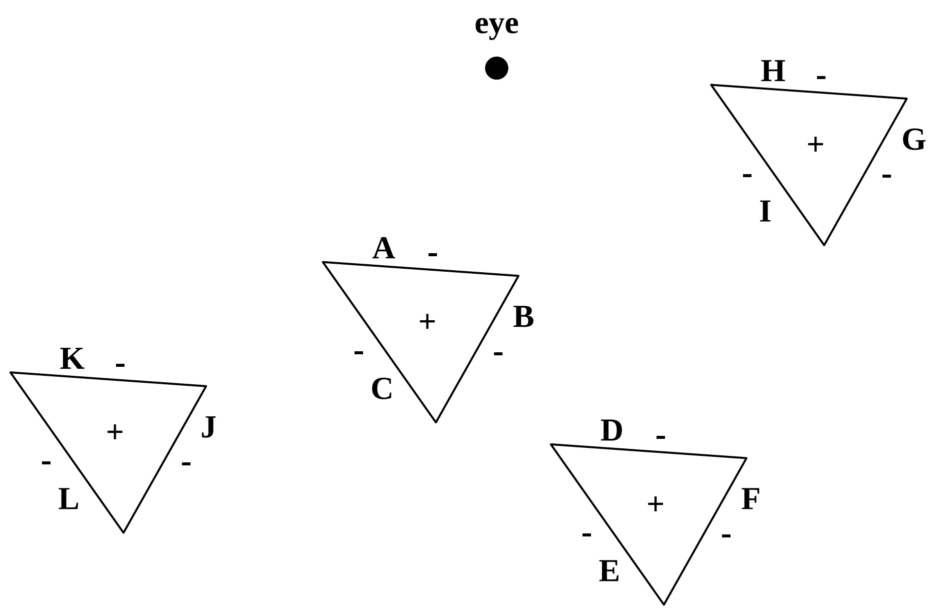
5. [*7 marks*] Suppose we perform the following *physical experiment*. In the first condition, a plane made of a Lambertian material (which is not black) is illuminated by a directional light source and the intensity of a point is recorded. In the second condition, we attach a second plane of the same material (that does not block the light), and record the intensity of the same point under the same illumination. Is the intensity of the point as seen by the camera the same in both conditions? If not, is it brighter under the first or second condition? Explain briefly.





6. [*3×5 marks*]

A. Build a BSP tree for the scene below. Insert the segments in alphabetical order. For each segment use the convention that the positive half-space is the half-space that contains the triangle (denoted by "+" in the diagram). Place the negative half-spaces to the left of each node, and the positive half-spaces to the right.



B. Use the BSP tree to construct a back-to-front ordering of the segments for the eye viewpoint.

C. In this question we compare two algorithms for hidden surface removal: Z-buffer and BSP. Explain which technique is more suited to support α-channel compositing of triangles whose projections overlap.

7. [*10 marks*] Recall that Catmull-Rom splines are computed using the formula

$$x(t) = \begin{bmatrix} 1 & t & t^2 & t^3 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -3 & 3 & -2 & -1 \\ 2 & -2 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_j \\ x_{j+1} \\ \kappa(x_{j+1} - x_{j-1}) \\ \kappa(x_{j+2} - x_j) \end{bmatrix}$$

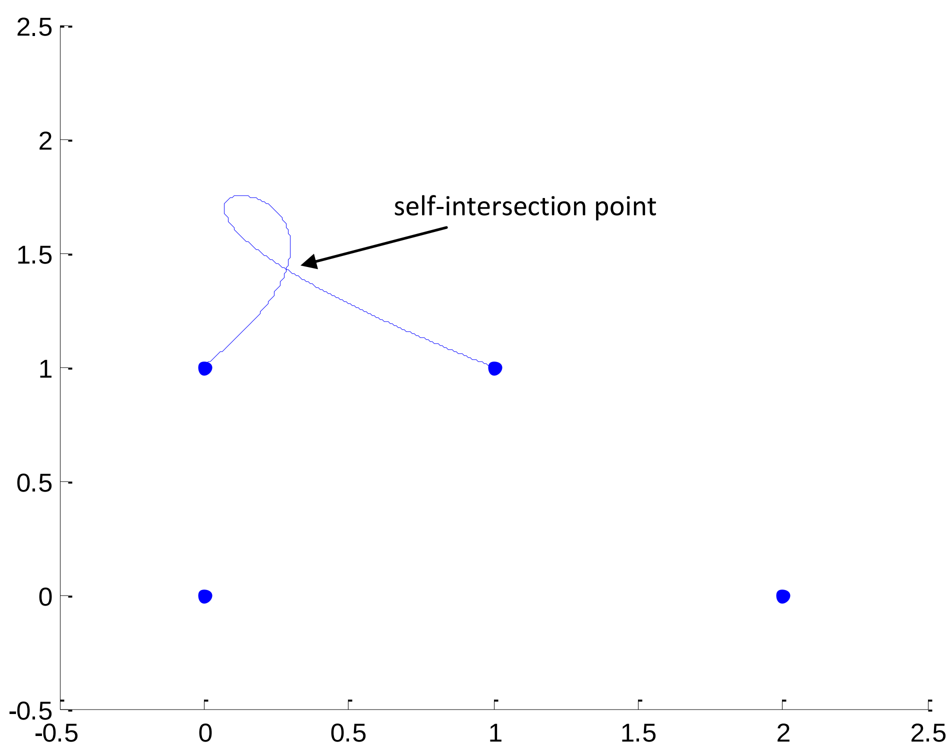
$$= \begin{bmatrix} 1 & t & t^2 & t^3 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\kappa & 0 & \kappa & 0 \\ 2\kappa & \kappa - 3 & 3 - 2\kappa & -\kappa \\ -\kappa & 2 - \kappa & \kappa - 2 & \kappa \end{bmatrix} \begin{bmatrix} x_{j-1} \\ x_j \\ x_{j+1} \\ x_{j+2} \end{bmatrix}.$$

Similar formula applies to $y(t)$.

In the figure below a segment of Catmull-Rom spline was computed using the parameters

$p_1 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$, $p_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$, $p_3 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$, $p_4 = \begin{pmatrix} 2 \\ 0 \end{pmatrix}$

$k = 3$



As you can see, the spline is self-intersecting. Find the coordinates of the self-intersection point.
Hint: the following formulas could be useful

$(a + b)(a - b) = a^2 - b^2$
$(a - b)(a^2 + b^2 + ab) = a^3 - b^3$

END OF EXAM