CSC 418/2504 Computer Graphics, Winter 2008

Assignment no. 1. Due: January 21 (beginning of Tutorial)

1. The cross product of two vectors in 3D is given by the formula:

$$a \times b = \begin{pmatrix} a_x \\ a_y \\ a_z \end{pmatrix} \times \begin{pmatrix} b_x \\ b_y \\ b_z \end{pmatrix} = \begin{pmatrix} a_y b_z - a_z b_y \\ a_z b_x - a_x b_z \\ a_x b_y - a_y b_x \end{pmatrix}$$

   A. Using the formula, show that a, b are orthogonal to $a \times b$.

   B. Using the formula, show that $\|a \times b\| = \|a\| \cdot \|b\| \cdot |\sin \varphi|$, where φ is the angle between the two 3D vectors a, b.

   C. Compute the area of the triangle whose vertices are $(1,2,3)^T, (2,3,1)^T, (3,1,2)^T$.

2. Consider a triangle in 3D with vertices at $(1,0,0)^T, (0,1,0)^T, (0,0,1)^T$. For each of the following line segments, determine whether the segment intersects the triangle, and if so find the intersection point. Explain every detail in your calculations (as if the procedure is run by a computer).

   A. Segment between $(1,1,1)^T$ and $(0.5,0.5,0.5)^T$.

   B. Segment between $(0,0,2)^T$ and $(1,1,-3)^T$.

   C. Segment between $(1,1,1)^T$ and $(0,-1,1)^T$.

3. A. Two intervals $[x_1,x_2]$ and $[x_3,x_4]$ on the real line overlap if they have at least one point in common. Explain how to determine whether two intervals overlap using as few comparisons (if statements) as you can. You may assume $x_1 \le x_2$, $x_3 \le x_4$.

   B. Checking whether objects intersect is a common operation in computer graphics. For example, in animations collision detection is performed to prevent objects from going through one another. To speed up the computation in situations where the scene has a large number of complex objects that don't intersect, it is common to bound the objects with axis-aligned boxes and check the intersection of the boxes first (an axis aligned box has sides parallel to one of the principle planes xy, yz, or xz, and can be described using two opposite corners $[x_1,y_1,z_1]$ and $[x_2,y_2,z_2]$, where $x_1 \le x_2$, $y_1 \le y_2$, $z_1 \le z_2$). If the boxes do not intersect we achieve quick rejection. Describe how to determine whether two axis-aligned boxes intersect using as few comparisons as you can.

4. A. Consider a linear transformation in 3D that maps the point $(1,3,0)^T$ to $(1,3,4)^T$, $(-3,1,0)^T$ to $(-3,1,-2)^T$ and $(0,0,10)^T$ to $(10,10,0)^T$. Where is the point $(1,1,1)^T$ mapped to under this transformation?

   B. Consider an affine transformation in 3D that maps the point $(1,1,1)^T$ to $(8,7,7)^T$, $(2,1,0)^T$ to $(6,8,8)^T$, $(1,3,1)^T$ to $(12,13,9)^T$, and $(2,1,2)^T$ to $(12,10,12)^T$. Where is the point $(1,2,3)^T$ mapped to under this transformation?