Shape from planar curves: a linear escape from Flatland
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Introduction
We revisit the problem of recovering 3D shape from a single photo containing planar curves. Previous work [1,2,3] did not explicitly explore the space of flat solutions and its orthogonal complement

For intersecting planar curves, we study the space of solutions and derive a stable linear method
For parallel planar curves, we demonstrate special cases where the shape is similarly solved
Our work unifies relevant literature on shape from contour, single view modeling and structured light

Intersecting curves
Consider planar curves
\( \alpha(x, y) = ax + by + d \)
At intersection points
\( \alpha(x_1, y_1) - \alpha(x_2, y_2) = (a_1 - a_2)x_1 + (b_1 - b_2)y_1 + (d_1 - d_2) = 0 \)
The homogenous linear system in vector form
\[
\begin{align*}
\mathbf{Av} &= 0 \\
\mathbf{v} &= (a_1, \ldots, a_N, b_1, \ldots, b_N, d_1, \ldots, d_N)^T
\end{align*}
\]
Let \( \mathbf{C} \) be a matrix so that \( \mathbf{Cv}^2 \) is the linear regression residue of fitting a plane to a sample of 3D points on the curves

\[
\mathbf{C}v = \begin{bmatrix}
\begin{array}{ccc}
\alpha & \beta & \gamma \\
\alpha_1 & \beta_1 & \gamma_1 \\
\vdots & \vdots & \vdots \\
\alpha_N & \beta_N & \gamma_N 
\end{array}
\end{bmatrix}
\]

parallel curves
sampled points
computed 3D surface

Global planarity
For curves arranged around a principle plane, the shape might be a singular vector of \( \mathbf{C} \)
\( \|\mathbf{Rv}\|^2 = \lambda(\text{Var}(a_i) + \text{Var}(b_i)) \)

Local planarity
Reduce to the intersecting case by assuming virtual planar facets

References