A Small World Threshold for Economic Network Formation & A Network Formation Game for Bipartite Exchange Economies

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Social Networks Seminar Fall 2007

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Introduction

- Structural properties in social networks

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- Decentralized network formation

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    - price of anarchy

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- The joint action of all players is $a = a_1 \times \cdots \times a_n$.
- The joint action a defines an undirected graph $G(a) = (V, E(a))$, where $V = \{v_1, \ldots, v_n\}$ and $E(a) = \cup_{i \in \{1,\ldots,n\}} E_i(a_i)$, $E_i(a_i) = \{(v_i, v_j) \mid a_i(j) = 1\}$

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Overall cost function c_i of player v_i is defined as

$$c_i(a) = c_i(a_i, a_{-i}) = \sum_{e \in E_i(a_i)} c(e) + \sum_{j=1}^{n} \Delta_{G(a)}(v_i, v_j)$$

where $\Delta_{G(a)}(u, v)$ is the shortest distance between u and v in $G(a)$.

Nash Equilibrium

Definition

A joint action $a = a_1 \times \cdots \times a_n$ is a *Nash equilibrium* if for every player v_i and any alternative action $\hat{a}_i \in \{0, 1\}^n$

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If a is a Nash equilibrium, we say that $G(a)$ is an equilibrium graph.

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Note: an equilibrium graph is a link stable graph.

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  - probability & cost both in a power law form

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Theorem

For any constant $\epsilon > 0$, if $\alpha = 2 - \epsilon$, then there exists a constant $c(\alpha)$ such that for any n, all Nash Equilibria or link stable graphs over n players have diameter at most $c(\alpha)$.

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  - if $\alpha = 1$, then $\Delta_{G(a)}(u, v) \leq 2\lceil C^2 + 4 \rceil$.

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Exchange Equilibrium

Let ω_j^s denote the exchange rate (or price), in terms of cash per unit wheat, that seller s_j is offering; Let ω_i^b denote the exchange rate, in terms of wheat per unit cash, that buyer b_i is offering. Let x_{ij} denote the amount of seller s_j's wheat that buyer b_i consumes.

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2. For each buyer b_i, the consumption plan $\{x_{ij}\}$ is optimal. That is, $x_{ij} > 0$ if and only if $\omega_j^s = \min_{s_k \in N(b_i)} \omega_k^s$.

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- The equilibrium exchange rates are unique, and at equilibrium, if $x_{ij} > 0$ then $\omega_j^s = 1/\omega_i^b$.
- We call each sellers(buyers) exchange rate $\omega_j^s(\omega_i^b)$ her *wealth*. There is *no wealth variation* at exchange equilibrium of a bipartite exchange economy when the wealth of all sellers are equal and the wealth of all buyers are equal.

Exchange Subgraph

Definition

Let $G = (B, S, E)$ be a bipartite exchange economy. Let $\{\omega_i^b\}$, $\{\omega_j^s\}$, and $\{x_{ij}\}$ be an exchange equilibrium. Then, the *exchange subgraph* of G is $G' = (B, S, E')$, where $E' = \{(i, j) \mid x_{ij} > 0\}$.

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- The exchange subgraph may not be unique.
- Exchange subgraph G' is minimal if the removal of any edge in G' from G changes the exchange equilibrium wealths.
- *Trading components*: the connected components of the exchange subgraph. We say that a trading component is (m, k) if there are m buyers and k sellers.

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- Other works have studied how the topology of G affects the variation in prices.
- The interest of this work is to allow players to construct the graph and to characterise the generated networks and the wealth variation.

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- Joint action of all players: $a = a_1^b \times \cdots \times a_n^b \times a_1^s \times \cdots \times a_n^s$, which defines a bipartite graph, $G(a) = (B, S, E)$, where E is the set of edges that the players bought.

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- $E_i^b(a) = \{(b_i, s_j) \mid a_i^b(j) = 1\}$ the set of edges b_i buys; $E_j^s(a) = \{(b_i, s_j) \mid a_j^s(i) = 1\}$ the set of edges s_j buys. Then, $E = \bigcup_{i \in [n], t \in \{b,s\}} E_i^t(a)$.

Nash Equilibrium

- Graph $G = G(a)$ defines a bipartite exchange economy, and an exchange equilibrium of the bipartite exchange economy determines the wealth each player earns, denoted by $\omega_i^t = \omega_i^t(G)$. The utility of each player is her wealth minus the cost for the edges she bought.

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- *Equilibrium graph*: the graph induced by a Nash equilibrium action a.

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The intuition: if redundant edges existed, the nodes that purchased them can remove them from the graph without affecting their wealth, and thus it is not a Nash equilibrium.

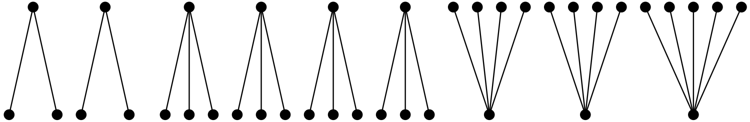
Structure of Equilibrium Graphs

Three types of graphs:

- *Perfect Matching* The class of all perfect matchings between the buyers and sellers. All exchange rates or wealths are equal to 1.

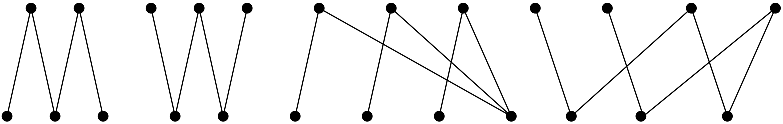
Structure of Equilibrium Graphs

- *Exploitation Graphs* Every trading component has a single player of one type "exploiting" a larger set of players of the other type. For any $k, l > 1$, G consists of the union of n_1 $(1, k)$, n_2 $(1, k+1)$, n_3 $(l, 1)$, and n_4 $(l+1, 1)$ trading components. At most 4 different seller wealth values: $1/k$, $1/(k+1)$, l, and $l+1$.



Structure of Equilibrium Graphs

- *Balanced Graphs* For any $k > 2$, G consists of the union of n_1 trading components either $(k-1, k)$ or $(k, k+1)$ and n_1 trading components either $(k, k-1)$ or $(k+1, k)$. At most 4 different seller wealth values: $k/(k-1)$, $(k+1)/k$, $(k-1)/k$, and $k/(k+1)$.



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Let $NE(n, \alpha)$ be the set of all Nash equilibrium graphs of the network formation game for a fixed population size n and edge cost α, and let NE be the union of $NE(n, \alpha)$ over all n and α. Then the set NE equals the union of classes Perfect Matchings, Exploitation Graphs, and Balanced Graphs.

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Proof.

- relate the edge cost α to the minimum exchange equilibrium wealth in any equilibrium graph

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- These bounds constrain the possible equilibrium graphs
- The remaining possibilities can all be realized

Discussion

Limitations in game-theoretical models

- one-shot, simultaneous move game
- how to reach a Nash equilibrium
- simulation results (Even-Dar and Kearns)
- convergence?
- approximation?
- bounded rationality: *satisfice* rather than *maximize* utility