On the Topologies Formed by Selfish Peers

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Social Networks Seminar Fall 2007

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Introduction

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  - Collaboration of peers to build scalable decentralized systems
  - e.g. BitTorrent for file distribution

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- Problem: many peers are *selfish*
  - Minimize own cost
- Use a *game-theoretic* approach to study the effect of selfish peers on the topology of a P2P system
  - Price of anarchy: $\Theta(\min(\alpha, n))$
  - Topologies may not converge to a stable state (Nash equilibrium)

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  - π_i maintains or establishes a link to π_j if $\pi_j \in s_i$, where $s_i \in S_i$
  - $s = (s_0, \ldots, s_{n-1}) \in S_0 \times \cdots \times S_{n-1}$ yields a directed graph $G[s] = \left(V, \bigcup_{i=0}^{n-1} (\{\pi_i\} \times s_i)\right)$ describing the P2P topology

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In plain English, each peer decides whether to establish a direct link to each of its other peers
$\Rightarrow$ a directed graph with peers as nodes and links as edges

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$$c_i(s) = \alpha \cdot |s_i| + \sum_{i \neq j} stretch_{G[s]}(\pi_i, \pi_j)$$

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  - α is a parameter measuring the relative importance between node degree and stretch in the cost function

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- Price of anarchy =

$$\frac{\text{social cost of worst NE}}{\text{social cost of optimal topology}}$$

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Theorem 4.1 (Upper bound)

For any metric space $\mathcal{M}$, the Price of Anarchy is $O(\min(\alpha, n))$

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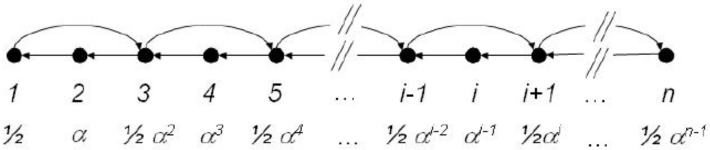

Figure 1: Example topology G where the Price of Anarchy is $\Theta(\min(\alpha, n))$ for $3.4 \le \alpha$.

Lower Bound

Lemma 4.2

The topology G shown in Figure 1 forms a Nash equilibrium for $\alpha \geq 3.4$

Lemma 4.3

The social cost $C(G)$ of the topology G shown in Figure 1 is $C(G) \in \Theta(\alpha n^2)$

Theorem 4.4

The Price of Anarchy of the peer topology G shown in Figure 1 is $\Theta(\min(\alpha, n))$

Main Theorem

Theorem 5.1

Regardless of the magnitude of α, there are metric spaces $\mathcal{M}$, for which there exists no pure Nash equilibrium, i.e. certain P2P networks cannot converge to a stable state. This is the case even if $\mathcal{M}$ is a 2-dimensional Euclidean space.

Illustration

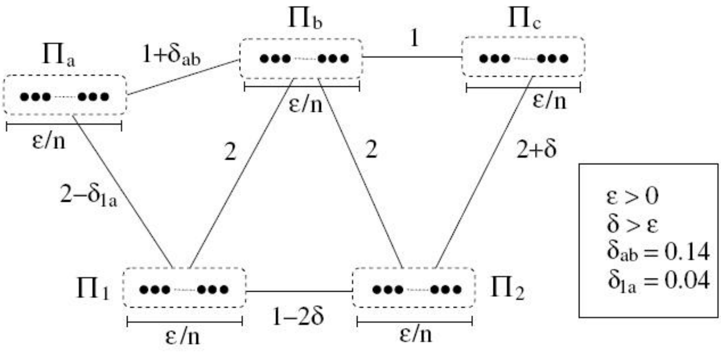

Figure 2: Instance I_k has no pure Nash equilibrium when $\alpha = 0.6k$, where $k = n/5$. The number of peers in each cluster is k.

Complexity

Theorem 6.1

Regardless of the magnitude of α, determining whether a given P2P network represented by a metric space $\mathcal{M}$ has a pure Nash equilibrium (and can therefore stabilize) is NP-hard.

Conclusion

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- Game-theoretic model of a P2P system
- Bounds of Price of Anarchy
- (Non-)Existence of Nash equilibria

Implications of Selfish Neighbor Selection in Overlay Networks

N. Laoutaris, G. Smaragdakis, A. Bestavros and J. Byers

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Scope and Contributions

- More practical model: for P2P and overlay routing application
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- Wiring Strategy: "Best Response"
- Experimental Results

Definitions

- Set of nodes: $V = \{v_1, v_2, \dots, v_n\}$
- Edge $e = \{v_i, v_j\}$: *Directed and Weighted*
- Preference vector: $p_i = \{p_{i1}, p_{i2}, \dots, p_{ii-1}, p_{ii+1}, p_{in}\}$
- Wiring of v_i: $s_i = \{v_{i_1}, v_{i_2}, \dots, v_{i_{k_i}}\}$
- Global Wiring: $S = \{s_1, s_2, \dots, s_n\}$
- Cost between v_i, v_j over S: $d_S(v_i, v_j)$
- Cost of v_i under S: $C_i(S) = \sum_{j=1, j \neq i}^{n} p_{ij} \times d_S(v_i, v_j)$

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  (Nash Equilibrium of the SNS game)

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- BR for v_i under S_{-i}, an *Integer Linear Program*
  - Minimize:

$$C_i(S_{-i}, X) = \sum_{j=1, j\neq i}^{n} p_{ij} \sum_{l=1, l\neq i}^{n} X_{lj} \times (d_{il} + d_{S_{-i}}(v_l, v_j))$$

  - Subject to:

$$\sum_{l=1, l\neq i}^{n} X_{lj} = 1, \forall j \neq i \textit{ and } \sum_{l=1, l\neq i}^{n} Y_l = k_i \textit{ and } X_{ij} \leq Y_l, \forall l, j \neq i$$

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To Find Stable Wirings

- **Iterative Best Response**

  At the m-th iteration, for all nodes, do

  1. v_i computes its best response $s_i^{(m)}$ to $S_{-i}^{(m,i-1)}$
  2. $S^{(m,i)} = S_{-i}^{(m,i-1)} + \{s_i^{(m)}\}$
  3. Do this until iteration $\mathcal{M}$: $s_i^{(\mathcal{M})} = s_i^{(\mathcal{M}-1)}$, $\forall v_i \in V$

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  - $\beta \in [0, 1]$: link density
    - Out-degree of each node: $k = \lceil n^{\beta} \rceil$

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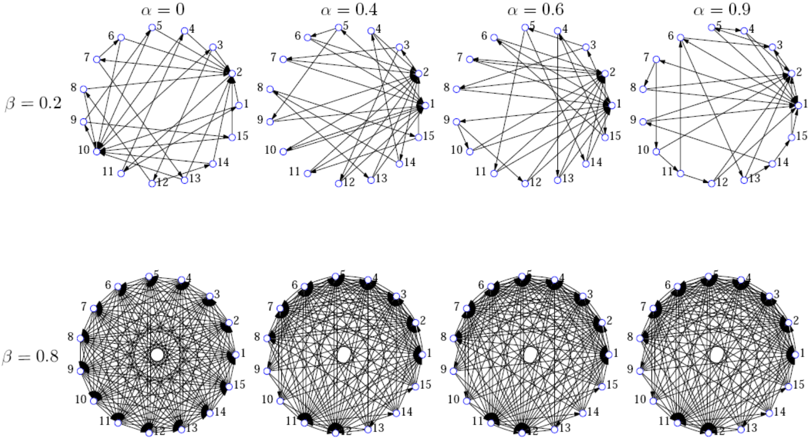
Note: "Hubs"



Performance of Newcomers: the cost

- Three kinds of Wiring Strategies
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- Newcomers using different strategies entering graphs constructed by different strategies.

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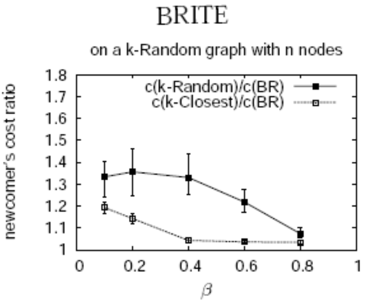
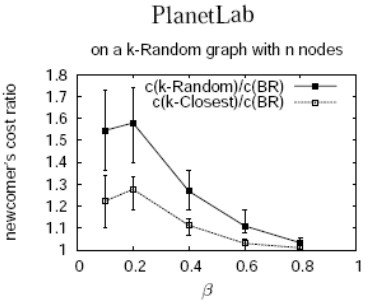
Newcomers to a *k*-random graph





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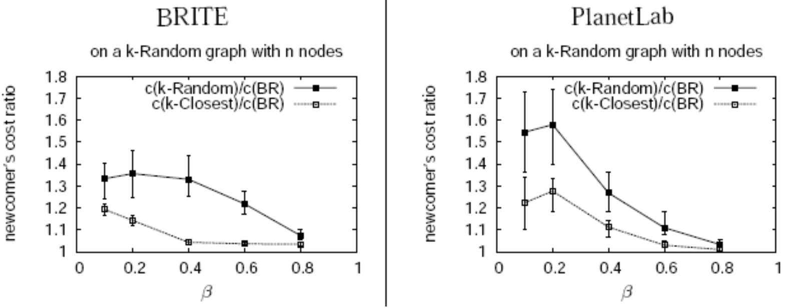

It pays to "cheat".

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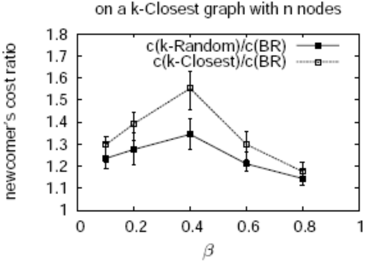
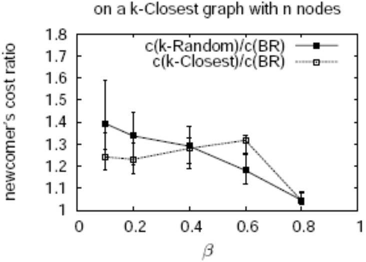
Newcomers to a *k*-closest graph





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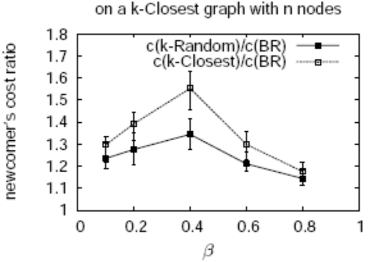

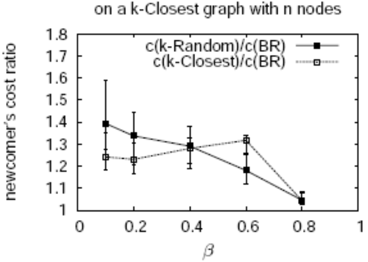

It does NOT pay to cheat, it may even hurt!

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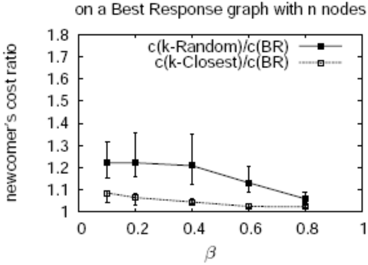

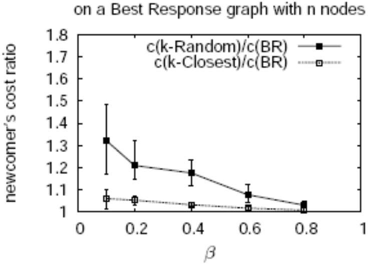

Knowledge Gained

- For prior arrivals: it is good to be selfish.
- For newcomers: should be careful.

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  - **B**: The optimization under certain restrictions

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