Efficient and Accurate Numerical Solutions for Helmholtz Equation

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This is a joint work with Kun Wang

The Helmholtz equation arises in many problems related to wave propagations, such as acoustic, electromagnetic wave scattering and models in geophysical applications. Developing efficient and highly accurate numerical schemes to solve the Helmholtz equation at large wave numbers is a very challenging scientific problem and it has attracted a great deal of attention for a long time. The difficulties in solving the Helmholtz equations are due to the construction of accurate numerical schemes for the equation and the boundary conditions, and efficient and robust numerical algorithms to solve the resulting indefinite linear systems. Moreover, it is a challenge to derive a numerical scheme which is capable of eliminating or minimizing the pollution effect. The pollution effect is the foremost difficulty which causes a serious problem as the wave number increases. Let k, h, and n denote the wave number, the grid size and the order of a finite difference or finite element approximations, then we could show that the relative error is bounded by $k^{\alpha}(kh)^{n}$, where $\alpha = 2$ or 1 for a finite difference or finite element method. It is clear that even using a fixed h with $kh < 1$, the error increases with k unless we apply a very fine mesh h such that $h < 1/k^{2}$. However, this will lead to an enormous size of ill-conditioned and indefinite system of linear equations. In this talk, we present a new finite difference scheme with an error estimate given by $k(h)^{2n}$ for a one-dimensional problem and the schemes are pollution free. Extension and numerical simulations for multi-dimensional problems will also be reported.