STA 410/2102 — Practice Questions for First Test

Please note that these questions do **not** cover all the topics that may be on the test.

1. For each of the following true statements about arithmetic on real numbers, give an example showing that it is not necessarily true for computer arithmetic on floating-point numbers in which the results are rounded to three decimal digits (ie, the mantissa consists of three decimal digits). In your examples, assume that overflow and underflow do not occur. (In other words, these should be examples of the effects of round-off error, not of the effects of overflow or underflow.)

   (a) If $x \neq y$ then $x + z \neq y + z$.

   (b) $x + (y + z) = (x + y) + z$.

   (c) If $x \neq 1$ and $x \neq 0$ then $x^2 \neq x$.

2. Write an R function to sum the first n terms of the following series:

   $$1 - \frac{1}{4} + \frac{1}{9} - \frac{1}{16} + \frac{1}{25} - \cdots$$

   The ith term of this series is $(-1)^{i+1}/i^2$.

   The function should take n as an argument, and should return the sum of the first n terms of the series as its value. Write the function so that it returns as accurate an answer as can be easily obtained. Although it might be most efficient to use the `sum` function built in to R, for this question, do *not* use `sum`, but instead write a loop. Otherwise, make the function as efficient as you reasonably can.

3. Consider the distribution on $(1, \infty)$ with density function $f(x) = 1/x^2$. Write an R function that takes as its argument a positive integer and which returns a vector of that length containing independent pseudo-random variates generated from this distribution. Show how you derived the formula or formulas required.

4. Find the Cholesky decomposition of the following matrix:

   $$\begin{bmatrix} 9 & -3 & 6 \\ -3 & 2 & -3 \\ 6 & -3 & 6 \end{bmatrix}$$

5. When writing a program, it is sometimes necessary to determine whether two floating-point numbers, held in variables `x` and `y`, have the same sign, or different signs, and do different things accordingly. (Assume that it makes no difference which thing is done when one or both variables are zero.) Here is a way that you might consider testing for this:

   ```
   if (x*y>0)
      Do things appropriate for when the signs are the same
   else
      Do things appropriate for when the signs are different
   ```

   Why is this not a reliable way of accomplishing this task?