CSC 2541, Small exercise #4, due in class February 28, worth 5% of the mark

Consider a Bayesian linear basis function model for the response associated with a single input, x, in which the basis functions are $\phi_0(x) = 1$ and $\phi_j(x) = \gamma \exp(-(x-\mu_j)^2/(2s^2))$, for $j = 1, 2, 3, \ldots$

Let the prior for β_0 be $N(0, \omega_0^2)$, and let the prior for all the β_j for $j = 1, \ldots, M-1$ be $N(0, \omega_j^2)$. (All these β_j are independent in the prior.)

Suppose that for a particular M, we independently draw μ_j for $j = 1, \ldots, M-1$ from the uniform distribution on the interval $(-\sqrt{M}/2, \sqrt{M}/2)$, and that we set all ω_j^2 for $j > 0$ to $1/\sqrt{M}$.

Find the limit of the covariance function that this setup defines as M goes to infinity. In other words, the limit, for any x and x', of

$$K(x, x') \quad = \quad \sum_{j=0}^{M-1} \omega_j^2 \phi_j(x) \phi_j(x')$$