CSC 486/2502, 2000 – Assignment 3

Due: Friday, December 1

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1. [**40 Marks**] (Default Reasoning)
   Consider the following statements:

   Echidnas are mammals.
   Mammals typically don't lay eggs.
   Echidnas usually lay eggs.
   Edwina is a mammal.
   Edwina is an echidna.

   (a) Represent these assertions in an inheritance network and determine what conclusion can be drawn about whether Edwina lays eggs.

   (b) Use default logic to represent the assertions above. Formalise any default assertions using *normal* default rules. What conclusions can be drawn (remember to show the default extensions that you obtain)?

   (c) Replace your default rule for "mammals typically don't lay eggs" with another *normal* default rule so that you draw the same conclusion as Question 1a.

   - Suppose the following assertions are now added to the KB:

     Eddy is an echidna.
     Eddy does not lay eggs.
     Either Edna or Elvis is an echidna.

   (d) Retaining your default rules from Question 1c, add the assertions above to your set of facts. In the resulting default logic, what conclusions can be drawn?

   (e) Now, suppose we replace the default rule representing the assertion "echidnas usually lay eggs" with the following rule in default logic:
   $$\langle \mathsf{TRUE} \Rightarrow (echidna(x) \supset lays\_eggs(x)) \rangle.$$
   What conclusions can be drawn in this case?

   (f) Formalise both sets of assertions given in English above using FOL together with appropriate Ab predicates. Adding a formula that says that echidnas are abnormal mammals, use circumscription (minimal entailment) to minimise abnormal echidnas *only*, what conclusions can you draw?

   (g) Using autoepistemic logic, we could represent the default "echidnas usually lay eggs" in two ways:

      i. (*strong* default) $\forall x(\mathsf{Echidna}(x) \wedge \neg\mathbf{B}\neg\mathsf{lay\_eggs}(x) \supset \mathsf{lay\_eggs}(x))$

      ii. (*weak* default) $\forall x(\mathbf{B}\mathsf{Echidna}(x) \wedge \neg\mathbf{B}\neg\mathsf{lay\_eggs}(x) \supset \mathsf{lay\_eggs}(x))$

   Using only the facts introduced thus far (you may disregard the other defaults), propositionalise and show that the strong and weak defaults lead to different conclusions. Does either correspond to the solutions you obtained using default logic or circumscription?

2. **[30 Marks]** (Probability)
Consider the following example:

> Headaches and blurred vision may be the result of sitting too close to a monitor. Headaches may also be caused by bad posture. Headache and blurred vision may cause nausea. Headaches may also lead to blurred vision.

(a) Represent the causal links in a belief network. Let H stand for "headache", B for "blurred vision", S for "sitting too close to a monitor", P for "bad posture", and N for "nausea".

(b) Write the formula for the full joint probability distribution over all 5 variables.

(c) Give an example of an independence assumption that is implicit in this network.

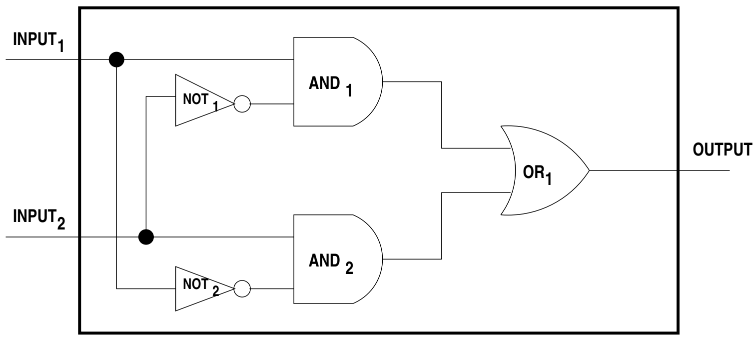
(d) Suppose the following probabilities are given:

$$\begin{array}{ll}
P(H \mid S,\ P) = 0.8 & P(H \mid \overline{S},\ P) = 0.4 \\
P(H \mid S,\ \overline{P}) = 0.6 & P(H \mid \overline{S},\ \overline{P}) = 0.02 \\
P(B \mid S,\ H) = 0.4 & P(B \mid \overline{S},\ H) = 0.3 \\
P(B \mid S,\ \overline{H}) = 0.2 & P(B \mid \overline{S},\ \overline{H}) = 0.01 \\
P(S) = 0.1 & \\
P(P) = 0.2 & \\
P(N \mid H,\ B) = 0.9 & P(N \mid \overline{H},\ B) = 0.3 \\
P(N \mid H,\ \overline{B}) = 0.5 & P(N \mid \overline{H},\ \overline{B}) = 0.7
\end{array}$$

Furthermore, assume that some patient is suffering from headaches but not from nausea. Calculate joint probabilities for the 8 remaining possibilities (that is, according to whether S, B, P are true or false).

(e) According to the numbers above, the *a priori* probability that the patient suffers from bad posture is 0.2. Given that the patient is suffering from headaches but not from nausea, are we now more or less inclined to believe the hypothesis? Explain.

3. **[30 Marks]** (Diagnosis)
Consider the following binary circuit for a half-adder.



(a) Write sentences describing this circuit: its components, connectivity, and normal behaviour.

(b) Write a sentence for a fault model saying that a faulty AND has its output the same as the inverse of its first input (the higher one in the diagram).

(c) Assuming the above fault model and that the output is 1 given inputs 0 and 0, generate the abductive explanations for this behaviour.

(d) Generate the consistency-based diagnoses for this circuit under the same conditions.

(e) Compare the two accounts and explain why they are different.

4. [**50 Marks**] (Prime Implicates)

Definition:
A clause c is a *prime implicate* of a KB if and only if

i. KB $\models c$; and,
ii. For no $c' \subset c$ does KB $\models c'$.

In the past decade there has been interesting progress in regard to the difficulty of determining the satisfiability of randomly generated clauses. Mitchell, Selman and Levesque (AAAI92 — `ftp://ftp.research.att.com/dist/ai/aaai92hsat.ps`) show that the Davis-Putnam satisfiability procedure has an easy-hard-easy nature: with a small set of clauses, Davis-Putnam terminates quickly; with about $4.2n$ clauses (where n is the number of atoms) Davis-Putnam often runs a long time; with a larger set of clauses, again Davis-Putnam usually terminates quickly. In this question you are to verify whether the generation of prime implicates exhibits an easy-hard-easy pattern.

(a) Implement a program to compile a set of clauses into prime implicate form. An algorithm (Tison 67) for compiling clauses into prime implicate form is as follows although you should feel free to implement an alternate algorithm by consulting the literature.

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Let F be the set of clauses KB = {c_1, ..., c_n}
//F represents the set of implicates of KB.
//At completion F contains all prime implicates of KB.

For each propositional variable p such that p ∈ c_i and ¬p ∈ c_j do
  For every pair of clauses c_i, c_j ∈ F do
    Add to F the resolvent of c_i, c_j on p if it is defined and not tautologous
    Delete from F every clause d such that d' ∈ F and d' ⊂ d.
```

For example:

If $\Gamma = \{a \vee b, \neg b \vee c, \neg c \vee d\}$, we end up with $F = \{a \vee b,\ \neg b \vee c,\ \neg c \vee d,\ a \vee c,\ \neg b \vee d,\ a \vee d\}$.

(b) Fix the number of propositional atoms at 10. Implement a program to generate random sets of clauses which are to be fed as input to your prime implicate program. In the next part, we consider firstly clauses of length 3 and then clauses of length 2. Clauses can be generated by randomly choosing 3 (respectively, 2) distinct atoms and negating each with 0.5 probability.

(c) Analyse the performance of your prime implicate generator on this input. Graph results for $10n$ clauses with $n = 1,\ 2,\ 3,\ 4,\ 5,\ 6,\ 7,\ 8$ (you should attempt at least 10 runs for each) for

i. clauses of length 3;
ii. clauses of length 2.