Computer Science 324
St. George Campus

20 September, 2008
University of Toronto

Homework Assignment #1

Due: Thursday, 16 October, 2008, by 6pm

No submissions will be accepted after 6pm Thursday, 16 October

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Silent Policy: *A silent policy will take effect 24 hours before this assignment is due. This means that no question about this assignment will be answered, whether it is asked on the newsgroup, by email, or in person.*

Handing in this Assignment

What to hand in on paper: Please use an **unsealed** envelope, attaching the cover page provided here to the front. Note that without a properly completed and **signed** cover page, your assignment will not be marked.

You must hand in the assignment in your tutorial on the due date.

Bulletin Board: Important corrections (hopefully few or none) and clarifications to the assignment will be posted on the bulletin board, linked from the course home page.

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Homework Assignment #1
Cover Sheet

Last Name:

First Name:

CDF login:

Email:

I have read, understood, and agree to the policies described in the Course Information handout, including the policy on collaboration.

Signature: ____________________

Question 1. (5 marks) Let $G_1 = \langle \Sigma_1, N_1, P_1, S_1 \rangle$ and $G_2 = \langle \Sigma_2, N_2, P_2, S_2 \rangle$ be context-free grammars. Prove that $L(G_1) \cup L(G_2)$ is a context-free language. **Hint:** build a context-free grammar for $L(G_1) \cup L(G_2)$.

Question 2. (10 marks) Given two context-free grammars, $G_1 = \langle \Sigma_1, N_1, P_1, S_1 \rangle$ and $G_2 = \langle \Sigma_2, N_2, P_2, S_2 \rangle$, it is *not* true that $L(G_1) \cap L(G_2)$ is necessarily context-free. Exhibit a choice for G_1 and G_2 such that $L(G_1) \cap L(G_2)$ is not context-free. **Hint:** we have only seen one non-context-free language in class. Choose that one as the intersection. Then you do not have to prove that it is not context-free.

Question 3. (10 marks) Consider the context-free grammar, G, for arithmetic expressions over $\{0, 1, 2, 3\}$:

$$
\begin{array}{lrcl}
\Sigma = \{0, 1, 2, 3, +, -, \times, /, (,)\} & & & \\
N = \{S, Op\} & & & \\
P = \{ & S & \longrightarrow & S \; Op \; S, \\
 & S & \longrightarrow & (\; S \;), \\
 & S & \longrightarrow & 0 \mid 1 \mid 2 \mid 3, \\
 & Op & \longrightarrow & + \mid - \mid \times \mid / \; \}
\end{array}
$$

(a) (2 marks) Exhibit all of the parse trees generated by G for the expression, $3 + 1 \times 2$.
(b) (1 marks) Is G ambiguous?
(c) (2 marks) Is $L(G)$ inherently ambiguous? Why (not)?
(d) (5 marks) Exhibit a grammar for $L(G)$ that generates only one parse tree for $3 + 1 \times 2$. The one parse tree that it generates should be the one licensed by the order of operations conventionally used for arithmetic expressions.

Question 4. (10 marks) Given a finite alphabet, $\Sigma = \{t_1, \ldots, t_k\}$, a finite-state automaton on n states can be specified as a set of edges, $E = \{\langle s_i^0, t_{j_i}, s_i^1 \rangle\}_{1 \le i \le e}$ along with a set of final states, $F \subseteq \{1, \ldots, n\}$, where e is the number of edges, 1 is the initial state, and for each $1 \le i \le e$, $1 \le s_i^0, s_i^1 \le n$ and $1 \le j_i \le k$. The s_i^0 are source states and the s_i^1 are destination states. Given such a specification, exhibit a context-free grammar, G, that generates all and only the same strings. Such a G exists because every regular language is a context-free language.

Question 5. (5 marks) Exhibit a context-free grammar for the language $\{a^n b^n c^m d^m \mid n \ge 1, m \ge 1\}$.

Question 6. (5 marks) Exhibit a context-free grammar for the language $\{a^n b^m c^m d^n \mid n \ge 1, m \ge 1\}$.

Question 7. (15 marks) Exhibit a context-free grammar for the language $\{a^n b^m c^m d^n \mid n \ge 1, m \ge 1, n + m \text{ is even}\}$.

Question 8. (10 marks) Exhibit a context-free grammar for the language $\{a^i b^n c^j \mid i, j \ge 1, i + j = n\}$.

Question 9. (15 marks) Exhibit a context-free grammar for the language $L \subset \{a, b\}^+$ in which every string has an equal number of a's and b's. **Note:** This is not the same language as $a^n b^n$, because the a's and b's in the strings of L can occur in any order.

Question 10. (15 marks) While it is not true that the intersection of two context-free languages is necessarily context-free, it is true that the intersection of a context-free language and a regular language is always context-free. Using this fact (which you do not have to prove), prove that the language $L \subset \{a, b, c\}^+$ in which every string has an equal number of a's, b's and c's is not context-free. **Hint:** see Question 2.