

# CSC 2517: Discrete Mathematical Models of Sentence Structure

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Tel: (416)978-7390
- This class will meet during A&S reading week!
- But we will not meet on Wednesday, 22<sup>nd</sup> March.

# CSC 2517: Discrete Mathematical Models of Sentence Structure

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- This is an advanced graduate seminar:
  - I will assume that you are familiar with the material of CSC 2501, although graduate seminars do not formally enforce prerequisites.
  - No programming assignments, although your final paper may involve some.
  - Classes will hopefully be more interactive than normal lectures.
  - **You** will do much of the presenting.
- If any of this doesn't sound like what you signed up for, then you probably belong in CSC 2511, Natural Language Computing, which is also being offered this term.

# CSC 2517: Discrete Mathematical Models of Sentence Structure

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- This year, presentations will be of papers chosen from among the following topics:
  - Algebraic Topology
  - Algebraic Invariance
  - Graphical/Algebraic Methods for Natural Language
  - Geometric Deep Learning

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- How I will compute your final mark for the class:
  - 30% your presentation(s) and participation in the seminar.
  - 70% a final paper, due on Friday, 28<sup>th</sup> April.
  - Paper proposals are due on Tuesday, 14<sup>th</sup> March.
  - Auditors are welcome, but they must present, just like everyone else.
  - Group papers must be approved in advance. If I approve, everyone in the group will receive the same mark.
  - Your final paper must be on the subject of mathematical linguistics, broadly construed. It needn't be on one of the presentation topics.
  - You may submit research you are conducting as part of your thesis or dissertation.
  - This is a Methods Area 1 class. In terms of length and style, think of your final papers as MOL or WoLLIC conference papers. Actually submitting to such a conference is encouraged but not required.

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- Rules for presentations:
  - Students will pick topics two weeks in advance of their presentation date.
  - I reserve the right to reject papers on the grounds that they are:
    - unsuitably difficult,
    - unsuitably bad,
    - insufficiently related to the topics of this seminar, or
    - excessively devoted to material already covered.
  - Unless invited, you may not present your own research.
  - I will also be offering pre-approved paper(s) to present.

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- More rules for presentations:
  - The papers that we will be reading are highly technical. Expect your presentations to be  $\approx 1$  hour long, or 1 hour of a 2-hour presentation that you will jointly give with another student (for longer papers and collections of related papers).
  - Your job as presenter is to *teach* the material. That not only includes the research presented in the paper, but the background material necessary to understand it.
    - Assume that no one has read the paper or understands what we have not discussed in class yet.
  - Your job as a non-presenter is to refute this assumption: read the paper, look up the background you are missing and come to class with questions.
  - Presenters must defend the work that they are presenting.
- Think of these as opportunities not to present a conference paper, but to teach a class – it's good practice.

# Combinatory Categorial Grammar

# Combinatory Categorical Grammar (CCG)

- Categorical grammar (CG) is one of the oldest grammar formalisms
- *Combinatory Categorical Grammar* now well established and computationally well founded (Steedman, 1996, 2000)
- Account of syntax; semantics; prosody and information structure; automatic parsers; generation



# Combinatory Categorical Grammar (CCG)

- CCG is a lexicalized grammar
- An elementary syntactic structure – for CCG a lexical category – is assigned to each word in a sentence

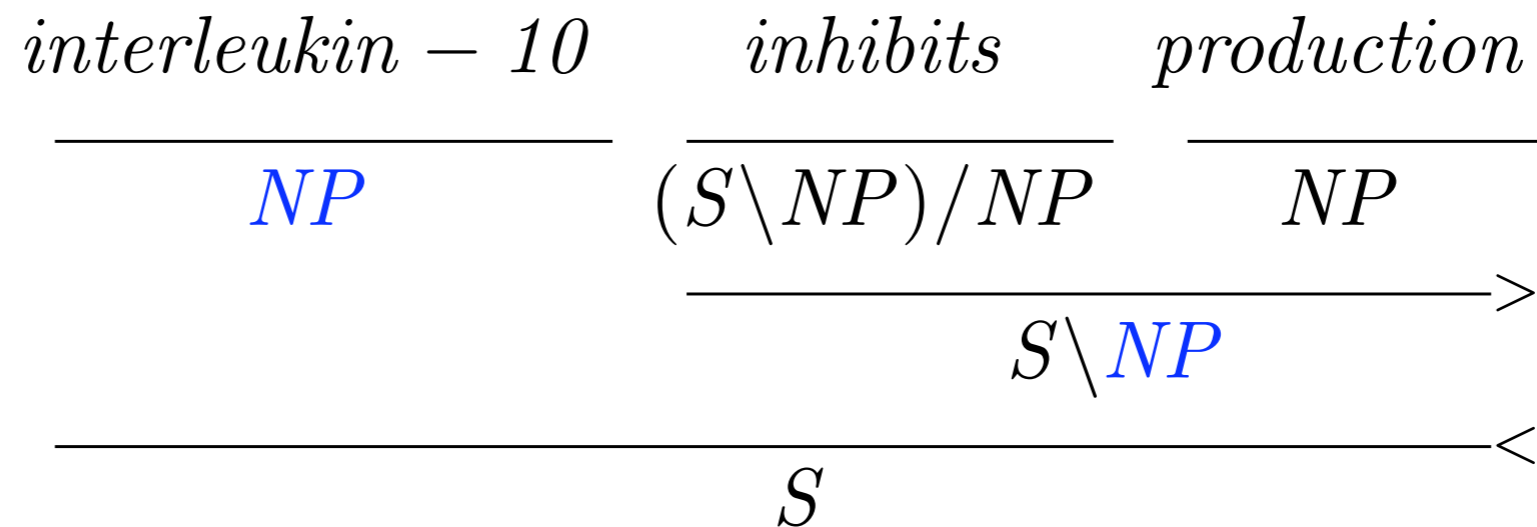
*walked*:  $S \backslash NP$  “give me an NP to my left and I return a sentence”

- A small number of rules define how categories can combine
- Rules based on the combinators from Combinatory Logic

# CCG Lexical Categories

- Atomic categories: S , N , NP , PP , ... (not many more)
- Complex categories are built recursively from atomic categories and slashes, which indicate the directions of arguments
- Complex categories encode subcategorisation information
  - intransitive verb:  $S \backslash NP$  *walked*
  - transitive verb:  $(S \backslash NP) / NP$  *respected*
  - ditransitive verb:  $((S \backslash NP) / NP) / NP$  *gave*
- Complex categories can encode modification
  - PP nominal:  $(NP \backslash NP) / NP$
  - PP verbal:  $((S \backslash NP) \backslash (S \backslash NP)) / NP$

# Simple CCG Derivation



- > forward application
- < backward application

# Function Application Schemata

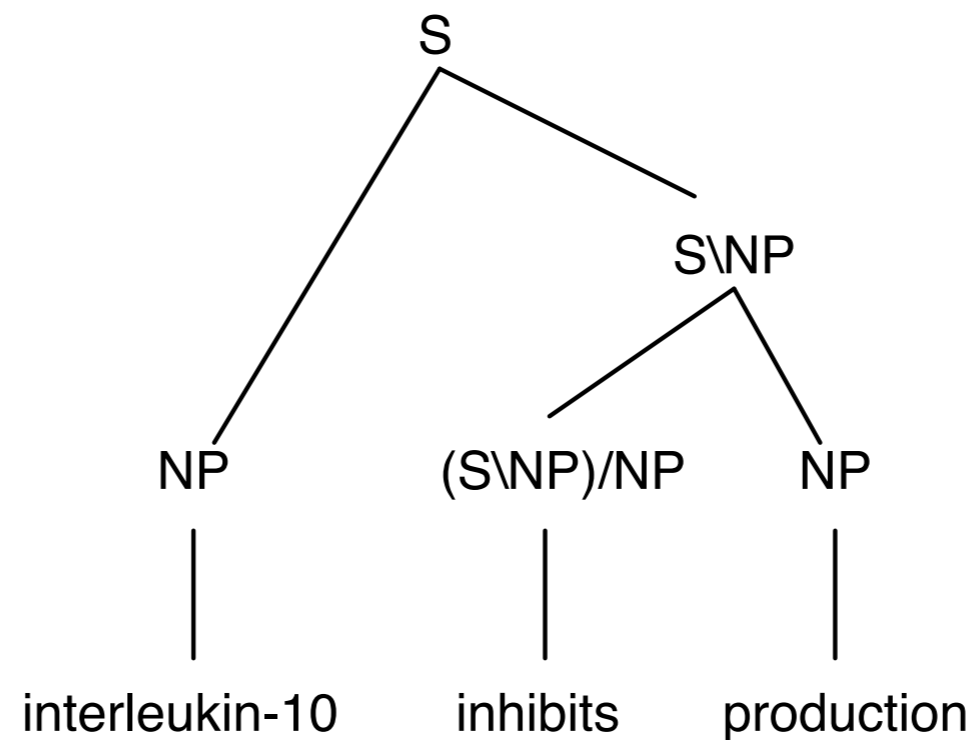
- Forward ( $>$ ) and backward ( $<$ ) application:

$$X/Y \quad Y \quad \Rightarrow \quad X \quad (>)$$

$$Y \quad X \setminus Y \quad \Rightarrow \quad X \quad (<)$$

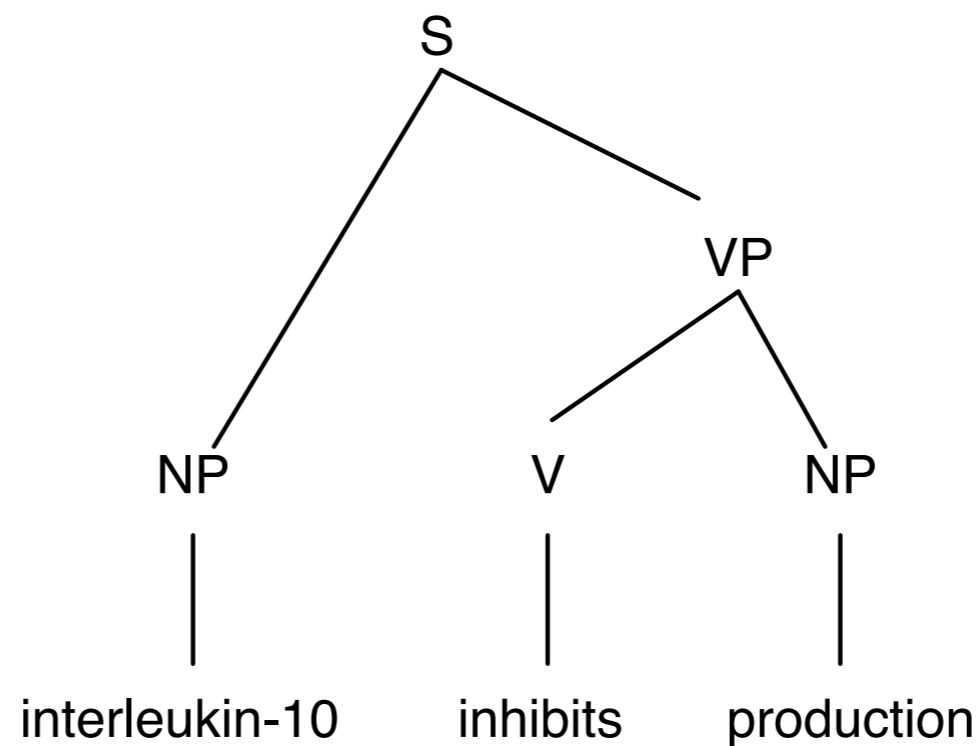
# Classical Categorical Grammar

- ‘Classical’ Categorical Grammar only has application rules
- Classical Categorical Grammar is context free



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# Extraction out of a Relative Clause

*The*     *company*     *which*     *Microsoft*     *bought*  
 $\overline{NP/N}$       $\overline{N}$       $\overline{(NP \setminus NP)/(S/NP)}$       $\overline{NP}$       $\overline{(S \setminus NP)/NP}$

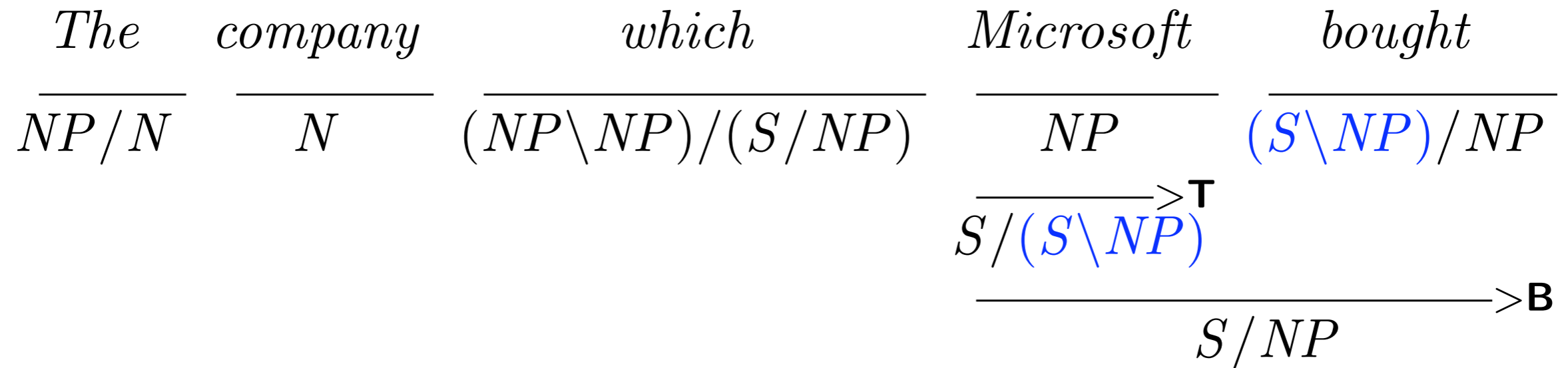
# Extraction out of a Relative Clause

*The*     *company*     *which*     *Microsoft*     *bought*  
 $\overline{NP/N}$       $\overline{N}$       $\overline{(NP \setminus NP)/(S/NP)}$       $\overline{NP}$       $\overline{(S \setminus NP)/NP}$   
 $\overline{S/(S \setminus NP)}^{>T}$

> **T**     type-raising

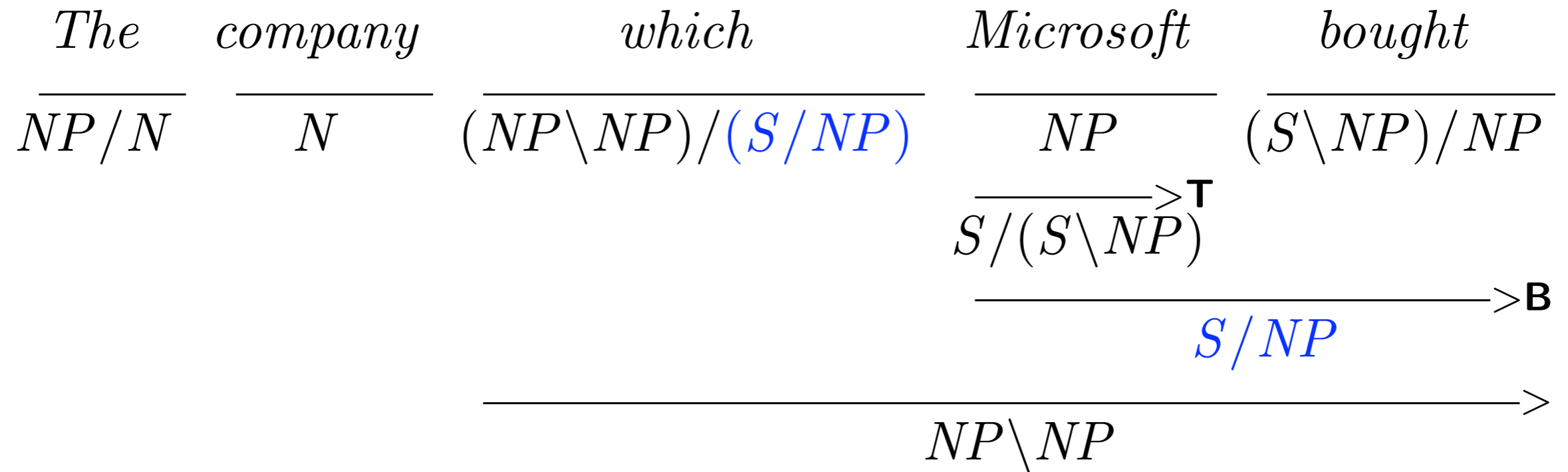


# Extraction out of a Relative Clause

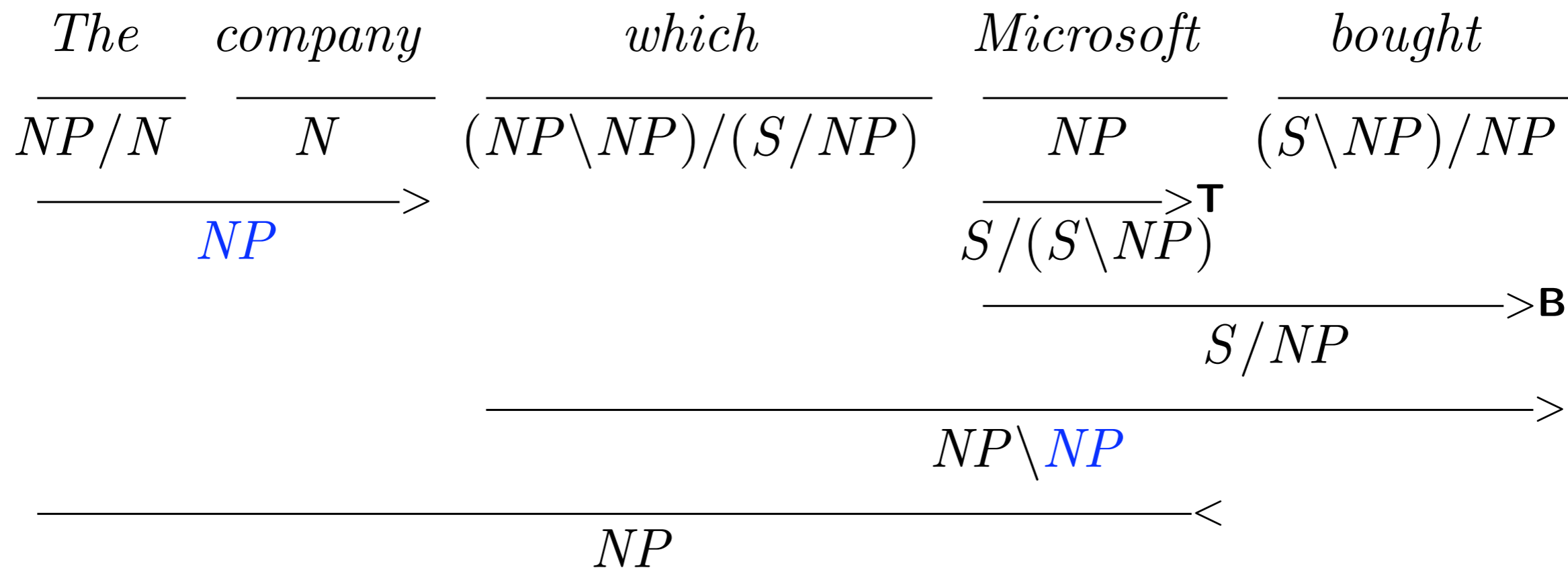


- > **T** type-raising
- > **B** forward composition

# Extraction out of a Relative Clause



# Extraction out of a Relative Clause



# Forward Composition and Type-Raising

- Forward composition ( $>_{\mathbf{B}}$ ):

$$X/Y \ Y/Z \Rightarrow X/Z \quad (>_{\mathbf{B}})$$

- Type-raising ( $\mathbf{T}$ ):

$$X \Rightarrow T/(T \setminus X) \quad (>_{\mathbf{T}})$$

$$X \Rightarrow T \setminus (T/X) \quad (<_{\mathbf{T}})$$

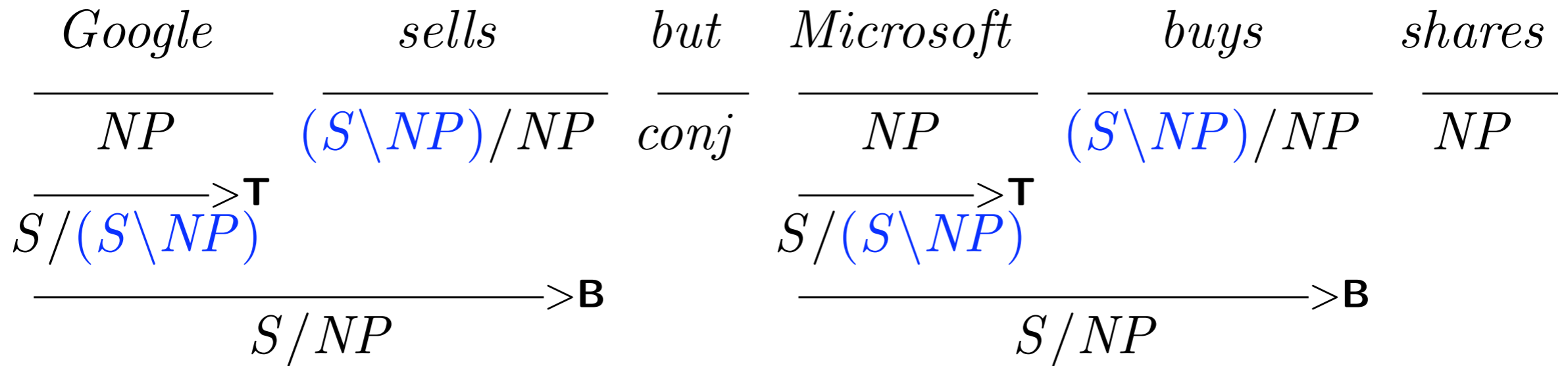
- Extra combinatory rules increase the weak generative power to mild context -sensitivity

# “Non-constituents” in CCG – Right Node Raising

<i>Google</i>	<i>sells</i>	<i>but</i>	<i>Microsoft</i>	<i>buys</i>	<i>shares</i>
$\overline{NP}$	$\overline{(S \setminus NP) / NP}$	$\overline{conj}$	$\overline{NP}$	$\overline{(S \setminus NP) / NP}$	$\overline{NP}$
$\overline{S / (S \setminus NP)} \xrightarrow{T}$			$\overline{S / (S \setminus NP)} \xrightarrow{T}$		

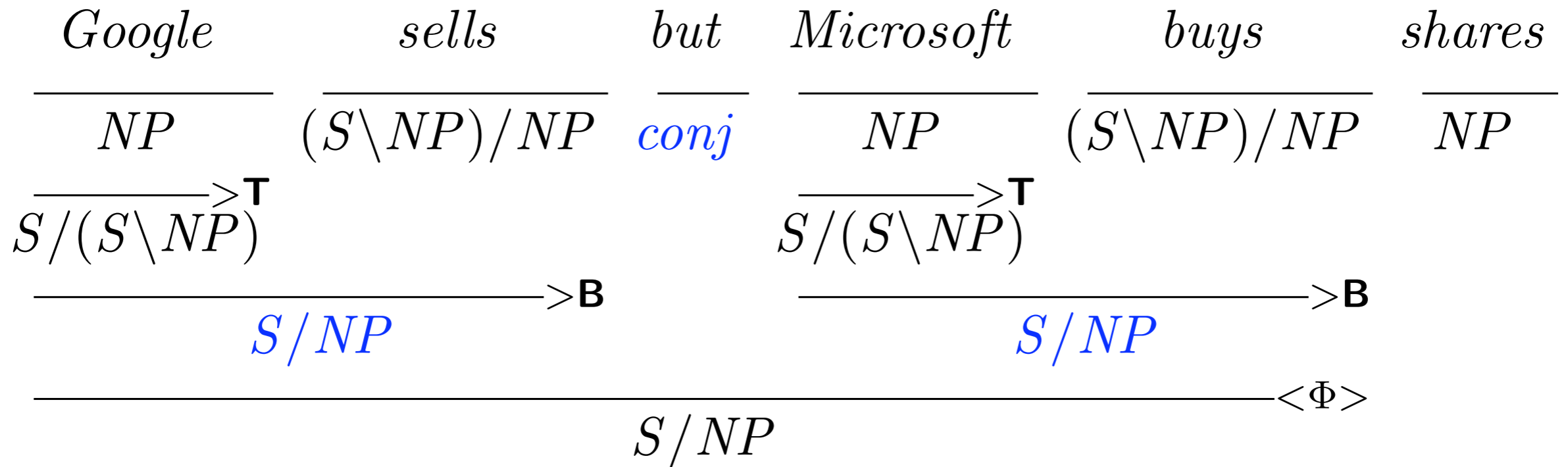
$\xrightarrow{T}$  type-raising

# “Non-constituents” in CCG – Right Node Raising

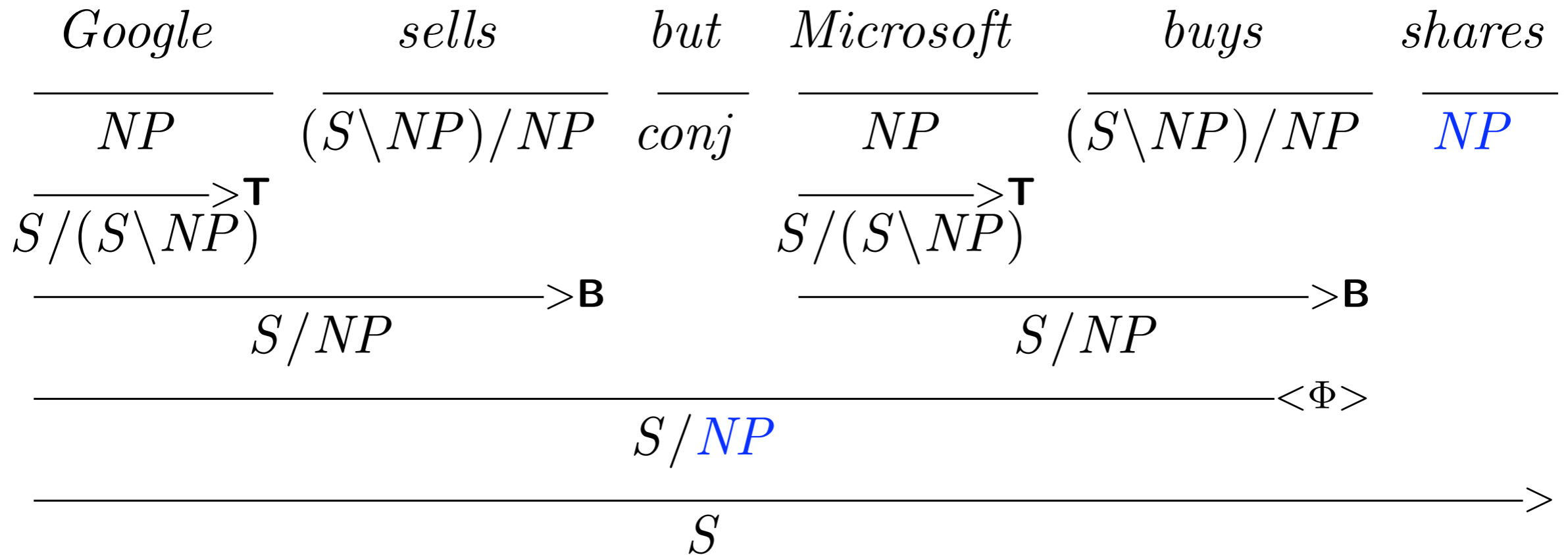


- > **T** type-raising
- > **B** forward composition

# “Non-constituents” in CCG – Right Node Raising



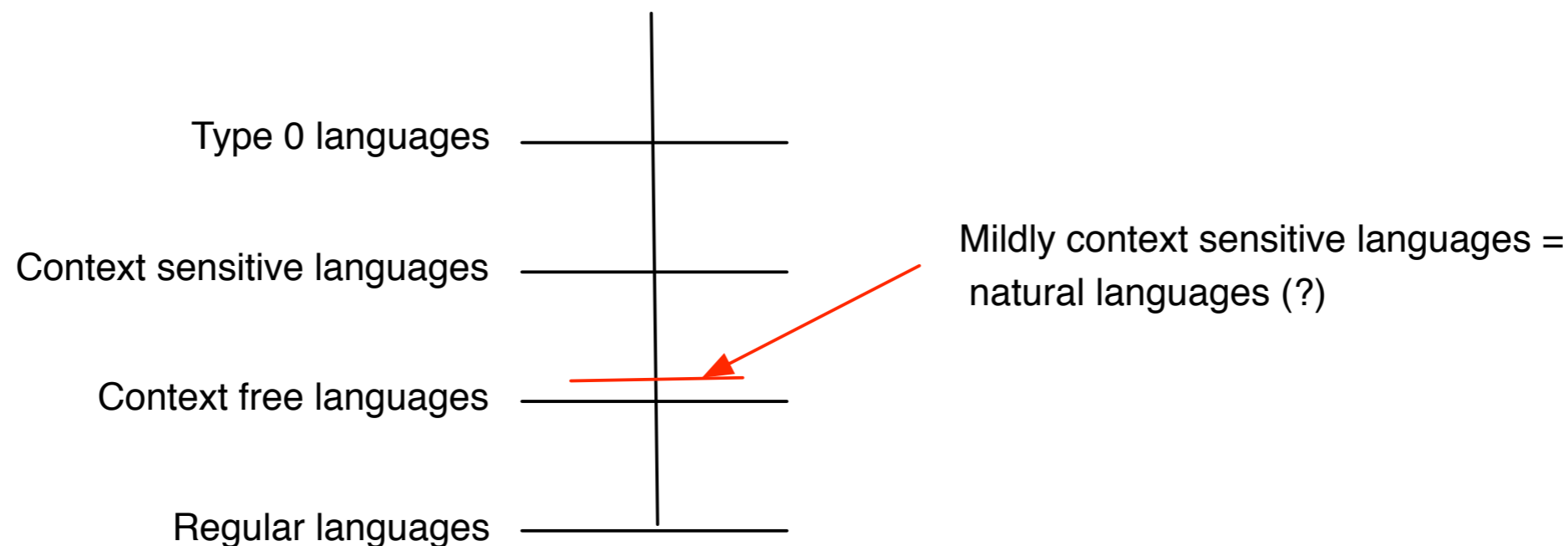
# “Non-constituents” in CCG – Right Node Raising





# Combinatory Categorical Grammar

- CCG is *mildly* context sensitive
- Natural language is provably non-context free
- Constructions in Dutch and Swiss German (Shieber, 1985) require more than context free power for their analysis
  - these have *crossing* dependencies (which CCG can handle)



# CCG Semantics

- Categories encode argument sequences
- Parallel syntactic combinator operations and lambda calculus semantic operations

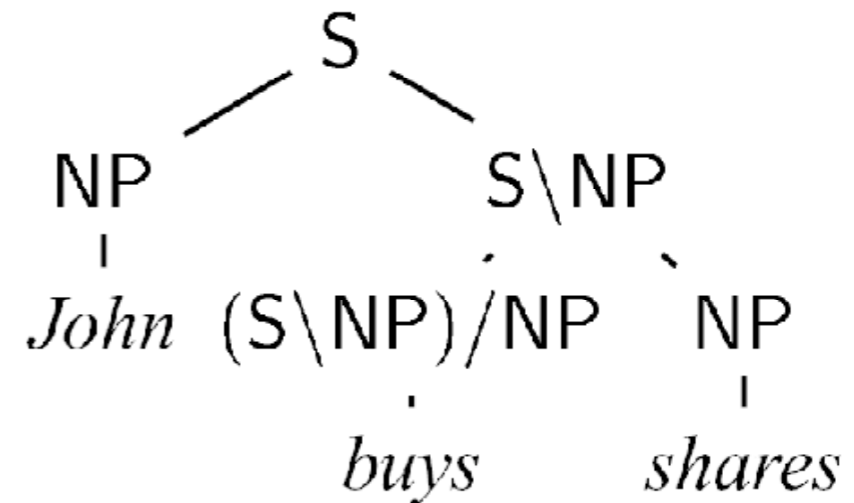
*John* ⊢ NP : *john'*

*shares* ⊢ NP : *shares'*

*buys* ⊢ (S\NP)/NP :  $\lambda x.\lambda y.buys'xy$

*sleeps* ⊢ S\NP :  $\lambda x.sleeps'x$

*well* ⊢ (S\NP)\(S\NP) :  $\lambda f.\lambda x.well'(fx)$



# CCG Semantics

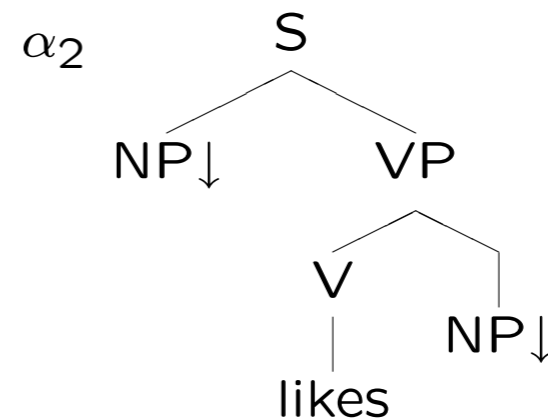
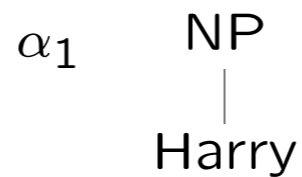
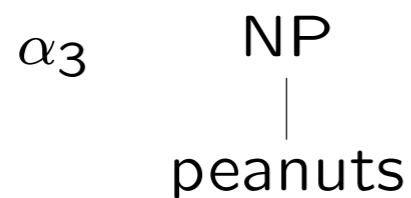
Left arg.	Right arg.	Operation	Result
$X/Y : f$	$Y : a$	Forward application	$X : f(a)$
$Y : a$	$X \backslash Y : f$	Backward application	$X : f(a)$
$X/Y : f$	$Y/Z : g$	Forward composition	$X/Z : \lambda x.f(g(x))$
$X : a$		Type raising	$T/(T \backslash X) : \lambda f.f(a)$

etc.

# Tree Adjoining Grammar

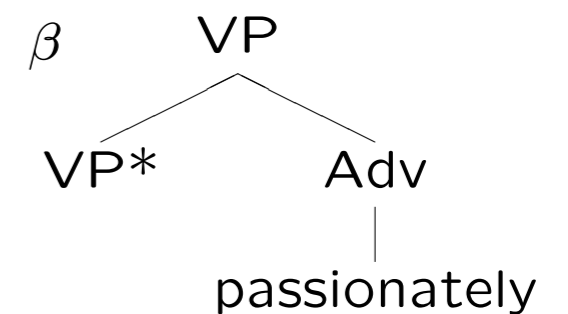
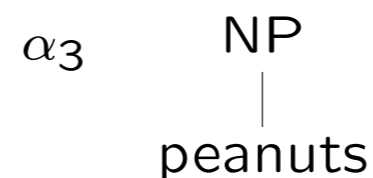
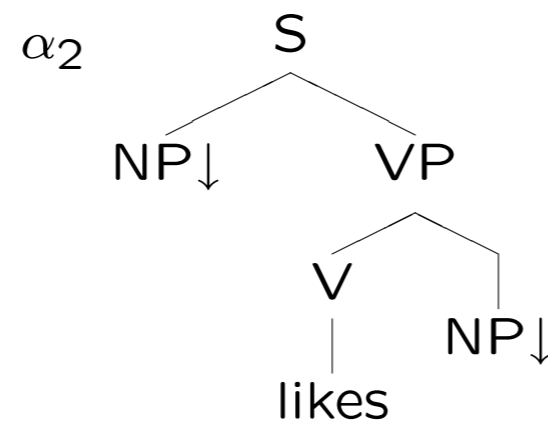
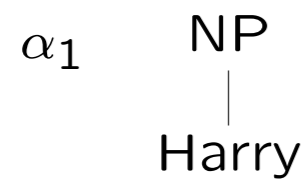
# TAG Building Blocks

- Elementary trees (of many depths)
- Substitution at ↓
- Tree *Substitution* Grammar equivalent to CFG

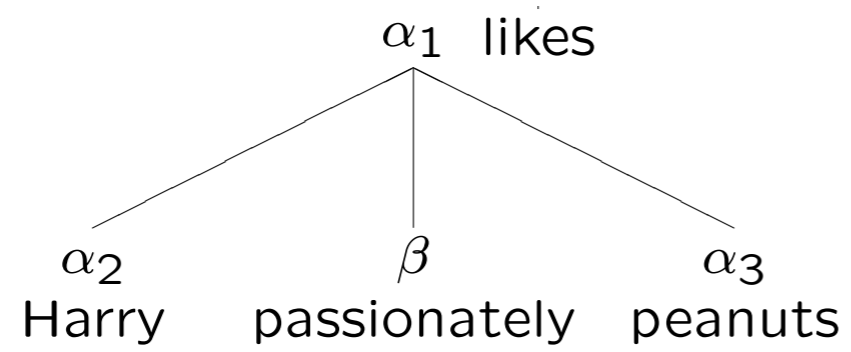


# TAG Building Blocks

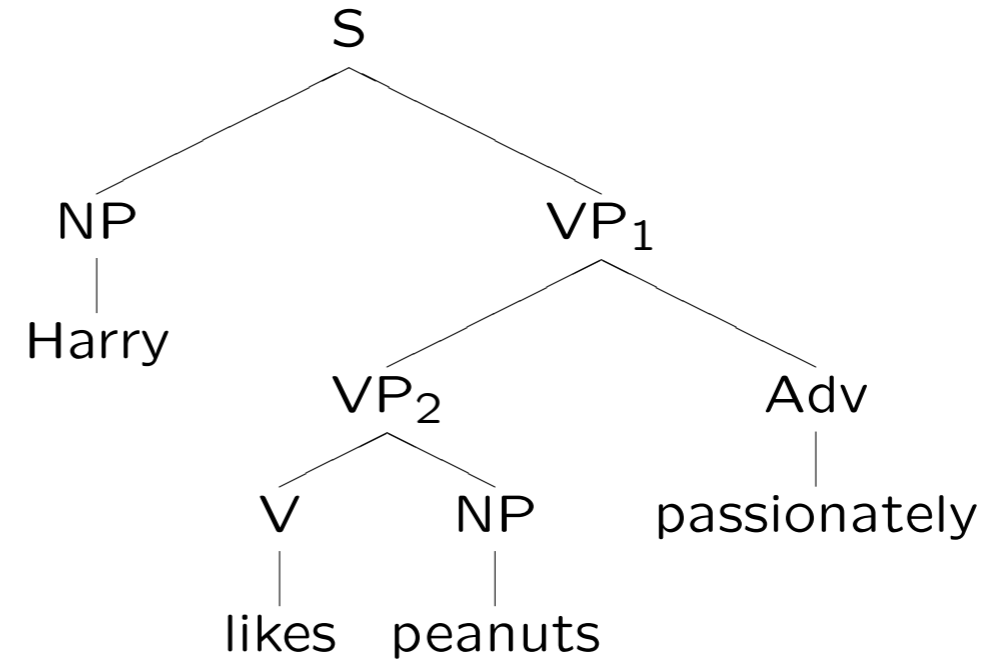
- Auxiliary trees for *adjunction*
- Adds extra power beyond CFG



## Derivation Tree



## Derived Tree



## Semantics

$Harry(x) \wedge likes(e, x, y) \wedge peanuts(y) \wedge passionately(e)$

# WHY SUPERTAG?

- If lexical items have more description associated with them, parsing is easier
  - Only useful if the supertag space is not huge
- Straightforward to compile parse from accurate supertagging
  - But impossible if there are any supertag errors
    - We can account for *some* supertag errors
    - Don't always want a full parse anyway





# WHAT IS SUPERTAGGING?

- Systematic assignment of supertags
- Supertags are:
  - Statistically selected
    - Robust
    - Tends to work
  - Linguistically motivated
    - This makes sense

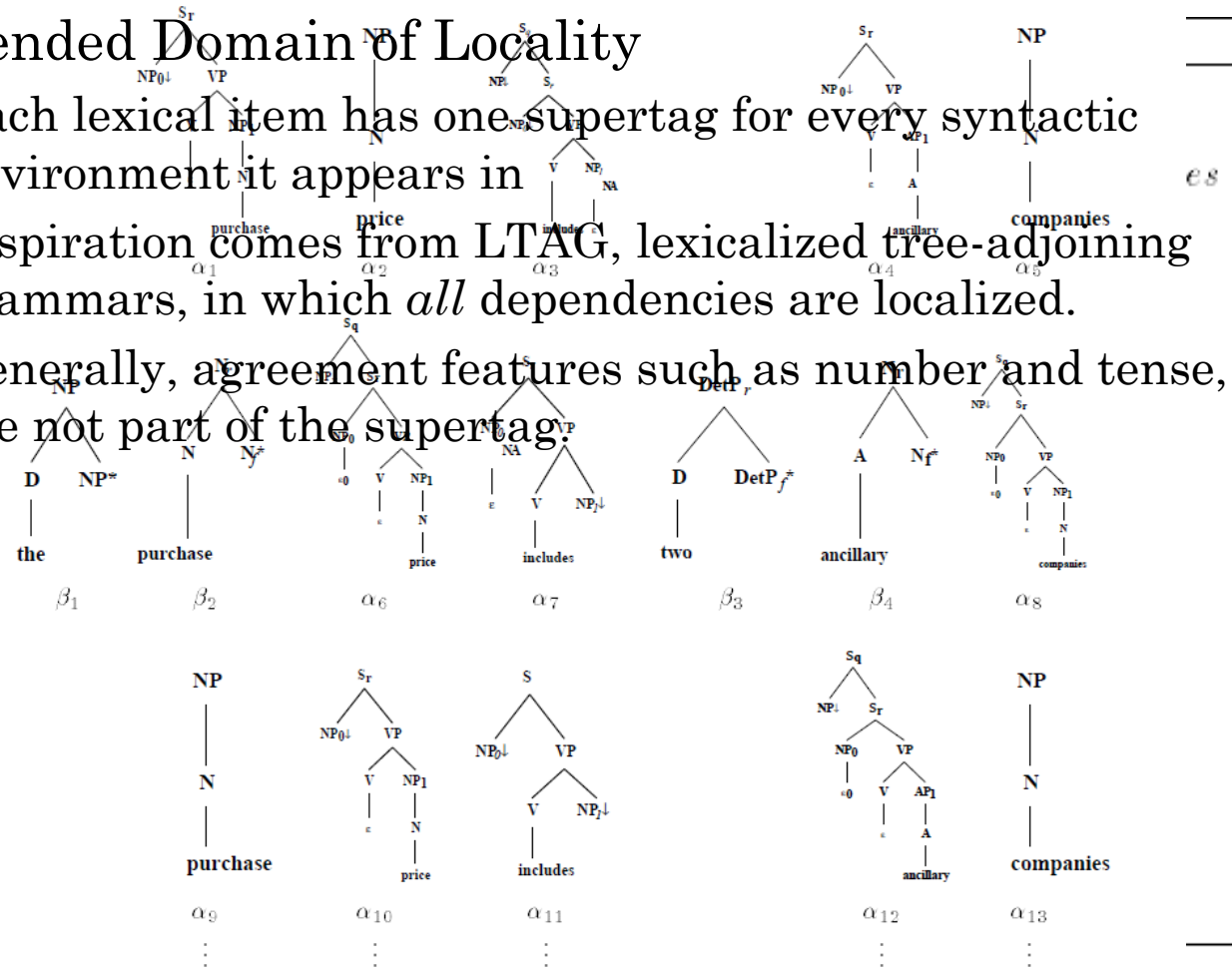


# WHAT IS SUPERTAGGING?

- Many supertags for each word

- Extended Domain of Locality

- Each lexical item has one supertag for every syntactic environment it appears in
- Inspiration comes from LTAG, lexicalized tree-adjoining grammars, in which *all* dependencies are localized.
- Generally, agreement features such as number and tense, are not part of the supertag.



# HOW TO SUPERTAG

“Alice opened her eyes and **saw**.”

- Supertags:

- Verb

- Transitive verb
- Intransitive verb
- Infinitive verb
- ...

- Noun

- Noun phrase (subject)
- Nominal predicative
- Nominal modifier
- Nominal predicative subject extraction
- ...



# HOW TO SUPERTAG

“Alice opened her eyes and saw.”

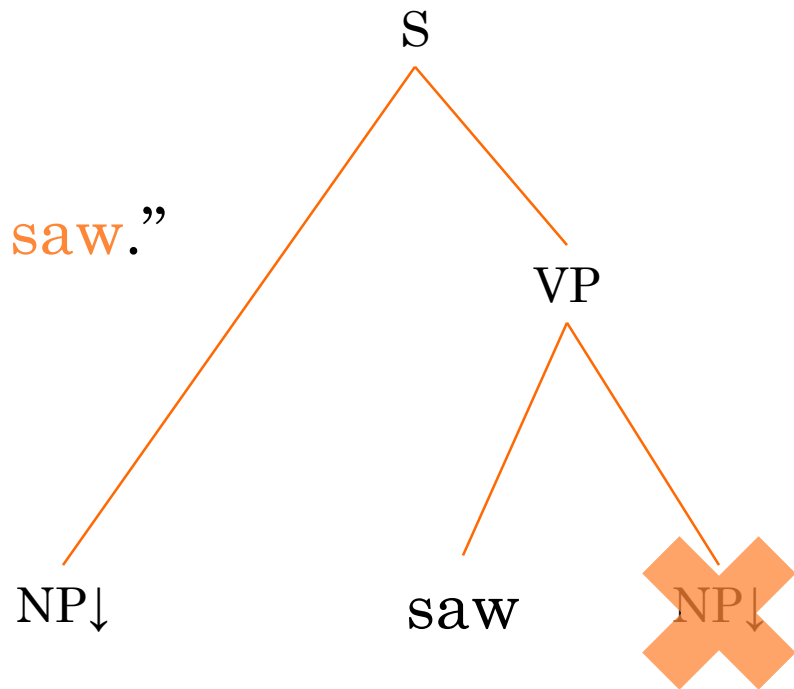
## ○ Supertags:

- Verb

- ~~Transitive verb~~
- Intransitive verb
- Infinitive verb
- ...

- Noun

- Noun phrase (subject)
- Nominal predicative
- Nominal modifier
- Nominal predicative subject extraction
- ...



# HOW TO SUPERTAG

- A supertag can be ruled out for a given word in a given input string...
  - Left and/or right context is too long/short for the input
  - If the supertag contains other terminals not found in the input

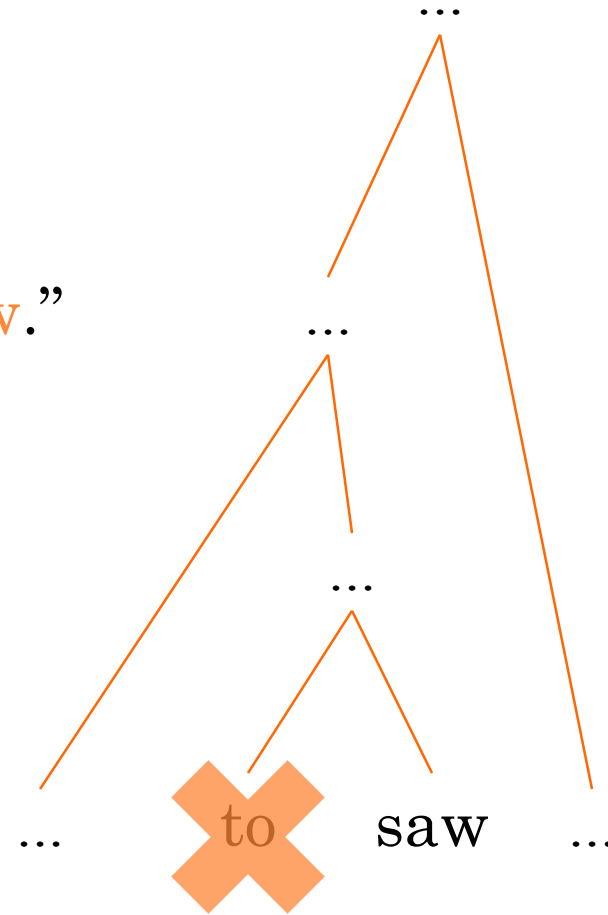


# HOW TO SUPERTAG

“Alice opened her eyes and **saw**.”

## ○ Supertags:

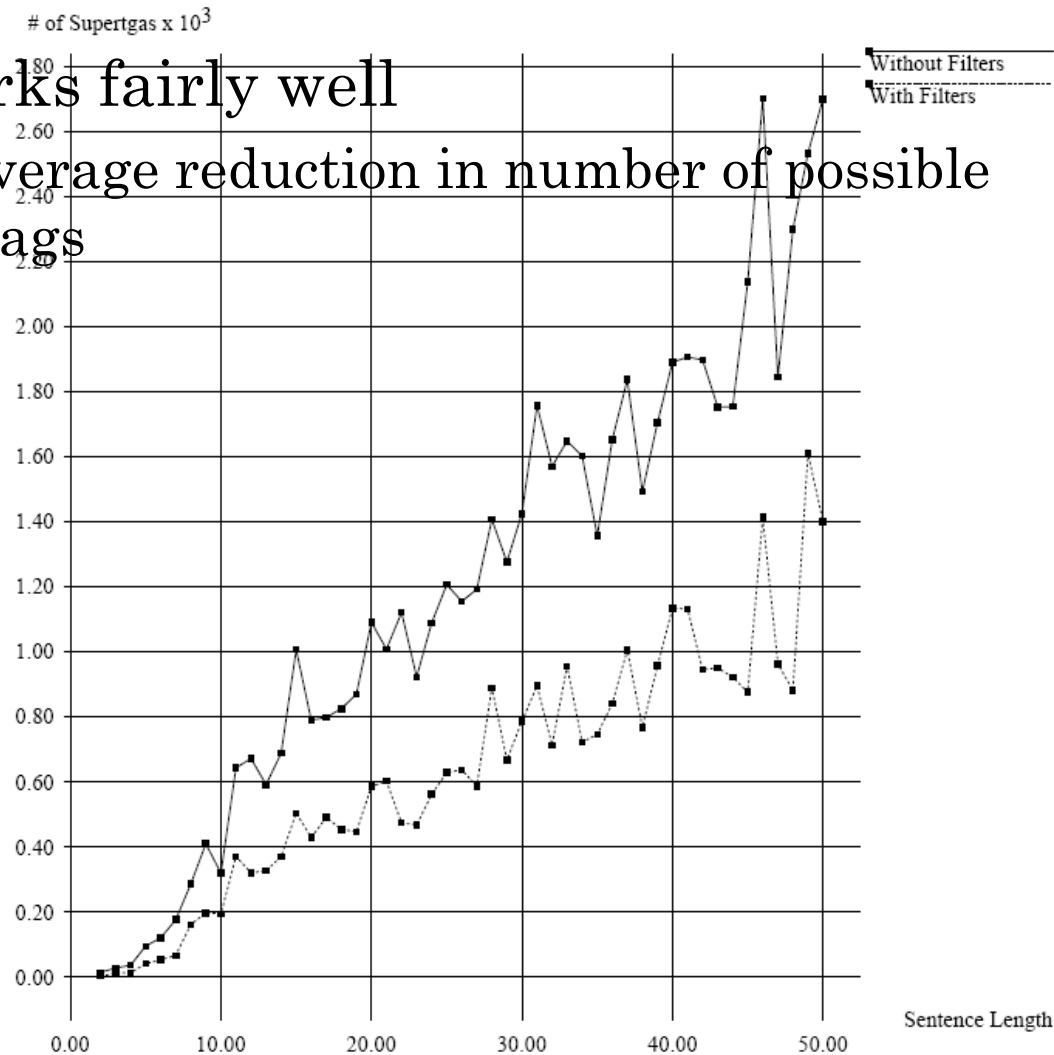
- Verb
  - ~~Transitive verb~~
  - Intransitive verb
  - ~~Infinitive verb~~
  - ...
- Noun
  - Noun phrase (subject)
  - Nominal predicative
  - Nominal modifier
  - Nominal predicative subject extraction
  - ...



# HOW TO SUPERTAG

○ This works fairly well

- 50% average reduction in number of possible supertags



# HOW TO SUPERTAG

- ...but there's more to be done
  - Good: average number of possible supertags per word reduced from 47 to 25
  - Bad: average of 25 possible supertags per word





# HOW TO SUPERTAG

- Disambiguation by unigrams?
  - Give each word its most frequent supertag after PoS tagging
    - ~75% accurate
      - Better results than one might expect given large number of possible supertags
      - Common words (determiners, etc.) usually correct
        - This helps accuracy
      - Back off to PoS for unknown words
        - Also usually correct



# HOW TO SUPERTAG

- Disambiguation by n-grams?

$$T = \operatorname{argmax}_T \Pr(T_1, T_2, \dots, T_N) * \Pr(W_1, W_2, \dots, W_N | T_1, T_2, \dots, T_N)$$

- We assume that subsequent words are independent

$$\Pr(W_1, W_2, \dots, W_N | T_1, T_2, \dots, T_N) \approx \prod_{i=1}^N \Pr(W_i | T_i)$$

- Trigrams plus Good-Turing smoothing
  - Accuracy around 90%
    - Versus 75% from unigrams
  - Contextual information more important than lexical
    - Reversal of trend for PoS tagging



# HOWEVER...

- Correctly supertagged text yields a 30X parsing speedup
  - But even one mistake can cause parsing to fail completely
    - This is rather likely
- Solution: n-best supertags?
  - When  $n=3$ , we get up to 96% accuracy...
    - Not bad at all for such a simple method
    - 425 lexical categories (PTB-CFG: ~50)
    - 12 combinatory rules (PTB-CFG: > 500,000)

