UNIVERSITY OF TORONTO
Faculty of Arts and Science

DECEMBER EXAMINATIONS 2001

CSC 238H1 F

Duration - 3 hours

Aids allowed: One 8.5×11 handwritten, non-photocopied sheet of notes (both sides)

Last Name:

First Name:

Student Number:

Rules:

1. This exam paper contains 13 pages. Legibly write your last name and student number on every page, as the pages of this exam will be separated during marking. *Unnamed pages will not be marked.*
2. There are 6 problems which you must solve, worth a total of 80 marks.
3. Write your answers directly on the exam in the space provided, or on the blank pages provided at the end. For each blank page that you use, write the number of the problem that you are solving on that page. Do not write the answer to more than one problem on the same blank page.
4. For rough work, you may use the back of any page. *These will not be marked.*
5. The instruction, "prove inductively," means that you must prove the given proposition by induction, complete induction, or structural induction. No credit will be given for (even very clever) non-inductive proofs.

Do not write below this line.

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6:

Total:

1. (15 marks)

(a) (5 marks) Write a program (either iterative or recursive) that, given an integer array of length $n \geq 1$, returns a position at which the minimum element in the array occurs. Specifically, your program should observe the following specification:

Precondition: an array of integers, `A[1...n]`, with $\texttt{n} \geq 1$.

Postcondition: the program returns an `x` such that:

1. $1 \leq \texttt{x} \leq \texttt{n}$, and
2. for all $1 \leq i \leq \texttt{n}$, $\texttt{A}[i] \geq \texttt{A}[\texttt{x}]$.

(b) (10 marks) Prove that your algorithm is correct.

2. (10 marks) State whether each of the following 10 statements is TRUE or FALSE.

1. $n^2 + n^3 \in \mathcal{O}(n^2)$
2. $n^2 + n^3 \in \Omega(n^3)$
3. $5 \cdot 2^n \in \Omega(3^n)$
4. $3^{2n} \in \Omega(8^n)$
5. $3^{2n} \in \mathcal{O}(8^n)$
6. $\log_5 n \in \Theta(\log_6 n)$
7. $\log_5 n \in \Omega(\log_5(n^2))$
8. $n^3 + n^5 \in \Theta(n^4)$
9. $\frac{1}{n} \in \mathcal{O}(\frac{1}{n^2})$
10. $\log_2 n \in \mathcal{O}(\frac{1}{n})$

3. (10 marks) Let the function $f : \mathbb{N} \to \mathbb{N}$ be defined by
$f(0) = 5$
$f(1) = 4$
$f(n) = 3f(\lfloor \frac{n}{2} \rfloor) + 2^n$ for $n \geq 2$

Use induction to prove that $f(n) \in \mathcal{O}(2^n)$. Do *not* use the general theorem about divide-and-conquer recurrences. (HINT: One way of doing this is by proving that $f(n) \leq 4 \cdot 2^n$ for sufficiently large n.)

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4. (15 marks) Consider the relational database for a library discussed in your notes, with the following predicate interpretations:

Book/3			Subscriber/3			Borrowed/3		
Call no.	Title	Author	SIN	Name	Address	SIN	Call no.	Due
1	Iliad	Homer	1	Penn	PT 276	1	1	2002/1/1
2	Odyssey	Homer	2	Hirst	PT 283A	2	1	2001/12/15
3	Odyssey	Homer	3	Reiter	PT 283D	2	2	2002/1/1
4	Oliver Twist	Dickens				3	1	2001/11/1
						3	2	2001/12/15
						3	3	2002/1/1

(a) (5 marks) Relative to the interpretation above, indicate what answer the following query will return, and briefly describe what it asks in English:

$$\exists s.(\texttt{Subscriber}(s, n, a) \wedge \forall b.\forall r.(\texttt{Book}(b, \text{Odyssey}, r) \rightarrow \exists d.\texttt{Borrowed}(s, b, d)))$$

(b) (5 marks each) For each of the following queries, write a formula that expresses it in ***any*** interpretation.

1. The names of all subscribers who have borrowed a book by Homer.
2. The call numbers of all books that have been borrowed by exactly one subscriber.

5. (10 marks) Convert the following into a PNF formula that uses only the $\wedge$ and $\vee$ connectives:

$$(\forall x.\neg T(z,x) \rightarrow S(x,y,z)) \wedge (\neg\exists y.\forall z.R(y,z,x) \rightarrow \exists x.U(x))$$

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6. (20 marks)

(a) (2 marks each) For each of the assertions below, state whether it is true or false and briefly justify your answer. ***No credit without correct justification.***

(i) If L is a regular language then every subset of L is also regular.

(ii) If L is accepted by a NFSA with k states then L is accepted by a DFSA with 2^k states.

(iii) If L and L' are regular languages then $L - L'$ (the set of strings in L but not in L') is also regular.

(iv) 1100 is in the language denoted by the regular expression $0^*1(11)^*10^*$.

(v) 01110 is in the language denoted by the regular expression $0^*1(11)^*10^*$.

(b) (5 marks each) Let L be the set of strings over $\{0, 1\}$ that contain both 01 and 10 as substrings.

(i) Draw the diagram of a DFSA that accepts L. (You need not prove that your DFSA is correct — but it should be!)

(ii) Give a regular expression that denotes L. (You need not prove that your expression is correct — but it should be!)

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