CSC 2534 — Decision Making Under Uncertainty
Assignment 1

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1. Suppose we've asked a decision maker to answer a few queries about her utility function over a small outcome space $Z = \{z_1, z_2, z_3, z_4, z_5\}$, in the form of preference queries comparing different lotteries. We've learned the following about her preferences:

   - $z_1 \succeq z_2 \succeq z_3 \succeq z_4 \succeq z_5$
   - $\langle 0.9, z_1; 0.1, z_5 \rangle \succ z_2 \succ \langle 0.8, z_1; 0.2, z_5 \rangle$
   - $\langle 0.42, z_1; 0.2, z_4; 0.38, z_5 \rangle \succ \langle 0.3, z_1; 0.6, z_4; 0.1, z_5 \rangle \succ \langle 0.38, z_1; 0.2, z_4; 0.42, z_5 \rangle$
   - $\langle 0.7, z_1; 0.3, z_5 \rangle \succ z_3 \succ \langle 0.5, z_2; 0.5, z_4 \rangle$

   (a) Assume a normalized utility function where the best outcome has utility 1 and the worst has utility 0. (i) Give a brief explanation why the best outcome must be strictly preferred to the worst. (ii) Derive (the tightest possible) upper and lower bounds on each of the five outcomes. Briefly explain why these bounds hold. (iii) Are there any other constraints on these utilities that can't be expressed as simple upper and lower bounds?

   (b) For each of the following pairs of lotteries, state whether the information elicited above is sufficient to determine which of the two is preferred to the other, and why this is (or is not) the case.

      i) $\langle 0.5, z_3; 0.5, z_4 \rangle$ and $\langle 0.5, z_2; 0.5, z_3 \rangle$
      ii) $\langle 0.3, z_1; 0.1, z_2; 0.5, z_3; 0.1, z_4 \rangle$ and z_3
      iii) $\langle 0.1, z_2; 0.6, z_3; 0.3, z_4 \rangle$ and $\langle 0.1, z_2; 0.7, z_3; 0.2, z_4 \rangle$
      iv) $\langle 0.5, z_1; 0.5, z_4 \rangle$ and $\langle 0.2, z_1; 0.6, z_3; 0.2, z_5 \rangle$

   (c) Consider the following four actions A, B, C, D, each of which induces a distribution over the outcomes (specified as lotteries):

   $$\bullet A : \langle 0.65, z_1; 0.35, z_5 \rangle \quad \bullet B : z_3 \quad \bullet C : \langle 0.3, z_1; 0.4, z_2; 0.3, z_4 \rangle \quad \bullet D : \langle 0.4, z_1; 0.6, z_3 \rangle$$

      i) What is the pairwise max regret (PMR) of action A relative to B (i.e., how much better *could* B be than A in the worst case)? What is the PMR of B relative to A? If required to choose between A and B, which decision has minimax regret (and what is its max regret)?
      ii) What is the PMR of action C relative to D? What is the PMR of D relative to C? If required to choose between C and D, which decision has minimax regret (and what is its max regret)?
      iii) Consider again the case where A and B are the only possible actions again. Is there a (single) bound query one could ask the decision maker about either action A or B so that the response to that query (no matter what it is) will allow an optimal decision to be made? Justify your answer.

2. Suppose we have a preference relation $\succeq$ over lotteries (defined over a finite set of states S) that satisfies the axioms: orderability, transitivity, continuity, substitutability, monotonicity, decomposability, and nontriviality (as defined in Lecture 2). Prove that there exists a utility function u defined over S such that for any two simple lotteries l_1 and l_2, we have $l_1 \succ l_2$ iff $EU(l_1) > EU(l_2)$. (Here EU denotes the expected utility function w.r.t. u.) To get started, consider the utility function that assigns zero to the worst state(s), one to the best state(s), and uses standard gambles to define all other utilities.

3. Let c be a choice function over some finite set X that satisifies these two properties (see Lecture 2):

   - For any $Y, Z \subseteq X$: if $y \in Y$, $Y \subseteq Z$, and $y \in c(Z)$, then $y \in c(Y)$.
   - For any $Y, Z \subseteq X$: if $Y \subseteq Z$, $y, z \in c(Y)$ and $z \in c(Z)$, then $y \in c(Z)$.

   Now let $\succeq$ be a connected, transitive preference relation over a finite set X, with induced strict preference $\succ$ and indifference $\sim$. For any $Y \subseteq X$, define choice function $c(Y, \succeq) = \{y \in Y : \nexists x \in Y \textit{ s.t. } x \succ y\}$. Prove that if choice function c satisfies the two properties above, there must exist a connected, transitive preference relation $\succeq$ such that $c = c(\cdot, \succeq)$.

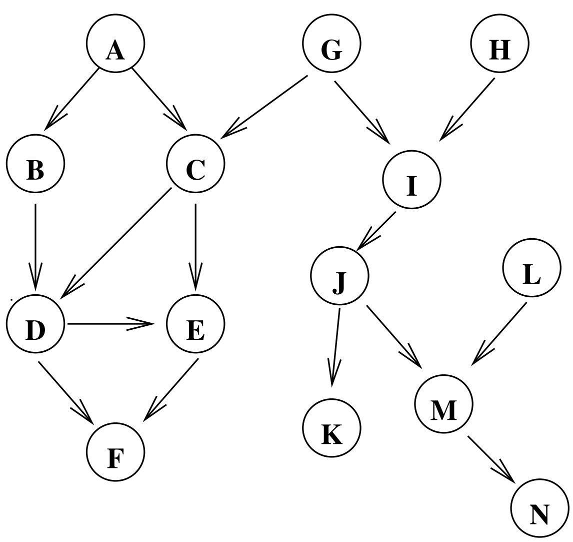
4. Suppose two investors Ali and Barb have invested \$2000 in a very early stage startup. Both anticipate the odds of success for the company to be 60%, and the chance of utter failure to be 40%. If the company is successful, they will both see a profit of \$10,000 (plus the return of their capital); if not, they will lose their investment. So they either realize a \$10,000 gain or a \$2000 loss.

   Ali and Barb have very different utility functions for the dollar amounts in this range. Given a (possibly negative) payoff of x dollars, they have the following utilities:

   - Ali: $u_a(x) = \ln(\frac{x}{500} + 6)$.
   - Barb: $u_b(x) = \exp(\frac{x}{3000} - 2)$.

   A third investor approaches each and wants to buy them out. For both Ali and Barb, derive (and explain) the minimum price they will accept from the new investor in order to sell their current stake (i.e., what is the certainty equivalent of their own investment)? Compare this to the expected monetary value of their investment. Which of them (if either) is risk-seeking, risk-averse or risk-neutral?

5. Consider the following Bayesian network, with the CPTs for a subset of the nodes listed beside. Each part of this question refers to it.



A	B	C	D	E	F	G
0.6	a 0.8	a g 0.2	b c 0.5	d c 0.1	d e 0.1	0.9
	$\bar{a}$ 0.4	a $\bar{g}$ 0.3	b $\bar{c}$ 0.5	d $\bar{c}$ 0.2	d $\bar{e}$ 0.7	
		$\bar{a}$ g 0.7	$\bar{b}$ c 0.7	$\bar{d}$ c 0.6	$\bar{d}$ e 0.2	
		$\overline{a\ g}$ 1.0	$\overline{b\ c}$ 0.0	$\overline{d\ c}$ 0.2	$\overline{d\ e}$ 0.4	

   (a) With respect to this network (ignoring the CPTs), which of the following statements of independence are true, which are false? (Give a brief justification for each you conclusion for reach.)

      i. H and G are independent
      ii. H and J are independent
      iii. H and L are independent

iv. H and L are independent given N
v. E and H are independent
vi. E and H are independent given I
vii. E and H are independent given I and C
viii. F and G are independent
ix. F and G are independent given C
x. F and G are independent given C and D

(b) Restrict your attention to the subnetwork involving nodes A through G. Using the numbers in the given CPTs, compute:
i. $\Pr(c)$ ii. $\Pr(b|a)$; iii. $\Pr(d|\overline{c}, a)$; iv. $\Pr(a|\overline{c}, d)$; v. $\Pr(a|\overline{c}, d, \overline{e})$.
Describe (very briefly) how you computed each value (e.g., what independence assumptions you used, if you did a "forward sweep" (i.e., simply applied the chain rule and independence assumptions, if you passed messages, if you implemented VE, etc.).